Name: ______________________________

Entry number: ______________________________

There are 2 questions for a total of 10 points.

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1. (5 points) Consider the following predicates:

   1. $D(x)$: x is a dog.
   2. $C(x)$: x is a cat.
   3. $M(x)$: x is a mouse.
   4. $B(x)$: x barks at night.
   5. $H(x, y)$: x has y.
   6. $L(x)$: x is a light sleeper.

   Express each of the statements using quantifiers and the predicates given above. Use the domain as the set of all living creatures.

	Statement	**Quantified expression**
S_1	All dogs bark at night.	$\forall x\ [D(x) \to B(x)]$
S_2	Anyone who has any cats will not have any mice.	$\forall x \forall y\ [(H(x, y) \wedge C(y)) \to \neg(\exists z\ (H(x, z) \wedge M(z)))]$
S_3	Light sleepers do not have anything which barks at night.	$\forall x\ [L(x) \to \neg(\exists y\ (H(x, y) \wedge B(y)))]$
S_4	John has either a cat or a dog.	$\exists x\ [H(John, x) \wedge (C(x) \vee D(x))]$
S_5	If John is a light sleeper, then John does not have any mice.	$L(John) \to \neg(\exists x\ (H(John, x) \wedge M(x)))$

2. (5 points) Consider the quantified expressions $S_1, ..., S_5$ obtained in the previous problem. Use the expressions obtained in the previous problem to replace $S_1, ..., S_5$ below and then determine whether it makes a valid argument form. Explain your answer. (*If your answer is "yes", then you need to show all steps while using rules of inference*)

$$\begin{array}{l} S_1 \\ S_2 \\ S_3 \\ S_4 \\ \hline \therefore S_5 \end{array}$$

Solution: We obtain the following argument form by replacing the $S_1, ..., S_5$ above.

$$\begin{array}{l} \forall x\ [D(x) \rightarrow B(x)] \\ \forall x \forall y\ [(H(x,y) \wedge C(y)) \rightarrow \neg(\exists z\ (H(x,z) \wedge M(z)))] \\ \forall x\ [L(x) \rightarrow \neg(\exists y\ (H(x,y) \wedge B(y)))] \\ \exists x\ [H(John,x) \wedge (C(x) \vee D(x))] \\ \hline \therefore L(John) \rightarrow \neg(\exists x\ (H(John,x) \wedge M(x))) \end{array}$$

We will show that the above argument form is valid using rules of inference:

1. $\forall x\ [D(x) \rightarrow B(x)]$ (Premise)
2. $\forall x \forall y\ [(H(x,y) \wedge C(y)) \rightarrow \neg(\exists z\ (H(x,z) \wedge M(z)))]$ (Premise)
3. $\forall x\ [L(x) \rightarrow \neg(\exists y\ (H(x,y) \wedge B(y)))]$ (Premise)
4. $\exists x\ [H(John,x) \wedge (C(x) \vee D(x))]$ (Premise)
5. $H(John,a) \wedge (C(a) \vee D(a))$ for a particular element a in domain (Existential instantiation using (4))
6. $H(John,a)$ (Simplification using (5))
7. $C(a) \vee D(a)$ (Simplification using (5))
8. $D(a) \rightarrow B(a)$ (Universal instantiation using (1))
9. $\neg D(a) \vee B(a)$ (From (8) using $p \rightarrow q \equiv \neg p \vee q$)
10. $(\neg H(John,a) \vee \neg C(a)) \rightarrow \neg\exists z\ (H(John,z) \wedge M(z))$ (Universal instantiation using (2))
11. $\forall z\ [\neg H(John,a) \vee \neg C(a) \vee \neg H(John,z) \vee \neg M(z)]$ (From (10) using DeMorgan's law for quantifiers and $p \rightarrow q \equiv \neg p \vee q$)
12. $\neg H(John,a) \vee \neg C(a) \vee \neg H(John,b) \vee \neg M(b)$ for an arbitrary b in domain (Universal instantiation using (11))
13. $\forall x \forall y\ [\neg L(x) \vee \neg H(x,y) \vee \neg B(y)]$ (From (3) using DeMorgan's law for quantifiers and $p \rightarrow q \equiv \neg p \vee q$)
14. $\neg L(John) \vee \neg H(John,a) \vee \neg B(a)$ (Universal instantiation using (13))
15. $C(a) \vee B(a)$ (Resolvent of (7) and (9))
16. $\neg C(a) \vee \neg H(John,b) \vee \neg M(b)$ (Resolvent of (6) and (12))
17. $\neg L(John) \vee \neg B(a)$ (Resolvent of (6) and (14))

18. $B(a) \vee \neg H(John, b) \vee \neg M(b)$ (Resolvent of (15) and (16))
19. $\neg L(John) \vee \neg H(John, b) \vee \neg M(b)$ (Resolvent of (17) and (18))
20. $\forall x\ [\neg L(John) \vee \neg H(John, x) \vee \neg M(x)]$ (Universal generalization using (19))
21. $L(John) \rightarrow \neg(\exists x\ (H(John, x) \wedge M(x)))$ (Using Demorgan's law for quantifiers and $p \rightarrow q \equiv \neg p \vee q$)