COL106: Data Structures and Algorithms

Ragesh Jaiswal, IITD

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- URGENT: Register on gradescope.
 - Use course code 9Z547M to add COL106.
 - Use your IIT Delhi email address.
 - Do this before the lecture tomorrow (Fri).
- Quiz 1 and 2 in the lecture tomorrow (Fri).

- <u>Data Structure</u>: Systematic way of organising and accessing data.
- Algorithm: A step-by-step procedure for performing some task.

- How do we describe an algorithm?
 - Using a pseudocode.
- What are the desirable features of an algorithm?
 - It should be correct.
 - We use proof of correctness to argue correctness.
 - It should run fast.
 - We do an asymptotic worst-case analysis noting the running time in Big-(O, Ω, Θ) notation and use it to compare algorithms.

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Problem

Given an integer array A with n elements output another array B such that for all i, $B[i] = \sum_{j=1}^{i} A[j]$. (That is find cumulative sum of elements in A.)

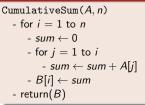
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Given an integer array A with n elements output another array B such that for all i, $B[i] = \sum_{j=1}^{i} A[j]$. (That is find cumulative sum of elements in A.)

Algorithm

CumulativeSum(A, n)	
- for $i = 1$ to n	3n operations
- $sum \leftarrow 0$	n operations
- for $j=1$ to i	$3 \cdot (1 + 2 + 3 + + n)$ operations
- $sum \leftarrow sum + A[j]$	$2 \cdot (1 + 2 + 3 + + n)$ operations
- $B[i] \leftarrow sum$	n operations
- return(<i>B</i>)	1 operation (assuming that only reference to the array is returned)
	Total : $\frac{1}{2} \cdot (5n^2 + 15n + 2)$ operations

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Algorithm	
CumulativeSum(A, n)	
- for $i = 1$ to n	2n operations
- $sum \leftarrow 0$	n operations
- for $j = 1$ to i	$2 \cdot (1 + 2 + 3 + + n)$ operations
- $sum \leftarrow sum + A[j]$	$2 \cdot (1 + 2 + 3 + + n)$ operations
- $B[i] \leftarrow sum$	n operations
 return(B) 	1 operation (assuming that only reference to the array is returned)
	Total : $\frac{1}{2} \cdot (5n^2 + 15n + 2)$ operations

• So, the asymptotic worst-case running time of the above algorithm is $O(n^2)$. Note that we can also say the running time is $\Omega(n^2)$ and $\Theta(n^2)$.

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- So, the asymptotic worst-case running time of the above algorithm is $O(n^2)$. Note that we can also say the running time is $\Omega(n^2)$ and $\Theta(n^2)$.
- Can you design a better O(n) (linear-time) algorithm for this problem?

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Problem

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Algorithm	
BetterCumulativeSum (A, n)	
- $sum \leftarrow 0$	O(1)
- for $i = 1$ to n	<i>O</i> (<i>n</i>)
- $sum \leftarrow sum + A[i]$	<i>O</i> (<i>n</i>)
- $B[i] \leftarrow sum$	<i>O</i> (<i>n</i>)
- return(<i>B</i>)	O(1)
	Total: O(n)

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Sorting: Given an integer array A with n elements, sort it.

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Problem

Sorting: Given an integer array A with n elements, sort it.

Algorithm

SelectionSort(A, n)

- for i = 1 to n 1
 - $min \leftarrow \texttt{FindMin}(A, n, i)$
 - Swap(A, i, min)

$$\begin{array}{l} \operatorname{FindMin}\left(A, n, i\right) \\ min \leftarrow i \\ \text{- for } j = i + 1 \text{ to } n \\ \text{- if } \left(A[j] < A[min]\right) \ min \leftarrow j \\ \text{- return}(min) \end{array}$$

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Problem

Sorting: Given an integer array A with n elements, sort it.

Algorithm

SelectionSort(A, n)	FindMin(A, n, i)
- for $i=1$ to $n-1$	$min \leftarrow i$
- $min \leftarrow \texttt{FindMin}(A, n, i)$	- for $j = i + 1$ to n
- Swap(<i>A</i> , <i>i</i> , <i>min</i>)	- if $(A[j] < A[min])$ min $\leftarrow j$
	- return(<i>min</i>)

What is an appropriate loop-invariant for the above algorithm?

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• What is running time of the above algorithm?

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Problem

Sorting: Given an integer array A with n elements, sort it.

Algorithm

BubbleSort
$$(A, n)$$

- for $i = 1$ to $(n - 1)$
- for $j = 1$ to $(n - i)$
- if $(A[j] > A[j + 1])$ Swap $(A, j, j + 1)$

• What is an appropriate loop-invariant for the above algorithm?

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Example problem

Maintain a record of students and their scores on some test so that queries of the following nature may be answered:

- Insert: Insert a new record of a student and his/her score.
- <u>Search</u>: Find the score of a given student.
- Suppose we maintain the information in a 2-dimensional array.
 - How much time does each insert operations take?
 - How much time does each search operation take?

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- <u>Search</u>: Find the score of a given student.
- Suppose we maintain the information in a 2-dimensional array.
 - How much time does each insert operations take? O(1)
 - How much time does each search operation take? O(n)
 - So, if the majority of the operations performed are search operations, then this data structure is perhaps not the right one.

Example problem

Maintain a record of students and their scores on some test so that queries of the following nature may be answered:

- Insert: Insert a new record of a student and his/her score.
- <u>Search</u>: Find the score of a given student.
- Suppose we maintain the information in a 2-dimensional array such that the array is sorted based on the names (dictionary order).
 - How much time does each insert operations take?
 - How much time does each search operation take?

• How do Data Structures play a part in making computational tasks efficient?

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- Insert: Insert a new record of a student and his/her score.
- <u>Search</u>: Find the score of a given student.
- Suppose we maintain the information in a 2-dimensional array such that the array is sorted based on the names (dictionary order).
 - How much time does each insert operations take? O(n)
 - How much time does each search operation take? $O(\log n)$ using **Binary Search**
 - In this case, if the majority of the operations performed are insert operations, then the previous one is better.

End

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