

Name: _____

Entry number: _____

There are 3 questions for a total of 30 points.

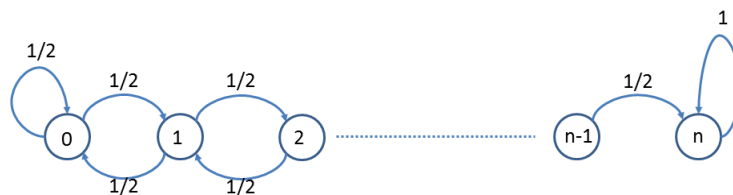
- (10) 1. n balls are thrown randomly into n bins. Show that the probability that the number of empty bins is more than $((3/4) \cdot n)$ is exponentially small in n .

- (10) 2. Let $\{0, 1, \dots, 2^n - 1\}$ be the universe of elements to be hashed into a table of size 2^m . Elements of the universe and table entries are represented as bit vectors of size n and m respectively. Consider the following family of hash functions:

$$\mathcal{H} = \{h_M \mid M \text{ is a randomly chosen } 0/1 \text{ matrix of size } m \times n \text{ and } h_M(x) = M \cdot x \pmod{2}\}$$

Show that $\mathcal{H}$ is a 2-universal family of hash function.

- (10) 3. Consider the following markov chain, that is very similar to the Markov Chain we studied when talking about the 2-SAT problem. Compute the expected time to reach the state n starting from any state i .



Extra credit question (5 points) Consider the lollipop graph consisting of a clique on $n/2$ vertices connected to a path on $n/2$ vertices. Argue that the expected cover time of a random walk starting at u is $\theta(n^3)$.

