

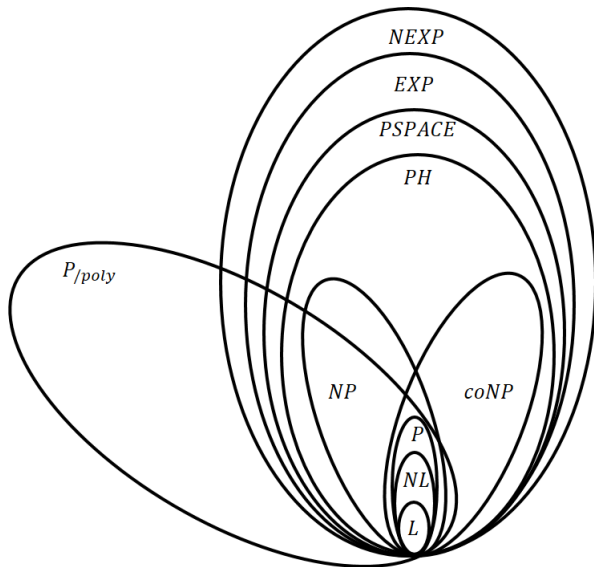
CSL853: Complexity Theory

Ragesh Jaiswal, CSE, IIT Delhi

Complexity Diagram

Complexity Diagram

Our current view of complexity classes



Boolean Circuits

Non-uniform hierarchy theorem

Theorem (Non-uniform hierarchy theorem)

For every function $T, T' : \mathbb{N} \rightarrow \mathbb{N}$ with $2^n/n > T'(n) > 10n > n$,

$$\text{SIZE}(T(n)) \subsetneq \text{SIZE}(T'(n))$$

NC and AC

Boolean Circuits

NC and AC

Definition (NC)

For every d , a language L is in NC^d if L can be decided by a family of circuits $\{C_n\}$ where C_n has *poly*(n) size and depth $O(\log^d n)$. The class NC is $\cup_{i \geq 1} NC^i$.

Definition (AC)

The class AC^i is defined similarly to NC^i except gates are allowed to have unbounded fan-in (i.e., the $\vee$ and $\wedge$ gates can be applied to more than two bits). The class $AC = \cup_{i \geq 0} AC^i$.

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- Show that $NC^i \subseteq AC^i \subseteq NC^{i+1}$.
- Show that addition is in NC^1 .
- Show that multiplication and division are in NC^1 .
- NC characterises languages with efficient *parallel algorithms*.

Boolean Circuits

NC and AC

Definition (Efficient parallel algorithm)

A computational problem has an efficient parallel algorithm if it can be solved for inputs of size n using a parallel computer with $n^{O(1)}$ processors in time $\log^{O(1)} n$.

Theorem (NC and parallel algorithms)

A language has efficient parallel algorithms if and only if it is in NC.

- An interesting result that we skipped.

Theorem (Kannan)

For every k , there is a language $L \in \Sigma_2^P$ that does not have n^k size circuits.

End