CSL202: Discrete Mathematical Structures (Semester-I-2018-19) Quiz-01

Name: _____

Entry number: _____

There are 3 questions for a total of 10 points.

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1. (a) (1 point) State true or false: The following compound proposition is a tautology.

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$$

(a) **True**

(b) (2 ½ points) Give reason for your answer to part (a).

Solution: We will show that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology using a truth table below.

p	q	r	$(p \vee q)$	$(\neg p \vee r)$	$(p \vee q) \wedge (\neg p \vee r)$	$(q \vee r)$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	F	T	T	T	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

2. (3 points) Let $C(p, q, r)$ denote a compound proposition involving simple propositions p, q, and r. Give a compound proposition $C(p, q, r)$ the truth table of which matches the one given below. (*Note that there may be multiple correct answers for this question*)

p	**q**	**r**	$\mathbf{C(p, q, r)}$
T	T	T	T
T	T	F	F
T	F	T	F
F	T	T	F
T	F	F	T
F	T	F	F
F	F	T	T
F	F	F	T

2. $C(p, q, r) \equiv (p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \vee \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$

You might have found a more simplified expression for this problem. The purpose of this exercise was to convey that any boolean function can be written as a compound propositions involving $\neg, \vee, \wedge$. There is a standard method for doing this. Consider all table entries that have T in the last column. For each such table entry, create a conjunction of variables or negations depending on whether the table entry is T or F. Finally, take a disjunction of all such conjunctions. Try

proving that the compound proposition created using this way will match the given truth table. (Furthermore, note that since $\vee$ can be written using $\neg$ and $\wedge$. We can write any boolean function using just $\neg$ and $\wedge$.)

3. (a) (1 point) State true or false: The following two compound propositions and are logically equivalent:
   - $(p \to q) \to (r \to s)$
   - $(p \to r) \to (q \to s)$

   (a) **False**

   (b) (2 ½ points) Give reason for your answer to part (a).

   Solution: Consider the following truth values to the propositional variables p, q, r, s:
   - $p = F, q = F, r = T, s = F$.
   For these truth values, $(p \to q) \to (r \to s)$ evaluates to F whereas $(p \to r) \to (q \to s)$ evaluates to T. Hence the compound propositions are not logically equivalent.