Special Topics in Multimedia System

Indian Institute of Technology Delhi
(IITD)
New Delhi

SIL801

Recap

Theory of Quantization

- Lloyd Max Quantizer
- Entropy as a constraint
- Scalar and Vector Quantization

Transform Coding

Image Compression

Transform Coding

In general 1D transform

$$\mathbf{f} = \sum_{k=0}^{N-1} t(k)\mathbf{h}_k = \mathbf{B}\mathbf{t},$$

$$where \quad \mathbf{B} = [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{N-1}], \mathbf{t} = \begin{bmatrix} t(0) \\ t(1) \\ \vdots \\ t(N-1) \end{bmatrix}.$$

⇒

$$\mathbf{t} = \mathbf{B}^{-1}\mathbf{f} = \mathbf{A}\mathbf{f}$$

f and t form a transform pair

Image Compression

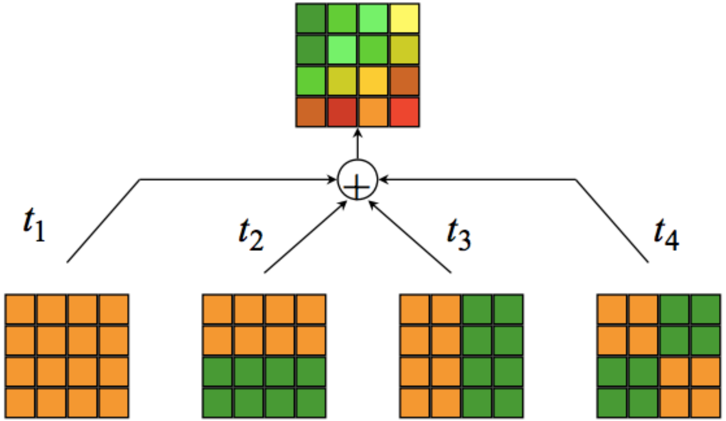
Transform Coding

In general

Represent an image (or an image block) as the linear combination of some basis images and specify the linear coefficients.

$$\mathbf{f} = \sum_{k=0}^{N-1} t(k)\mathbf{h}_k = \mathbf{B}\mathbf{t},$$

$$where \quad \mathbf{B} = [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{N-1}], \mathbf{t} = \begin{bmatrix} t(0) \\ t(1) \\ \vdots \\ t(N-1) \end{bmatrix}.$$



Source: http://eeweb.poly.edu/~yao/EL5123/lecture10_TransformCoding_JPEG.pdf

Image Compression

Transform Coding

In general

Forward transform $t = Af$
Inverse transform $f = A^T t$
Considering A orthonormal $A^{-1} = A^T$

$$\begin{bmatrix} f_0 \\ . \\ . \\ f_{N-1} \end{bmatrix} = \begin{bmatrix} a_{0,0} & . \; . & a_{N-1,0} \\ . & . \; . & . \\ . & . \; . & . \\ a_{0,N-1} & . \; . & a_{N-1,N-1} \end{bmatrix} \begin{bmatrix} t_0 \\ . \\ . \\ t_{N-1} \end{bmatrix}$$

Coefficients

$$\begin{bmatrix} f_0 \\ . \\ . \\ f_{N-1} \end{bmatrix} = \begin{bmatrix} a_{0,0} \\ . \\ . \\ a_{0,N-1} \end{bmatrix} t_0 + .. + \begin{bmatrix} a_{N-1,0} \\ . \\ . \\ a_{N-1,N-1} \end{bmatrix} t_{N-1}$$

Basis Vectors

Image Compression

Transform Coding

Discrete Cosine Transform

1D transform

$$d_{ij} = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } i = 0 \\ \sqrt{\frac{2}{N}} \cos \frac{(2j+1)i\pi}{2N} & \text{if } i > 0 \end{cases}$$

$N = 4$

$$D = \begin{bmatrix} .5 & .5 & .5 & .5 \\ .65328 & .270598 & -.270598 & -.65328 \\ .5 & -.5 & -.5 & .5 \\ .270598 & -.65328 & .65328 & -.270598 \end{bmatrix}$$

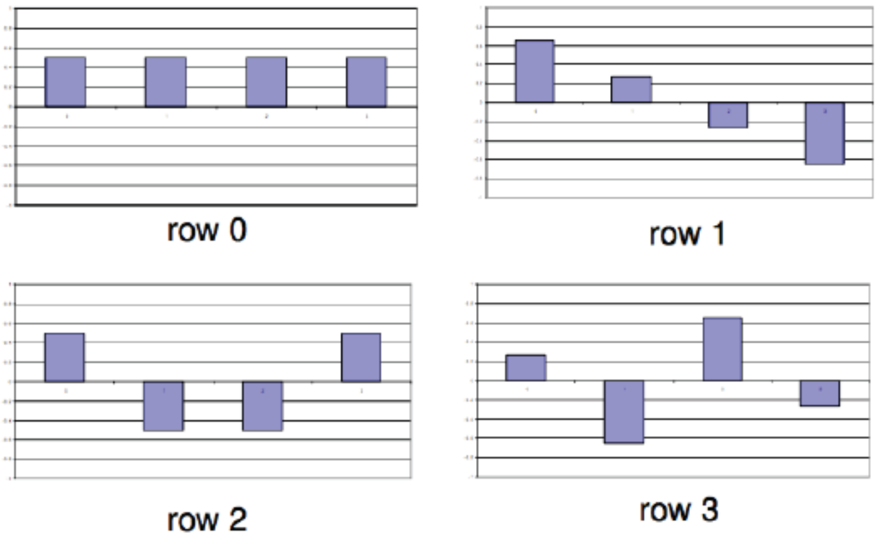


Image Compression

Transform Coding

Discrete Cosine Transform

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1D transform

$$\begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} c_0 + \begin{bmatrix} .653281 \\ .270598 \\ -.270598 \\ -.653281 \end{bmatrix} c_1 + \begin{bmatrix} .5 \\ -.5 \\ -.5 \\ .5 \end{bmatrix} c_2 + \begin{bmatrix} .270598 \\ -.653281 \\ .653281 \\ -.270598 \end{bmatrix} c_3 = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

DC coefficient (c_0)

AC coefficients (c_1, c_2, c_3)

Image Compression

Transform Coding

In general 2D transform

Input

$$\mathbf{F} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)\mathbf{H}_{k,l},$$

Inverse Transform

$$F(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)H_{k,l}(m,n)$$

Transform Coefficients

Image Compression

Transform Coding

2D transform

$$\mathbf{F} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)\mathbf{H}_{k,l},$$

$$F(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)H_{k,l}(m,n)$$

Discrete Cosine Transform

$$H_{k,l}(m,n) = \alpha(k)\alpha(l)\cos\left[\frac{(2m+1)k\pi}{2N}\right]\cos\left[\frac{(2n+1)l\pi}{2N}\right]$$

$$\text{where } \alpha(k) = \begin{cases} \sqrt{1/N} & k = 0 \\ \sqrt{2/N} & k = 1,\dots,N-1 \end{cases}$$

Image Compression

Transform Coding

2D transform

Discrete Cosine Transform

8x8 DCT Basis Images

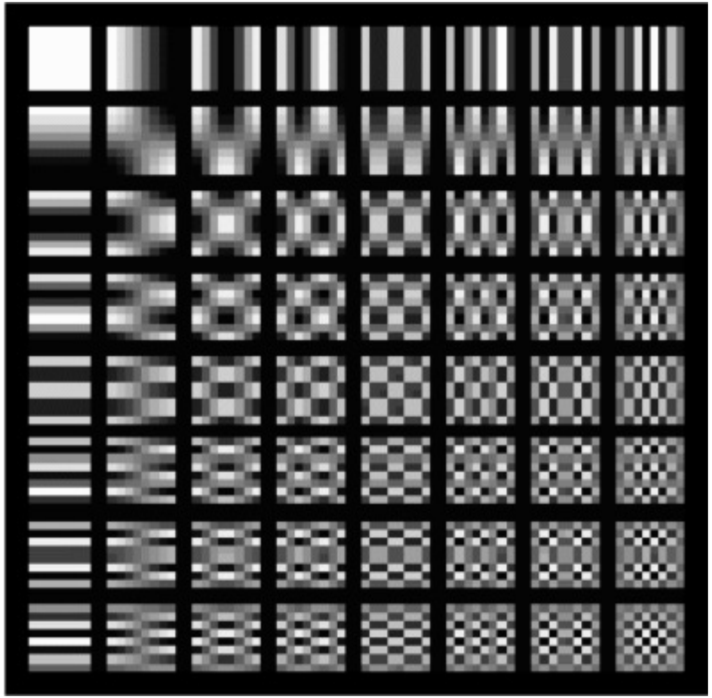


Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

Covariance Matrix

$$C_x = E\{(x - m_x)(x - m_x)^T\}$$

$$m_x = E\{x\}$$

$$m_x = \frac{1}{M}\sum_{k=0}^{M-1} x_k$$

$$C_x = \frac{1}{M}\sum_{k=0}^{M-1} (x_k - m_x)(x_k - m_x)^T$$

$$C_x = \frac{1}{M}\sum_{k=0}^{M-1} x_k x_k - m_x m_x^T$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

$$Y = A(X - m_x)$$

Inverse Transform

$$X = A^T Y + m_x$$

$$C_x T_i = \lambda_i T_i$$

$$A = \text{Matrix with rows of eigen vectors } T_i s \text{ with}$$
$$\lambda_0 > \lambda_1 > \lambda_2 \ldots$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

$$m_y = E\{Y\} = E\{A(X - m_x)\} = AE\{X\} - Am_x = 0$$

$$\begin{aligned} C_y &= E\{(Y - m_y)(Y - m_y)^T\} = E\{(Y)(Y)^T\} \\ &= E\{(AX - Am_x)(AX - Am_x)^T\} \\ &= E\{A(X - m_x)(X - m_x)^T A^T\} \\ &= AE\{(X - m_x)(X - m_x)^T\}A^T \\ &= AC_xA^T = AA^TD = D \end{aligned}$$

Covariance Matrix is a Diagonal Matrix

$$D = \begin{bmatrix} \lambda_1 & . & . & 0 \\ . & \lambda_2 & . & . \\ . & . & .. & . \\ 0 & . & . & \lambda_N \end{bmatrix}$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

Inverse Transform $X = A^T Y + m_x$

$$X' = A_k^T Y + m_x$$

$$e_{ms} = \sum_{j=0}^{N-1} \lambda_j - \sum_{j=0}^{k-1} \lambda_j = \sum_{j=K}^{N-1} \lambda_j$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

Geometrically

Principal Component Analysis

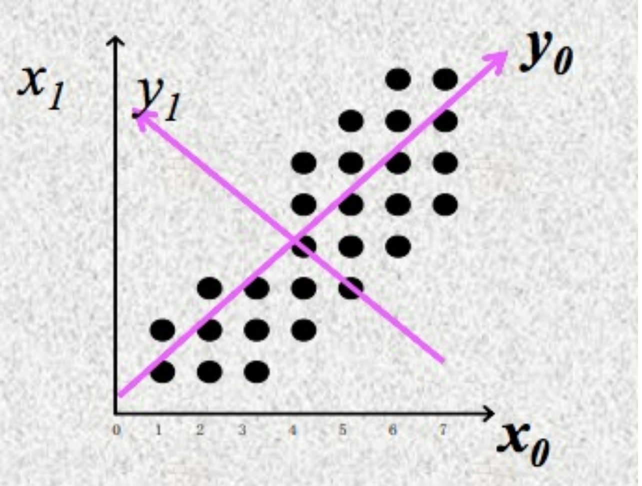
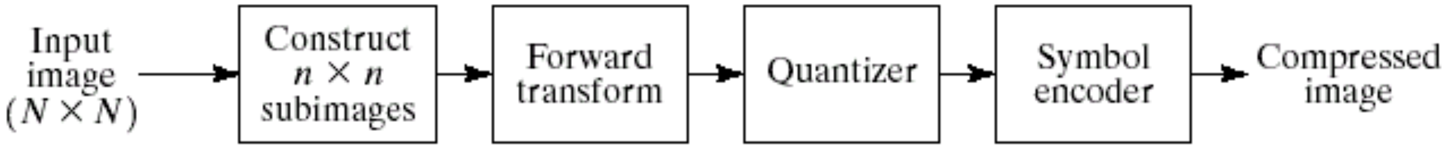


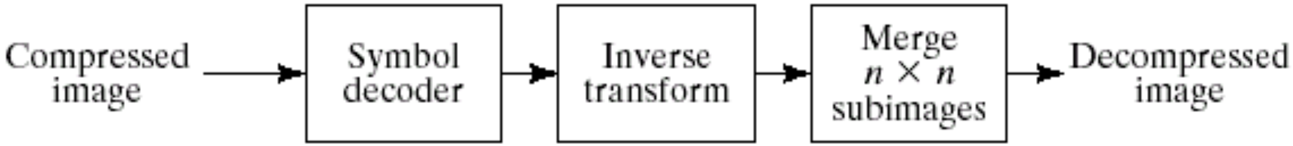
Image Compression

Transform Coding

Pipeline

Compression





Decompression

Image Compression

Transform Coding

Pipeline

Compression

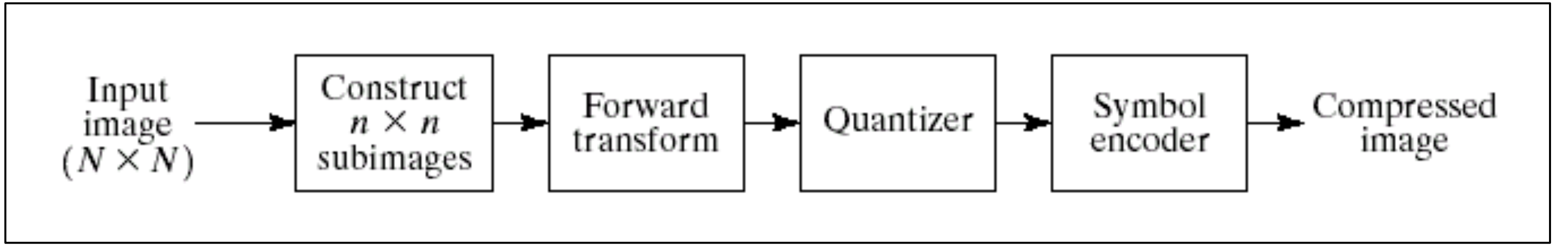
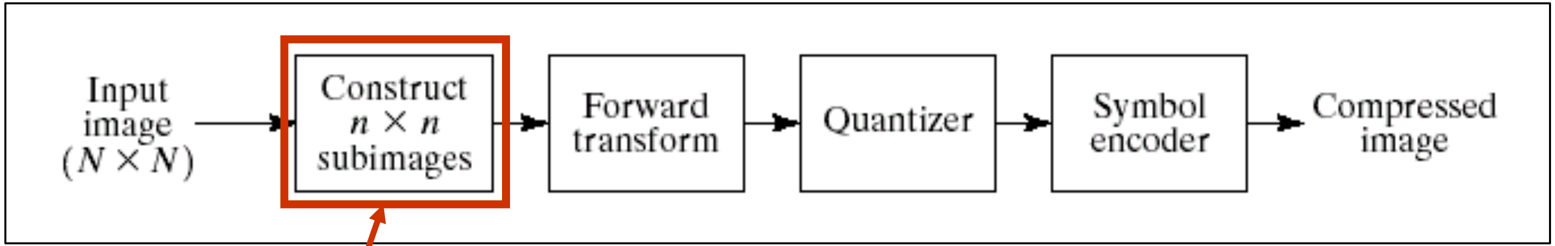


Image Compression

Transform Coding

Pipeline

Compression



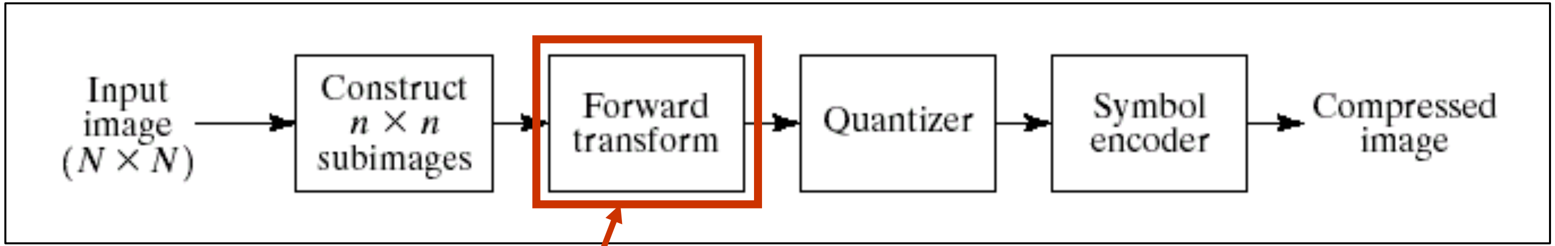
Typically
8x8 or 16x16

Image Compression

Transform Coding

Pipeline

Compression



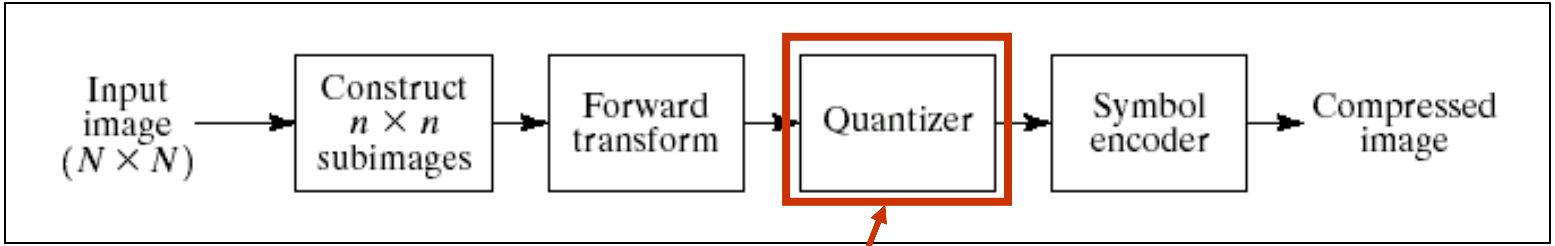
Discrete Fourier Transform
Discrete Cosine Transform
Karhunen-Loeve Transform
…..

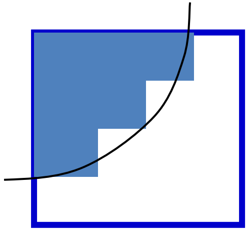
Image Compression

Transform Coding

Pipeline

Compression





Threshold Coding
Zonal Coding

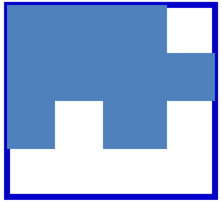


Image Compression

Transform Coding

Pipeline

Compression

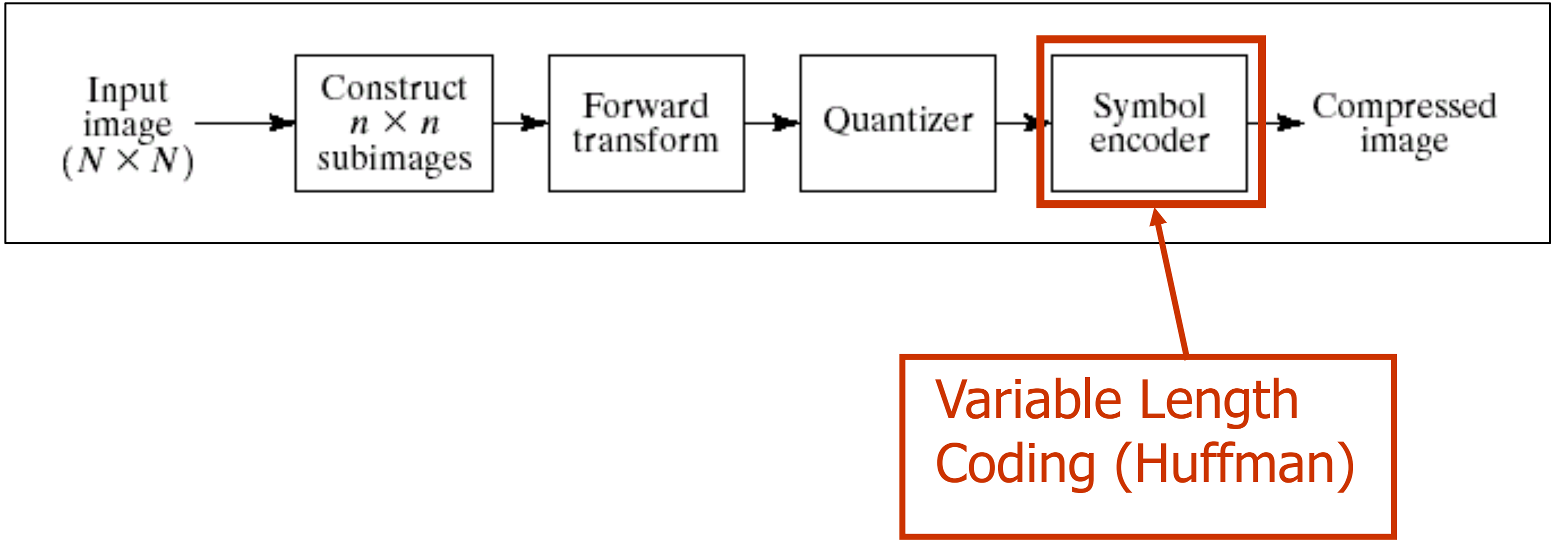
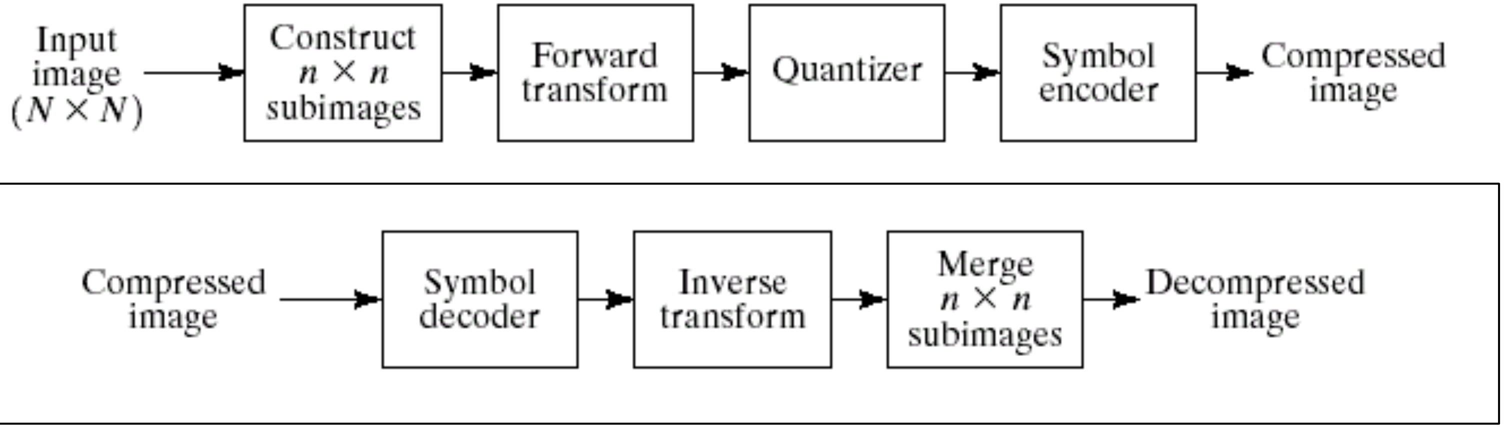


Image Compression

Transform Coding

Pipeline

Compression



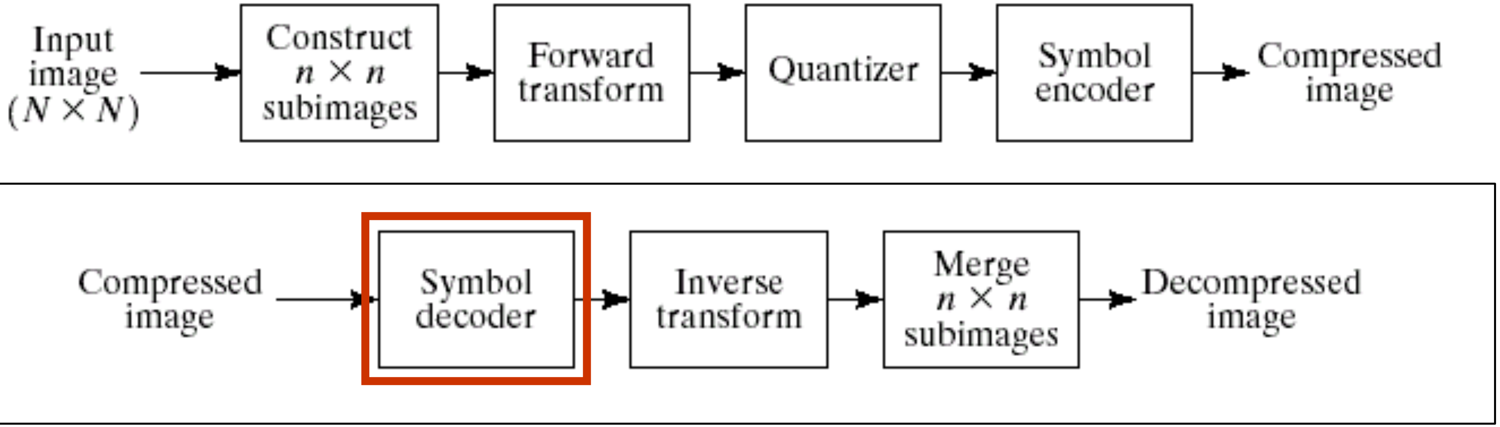
Decompression

Image Compression

Transform Coding

Pipeline

Compression



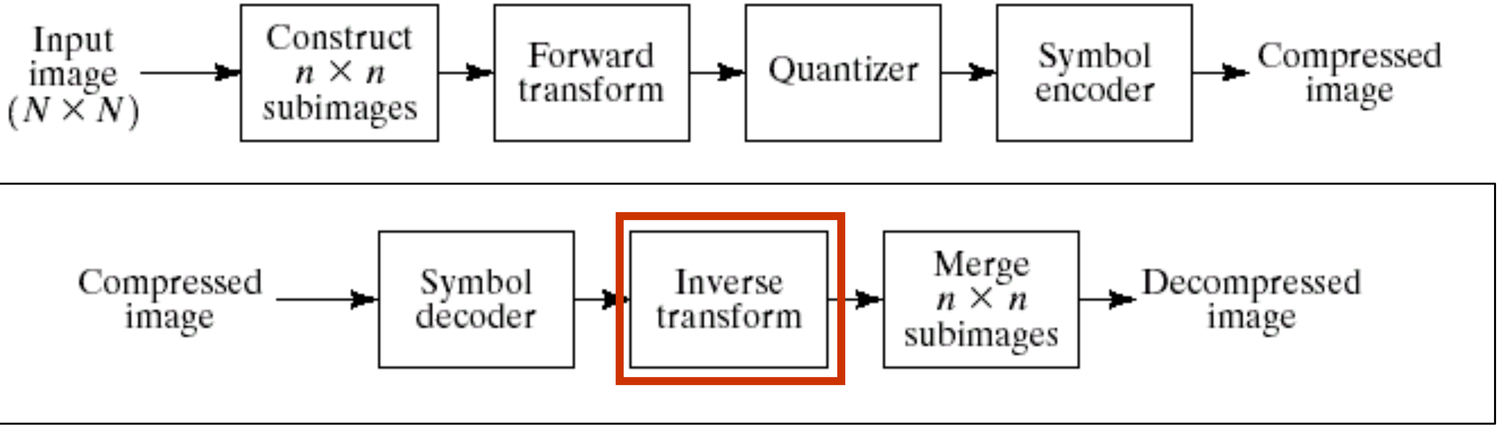
Decompression

Image Compression

Transform Coding

Pipeline

Compression



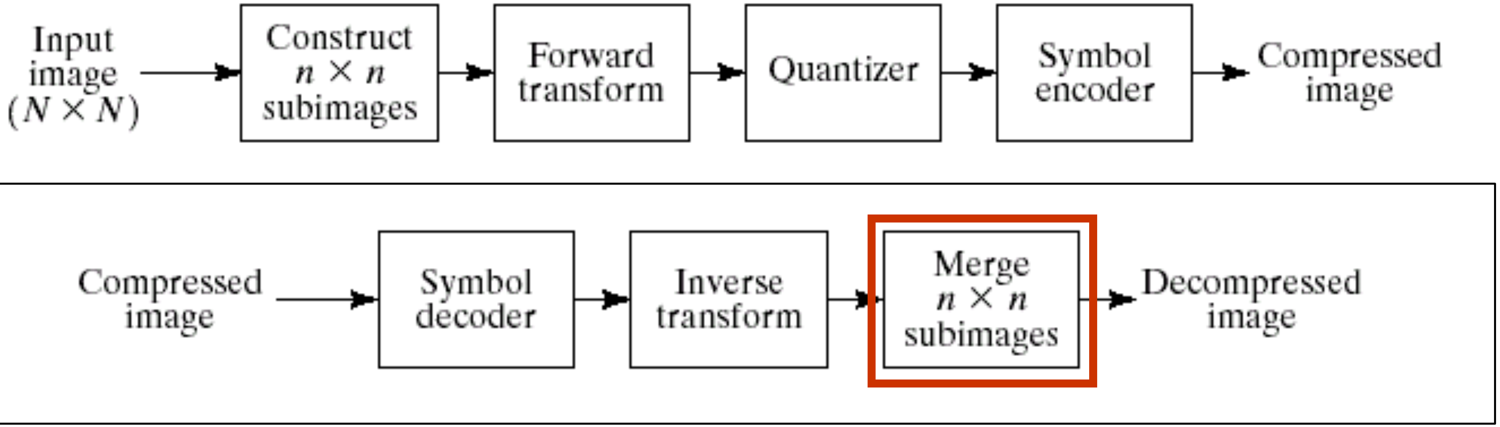
Decompression

Image Compression

Transform Coding

Pipeline

Compression



Decompression