COL351: Analysis and Design of Algorithms

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Computational Intractability: NP-complete problems

Computational Intractability Many-one reduction

- Most of the polynomial-time reductions X ≤_p Y that we have seen are of the following general nature: We give an efficient mapping from instances of X to instances of Y such that "yes" instances of X map to "yes" instances of Y and "no" instances of X map to "no" instances of Y.
- Such reductions have special name. They are called many-one reductions.

Many-one reduction

In order to show that $X \leq_p Y$ we design an efficient mapping f from the set of instances of X to set of instances of Y such that $s \in X$ iff $f(s) \in Y$.



NP-complete problems: 3D-Matching

3D-MATCHING

Given disjoint sets X, Y, and Z each of size n, and given a set T of triples (x, y, z), determine if there exist a subset of n triples in T such that each element of $X \cup Y \cup Z$ is contained in exactly one of these triples.



Figure: Let $T = \{(a, x, p), (a, y, p), (b, y, q), (c, z, r)\}$. Does there exist a 3D-Matching?

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Computational Intractability NP-complete problems: 3D-Matching

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• <u>Claim 1</u>: 3D-MATCHING \in NP.

Computational Intractability NP-complete problems: 3D-Matching

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- Claim 1: 3D-MATCHING \in NP.
- <u>Claim 2</u>: 3D-MATCHING is NP-complete.

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- Claim 1: 3D-MATCHING \in NP.
- <u>Claim 2</u>: 3D-MATCHING is NP-complete.
 - Claim 2.1: 3-SAT \leq_p 3D-MATCHING.
 - <u>Proof sketch of Claim 2.1</u>: We will show an efficient many-one reduction.

NP-complete problems: 3D-Matching



Figure: Example construction for $(x_1 \lor \bar{x}_2 \lor x_3), (\bar{x}_1 \lor x_2 \lor \bar{x}_3)$

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NP-complete problems: 3D-Matching



Figure: Example construction for $(x_1 \lor \bar{x}_2 \lor x_3), (\bar{x}_1 \lor x_2 \lor \bar{x}_3)$. k denotes the number of clauses.

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NP-complete problems: Subset-sum

SUBSET-SUM

Given natural numbers $w_1, ..., w_n$ and a target number W, determine if there is a subset S of $\{1, ..., n\}$ such that $\sum_{i \in S} w_i = W$.

• <u>Claim 1</u>: SUBSET-SUM \in NP.

NP-complete problems: Subset-sum

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- <u>Claim 1</u>: SUBSET-SUM \in NP.
- <u>Claim 2</u>: SUBSET-SUM is NP-complete.

NP-complete problems: Subset-sum

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- <u>Claim 1</u>: SUBSET-SUM \in NP.
- <u>Claim 2</u>: SUBSET-SUM is NP-complete.
 - <u>Claim 2.1</u>: 3D-MATCHING \leq_p SUBSET-SUM.
 - <u>Proof sketch</u>: We will show an efficient many-one reduction. Given an instance (X, Y, Z, T) of the 3D-MATCHING problem, we construct an instance of the SUBSET-SET problem.
 - We first construct a 3*n*-bit vector. Given a triple $t_i = (x_1, y_3, z_5)$, we construct the following vector v_i :



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	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
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- Let w_i be the value of v_i in base (|T|+1) and $W = \sum_{i=0}^{3n-1} (|T|+1)^i$.
- <u>Claim 2.1.1</u>: There is a 3D-Matching iff there is a subset $\{w_1, ..., w_{|T|}\}$ that sums to W.

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