

CSL853: Complexity Theory

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PCP Theorem and Hardness of Approximation

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Two Prover one round proof systems (2P1R)

Theorem (Weak hardness for Label Cover)

There exists a universal constant $\epsilon > 0$ and Σ_0 of constant size such that $\text{GapLabelCover}_{1,1-\epsilon}(\Sigma_0)$ is NP-hard.

Theorem (2P1R proof system characterisation of NP)

There is a constant $\epsilon > 0$ such that $\text{NP} = 2\text{P1R}_{1,1-\epsilon}(\log n)$.

Theorem (Ran Raz's Parallel repetition theorem (informal version))

Let the soundness error probability of a two prover one round proof system be $\delta < 1$. Then the error probability on k parallel repetitions is δ^{dk} , where d is a constant that depends only on the length of the answers of the original proof system.

Theorem (Strong hardness for Label Cover)

*For **all** constant $\epsilon > 0$ there exists a constant $|\Sigma|$ such that $\text{GapLabelCover}_{1,\epsilon}(\Sigma)$ is NP-hard.*

Theorem

The problem $\text{GapLabelCover}_{1, \frac{1}{(\log n)^3}}$ is not in P unless $\text{NP} \subseteq \text{DTIME}(n^{O(\log \log n)})$.

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Set cover

Theorem

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Theorem

There exists $c > 0$, such that no polynomial time $c \log n$ approximation algorithm exists for MIN-SC unless $\text{NP} \subseteq \text{DTIME}(n^{O(\log \log n)})$.

End