# Special Topics in Multimedia System

Indian Institute of Technology Delhi
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SIL801

# Recap

Arithmetic Coding

Dictionary based methods

- LZ77
- LZW

Run Length Coding

Predictive Coding

# Recap: Image Compression

## Predictive Coding

Basic premise: Current pixel is similar to the previous pixel (coherence)

### Differential Coding

$$d(x,y) = I(x,y) - I(x-1,y)$$

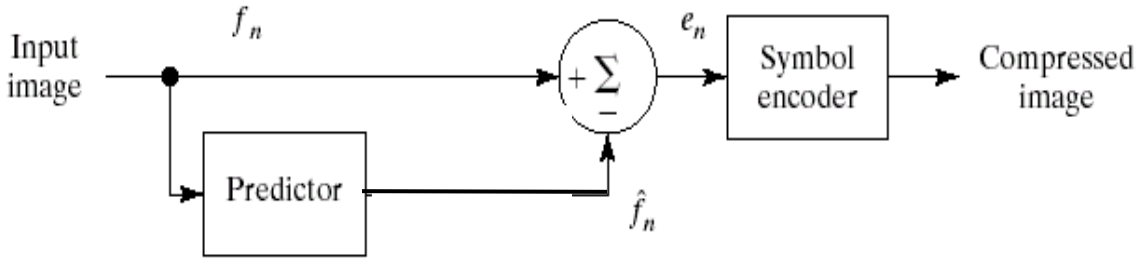
d(x,y) prediction error which is to be encoded.

# Recap: Image Compression

## Predictive Coding

### Compression

$$e_n = f_n - \hat{f}_n,$$

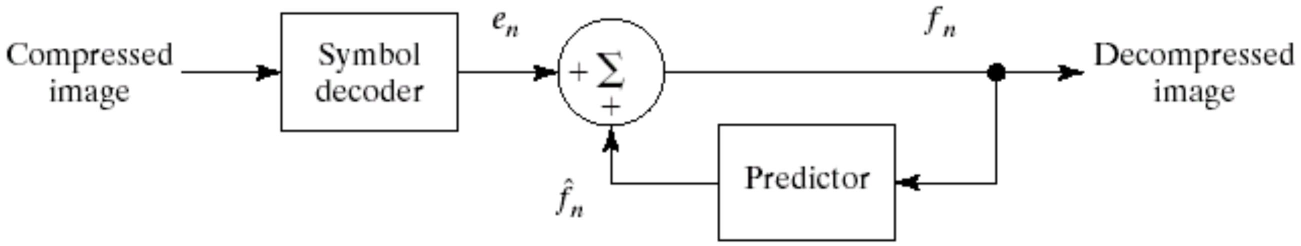



Source: Digital Image Processing, Gonzalez and Woods.

# Recap: Image Compression

## Predictive Coding

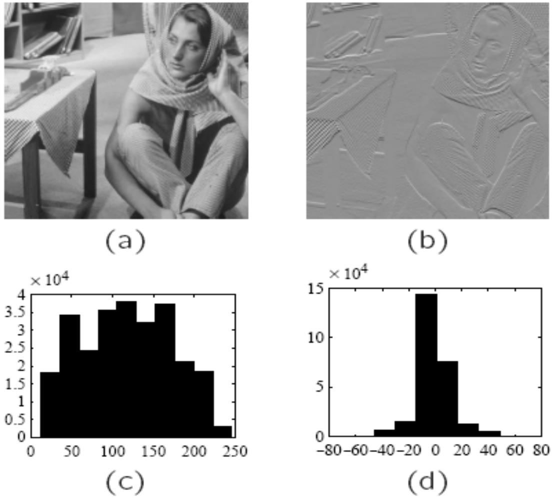
### Decompression




Source: Digital Image Processing, Gonzalez and Woods.

# Recap: Image Compression

## Predictive Coding




Distributions for Original versus Derivative Images. (a,b): Original gray-level image and its partial derivative image; (c,d): Histograms for original and derivative images.

# Recap: Image Compression

## Predictive Coding

$$E\{e_n^2\} = E\left\{\left[f_n - \sum_{i=1}^{m} \alpha_i f_{n-i}\right]^2\right\}.$$

$$\boldsymbol{\alpha} = \mathbf{R}^{-1}\mathbf{r}$$

where $\mathbf{R}^{-1}$ is the inverse of the $m \times m$ autocorrelation matrix

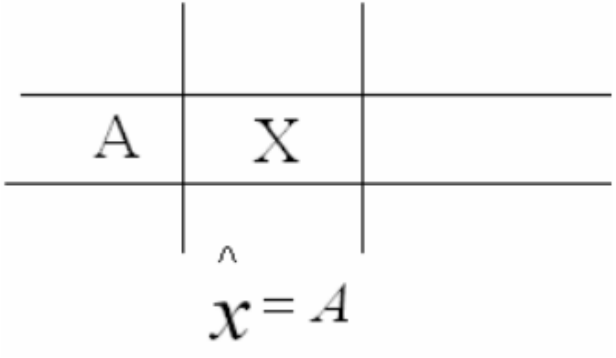
$$\mathbf{R} = \begin{bmatrix} E\{f_{n-1}f_{n-1}\} & E\{f_{n-1}f_{n-2}\} & \cdots & E\{f_{n-1}f_{n-m}\} \\ E\{f_{n-2}f_{n-1}\} & \cdots & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & & & \\ E\{f_{n-m}f_{n-1}\} & E\{f_{n-m}f_{n-2}\} & \cdots & E\{f_{n-m}f_{n-m}\} \end{bmatrix}$$

and $\mathbf{r}$ and $\boldsymbol{\alpha}$ are the $m$-element vectors

$$\mathbf{r} = \begin{bmatrix} E\{f_nf_{n-1}\} \\ E\{f_nf_{n-2}\} \\ \vdots \\ E\{f_nf_{n-m}\} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{bmatrix}.$$

# Recap: Image Compression

## Predictive Coding




| | | |
|---|---|---|
| 91 | 99 (+7) | 96 (-3) |
| 93 | 101 (+8) | 97 (-4) |
| 101 | 103 (+2) | 104 (+1) |

# Recap: Image Compression

## Predictive Coding

| | | |
|---|---|---|
| B | C | D |
| A | X | |
| | | |

$$\hat{x} = k_1 A + k_2 B + k_3 C + K_4 D$$

# Image Compression

## Lossy

- Psychovisual redundancy
- Keep more important information
- Trade off between loss (degradation) and compression

**Original**

**Compression Ratio: 7.7**

**Compression Ratio: 12.3**

**Compression Ratio: 33.9**

# Image Compression

## Lossy

Original

# Image Compression

## Lossy

Compression Ratio 7.7

# Image Compression

## Lossy

Compression Ratio 33.9

# Image Compression

## Lossy

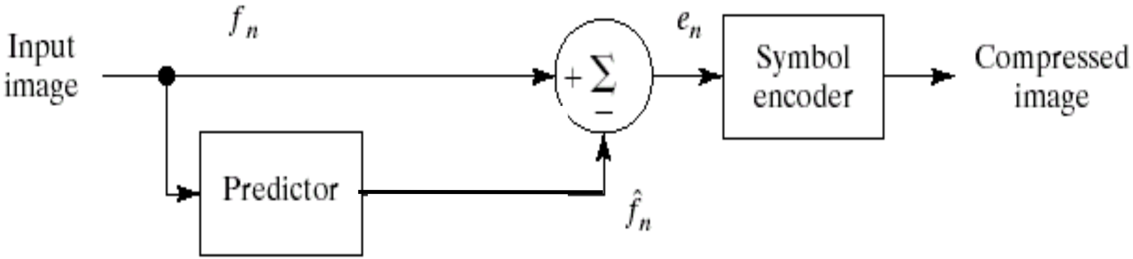
- Recall Quantization
  - Discrete value to represent range of values
  - Irreversible operation
  - Information loss !

- Predictive Coding
- Transform Coding

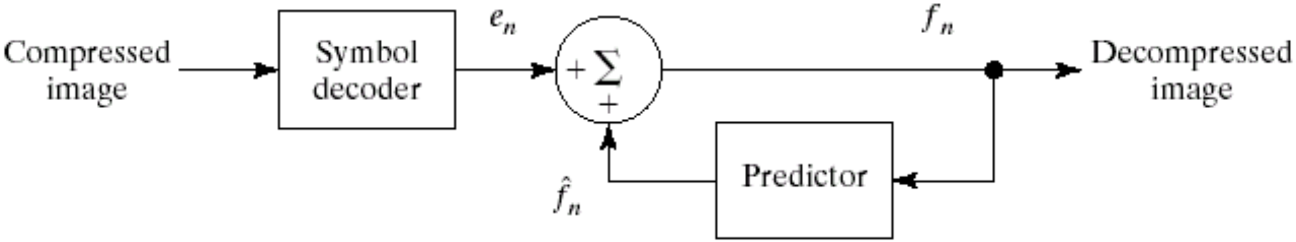
# Image Compression

## Predictive Coding: Loss-less (Revisit)

Compression



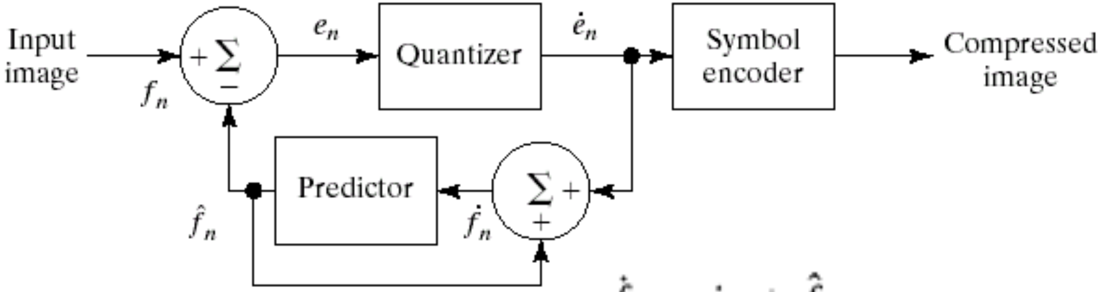

Decompression



# Image Compression

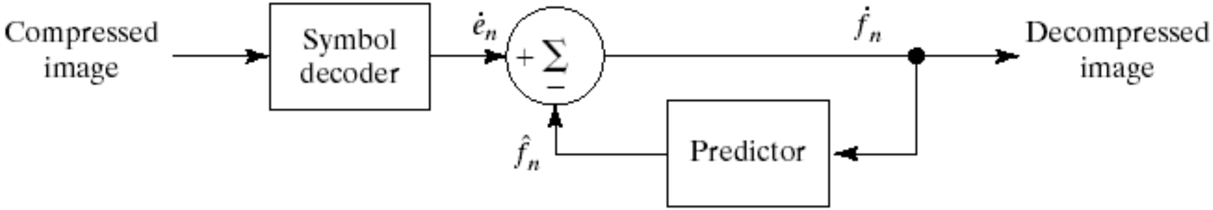
## Predictive Coding: Lossy

### Compression




$$\dot{f}_n = \dot{e}_n + \hat{f}_n$$

### Decompression



# Image Compression

## Predictive Coding: Lossy

### Delta Modulation

**Example:**

$$\hat{f}_n = \alpha \, \dot{f}_{n-1}$$

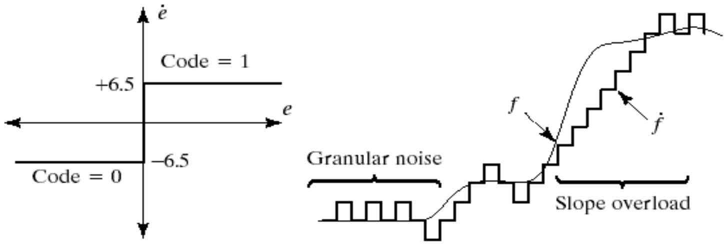
$0 < \alpha < 1$
prediction coefficient

$$\text{and } \dot{e}_n = \begin{cases} +\xi & e_n > 0 \\ -\xi & e_n < 0 \end{cases}$$

$$\dot{f}_n = \dot{e}_n + \hat{f}_n$$
$$= \dot{e}_n + \alpha \, \dot{f}_{n-1}$$

# Image Compression

## Predictive Coding: Lossy




a b
c

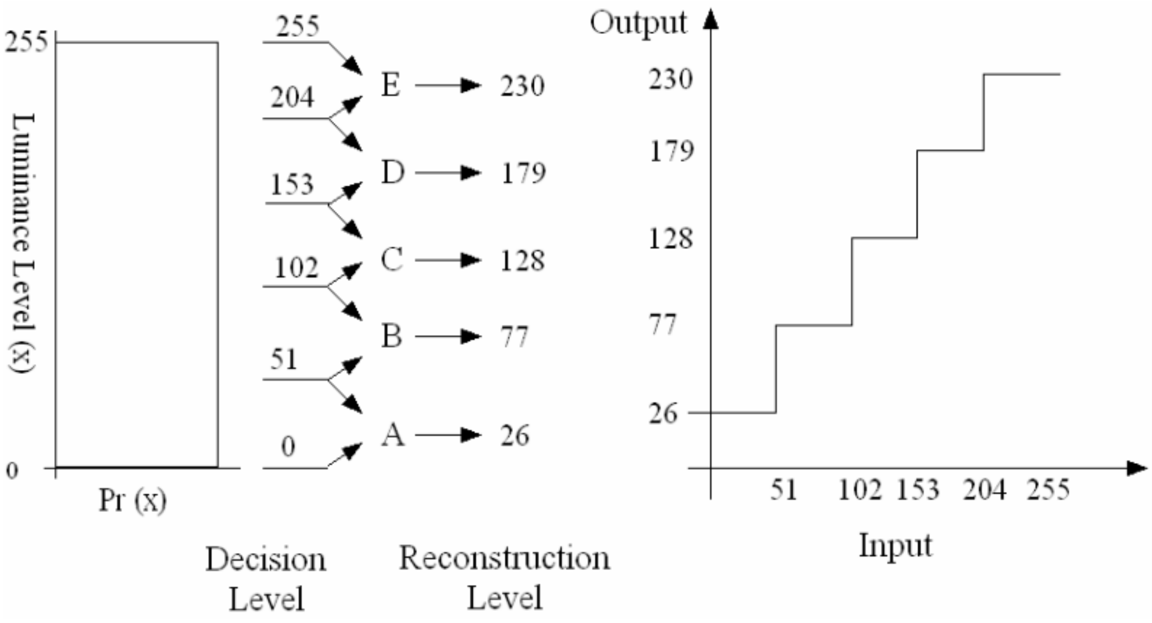
**FIGURE 8.22** An example of delta modulation.

| Input | | Encoder | | | | Decoder | | Error |
|---|---|---|---|---|---|---|---|---|
| $n$ | $f$ | $\hat{f}$ | $e$ | $\dot{e}$ | $\dot{f}$ | $\hat{f}$ | $\dot{f}$ | $[f - \dot{f}]$ |
| 0 | 14 | — | — | — | 14.0 | — | 14.0 | 0.0 |
| 1 | 15 | 14.0 | 1.0 | 6.5 | 20.5 | 14.0 | 20.5 | –5.5 |
| 2 | 14 | 20.5 | –6.5 | –6.5 | 14.0 | 20.5 | 14.0 | 0.0 |
| 3 | 15 | 14.0 | 1.0 | 6.5 | 20.5 | 14.0 | 20.5 | –5.5 |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| 14 | 29 | 20.5 | 8.5 | 6.5 | 27.0 | 20.5 | 27.0 | 2.0 |
| 15 | 37 | 27.0 | 10.0 | 6.5 | 33.5 | 27.0 | 33.5 | 3.5 |
| 16 | 47 | 33.5 | 13.5 | 6.5 | 40.0 | 33.5 | 40.0 | 7.0 |
| 17 | 62 | 40.0 | 22.0 | 6.5 | 46.5 | 40.0 | 46.5 | 15.5 |
| 18 | 75 | 46.5 | 28.5 | 6.5 | 53.0 | 46.5 | 53.0 | 22.0 |
| 19 | 77 | 53.0 | 24.0 | 6.5 | 59.6 | 53.0 | 59.6 | 17.5 |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |

# Image Compression

## Predictive Coding: Lossy – Quantization

Uniform Quantization

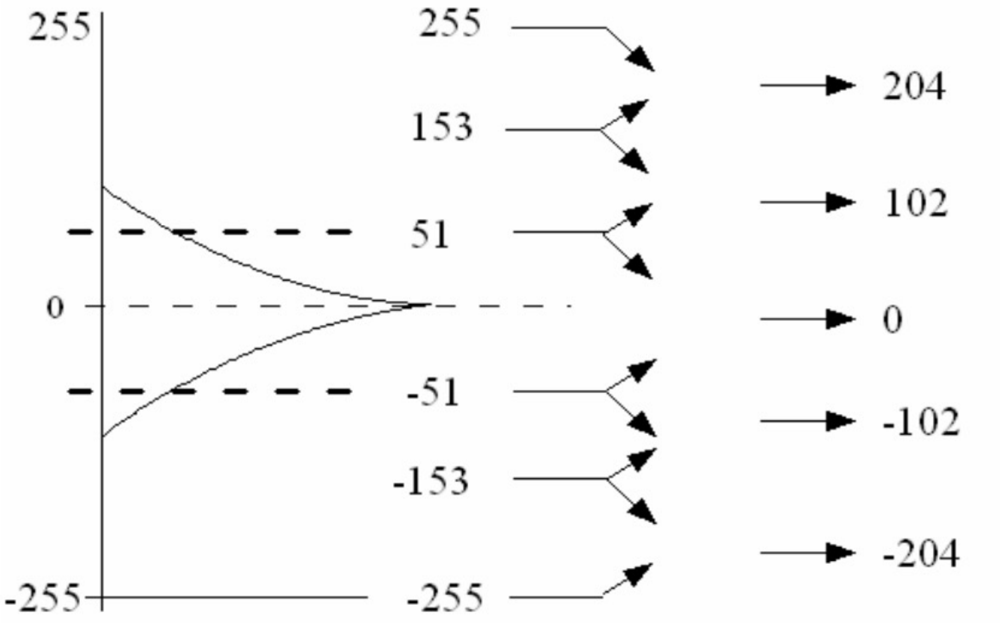


# Image Compression

## Predictive Coding: Lossy – Quantization

Uniform Quantization

Difference Image

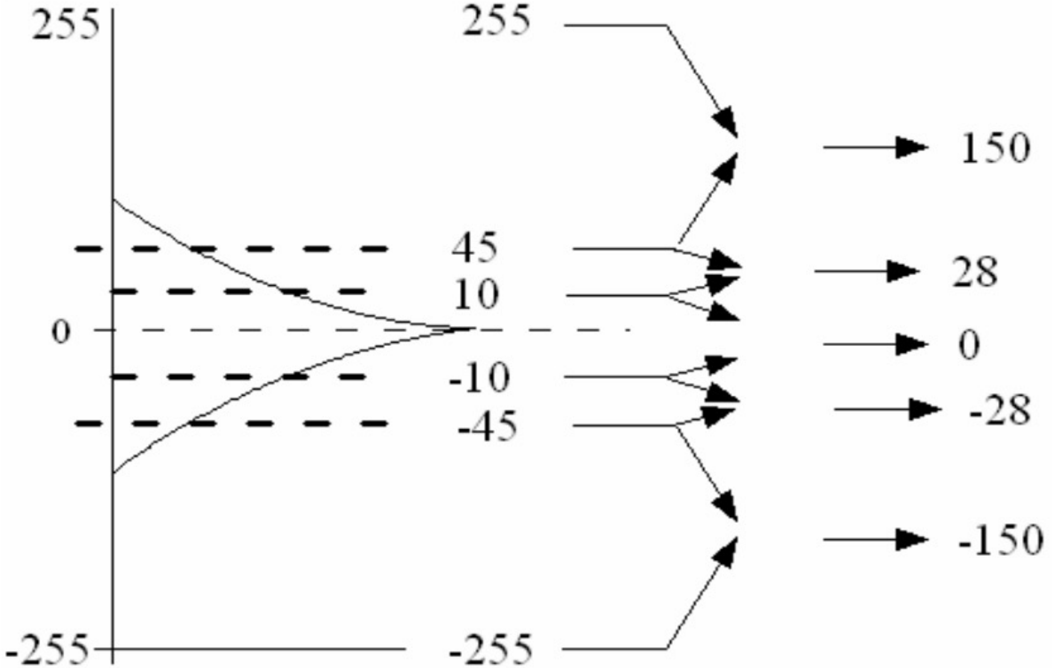


# Image Compression

## Predictive Coding: Lossy – Quantization

Non-Uniform Quantization

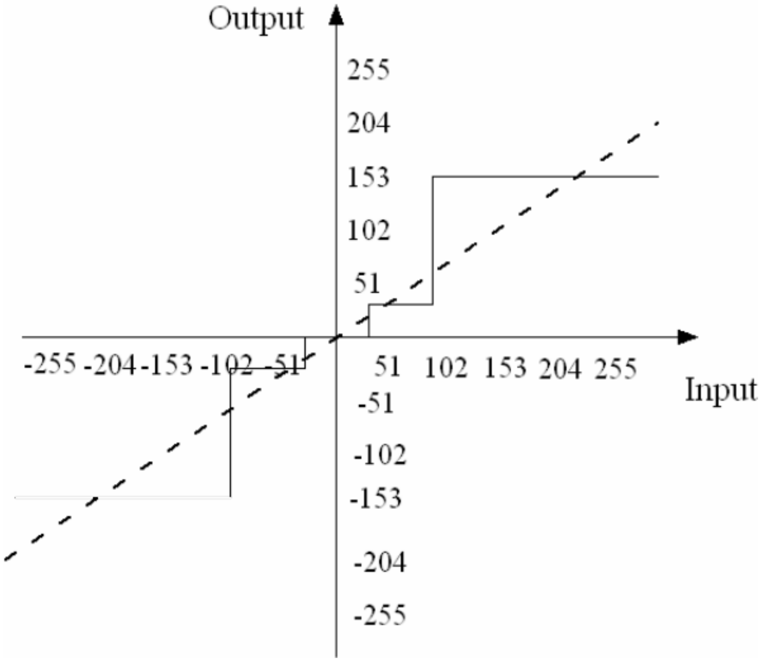
Difference Image



# Image Compression

## Predictive Coding: Lossy – Quantization

Non-Uniform Quantization



# Image Compression

## Transform Coding

Transformation

- Represent information in anther space
- Identify and remove correlation
- Quantization of transform coefficients

Information loss!

Example

time/space frequency

(Fourier Transform)

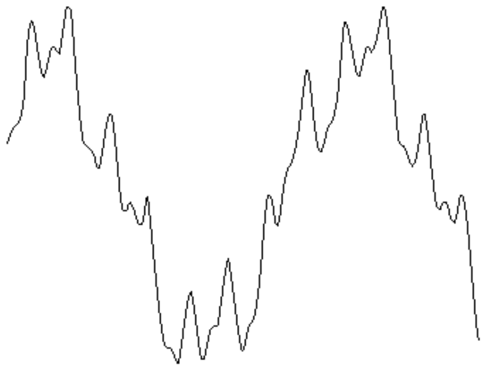
Inverse transformation

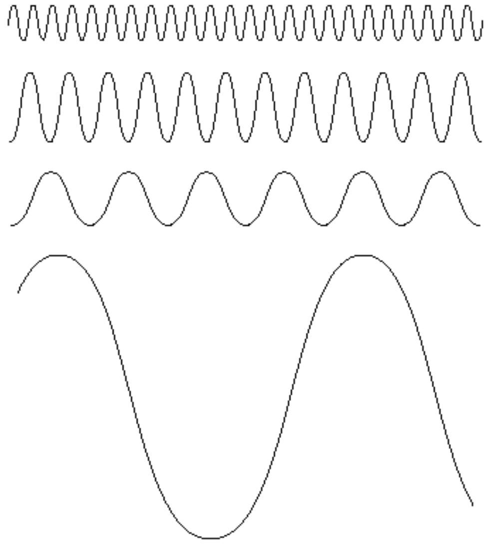
- Bring back information in the original space

# Image Compression

## Transform Coding

### Fourier Transform (Review)





A function as sum of sines and cosines

# Image Compression

## Transform Coding

### Fourier Transform (Review)

Mathematically

Forward

1-D:

$$F(u) \equiv \Im\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

Inverse

$$f(x) \equiv \Im^{-1}\{F(u)\} = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

2-D:

$$F(u,v) = \iint f(x,y) e^{-j2\pi(ux+vy)} \, dx \, dy$$

$$f(x,y) = \iint F(u,v) e^{j2\pi(ux+vy)} \, du \, dv$$

Discrete Fourier Transform

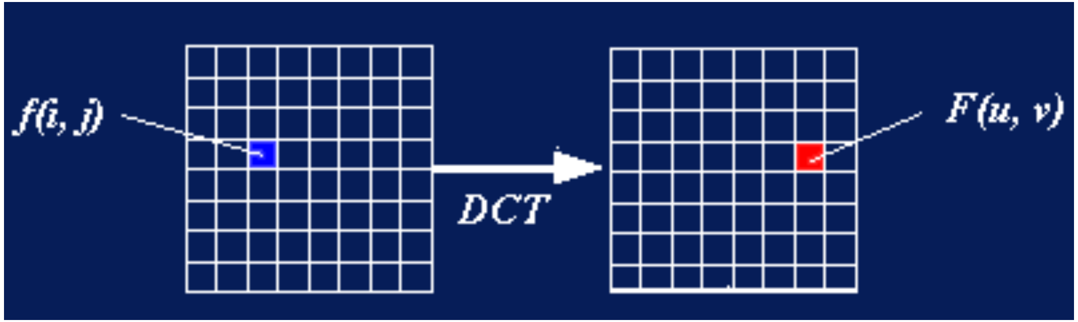
Fast Fourier Transform (FFT)

# Image Compression

## Transform Coding

### Discrete Cosine Transform

Popular transform for its performance and efficiency



# Image Compression

## Transform Coding

### Discrete Cosine Transform

Forward transform

$$F(u,v) = \frac{2}{N} C(u)C(v) \sum_{y=0}^{N-1} \sum_{x=0}^{N-1} f(x,y) \cos[\frac{(2x+1)u\pi}{2N}] \cos[\frac{(2y+1)v\pi}{2N}]$$

Inverse transform

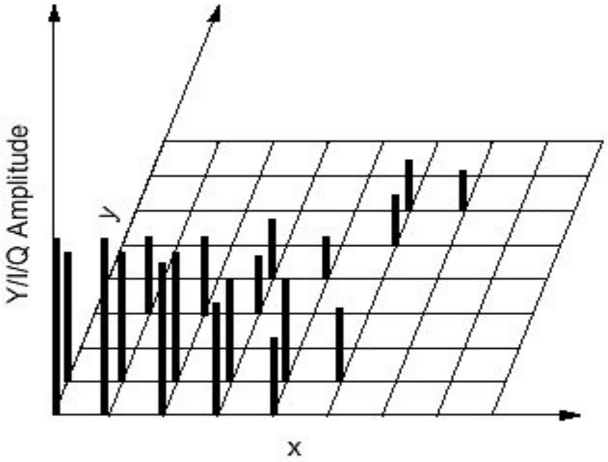
$$f(x,y) = \frac{2}{N} \sum_{v=0}^{N-1} \sum_{u=0}^{N-1} C(u)C(v)F(u,v) \cos[\frac{(2x+1)u\pi}{2N}] \cos[\frac{(2y+1)v\pi}{2N}]$$

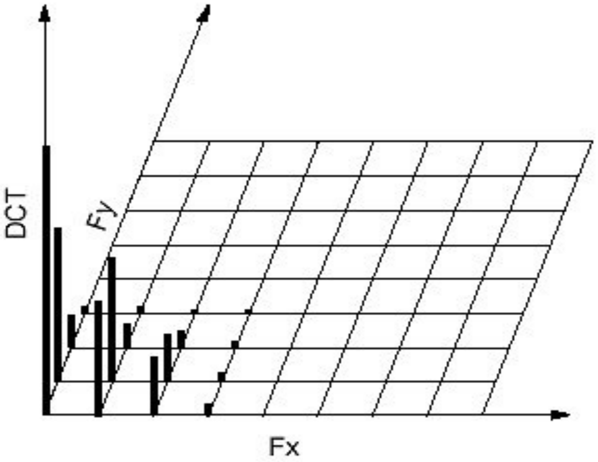
# Image Compression

## Transform Coding

### Discrete Cosine Transform

Energy compaction



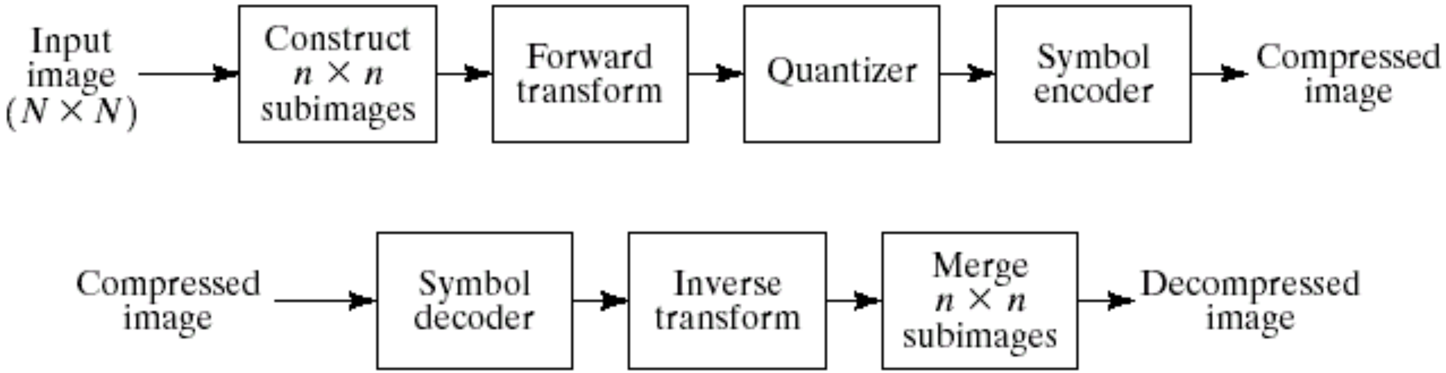



# Image Compression

## Transform Coding

### Pipeline

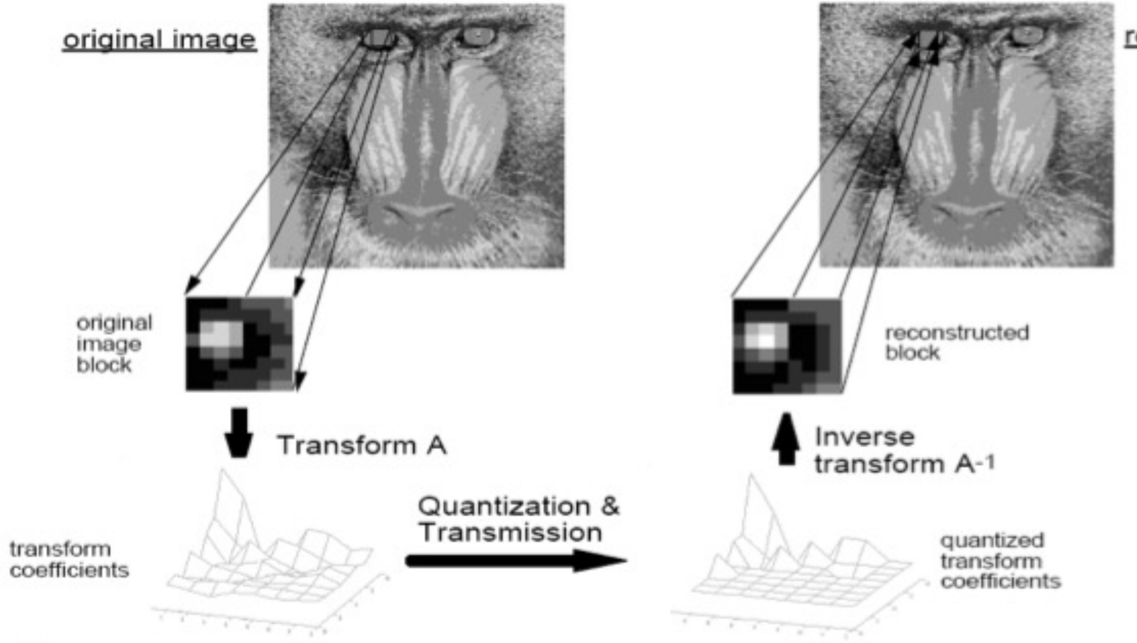
Compression




Decompression

# Image Compression

## Transform Coding

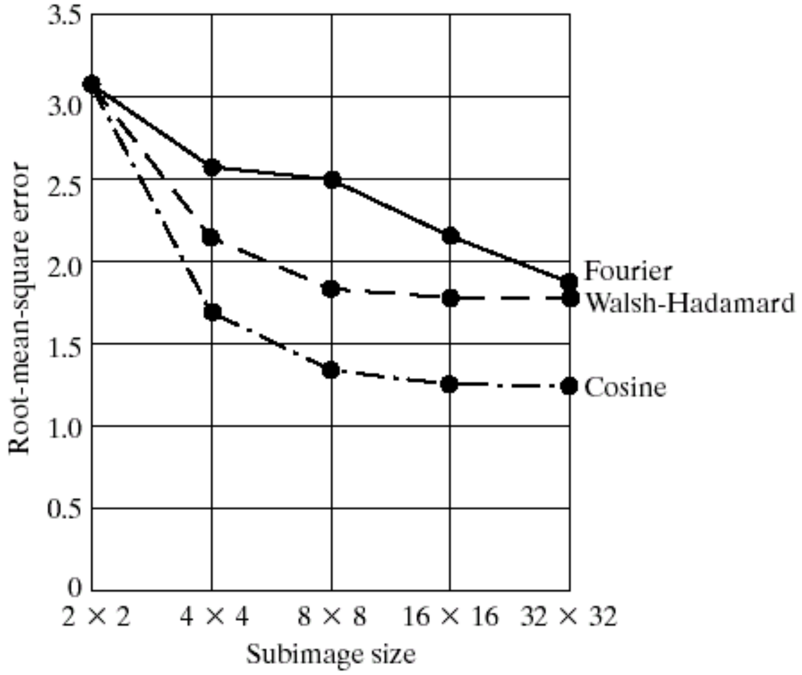


# Image Compression

## Transform Coding

Why sub image (nxn)?

- Computational benefit
- Typically 8x8, 16x16
- Error is not very high



# Image Compression

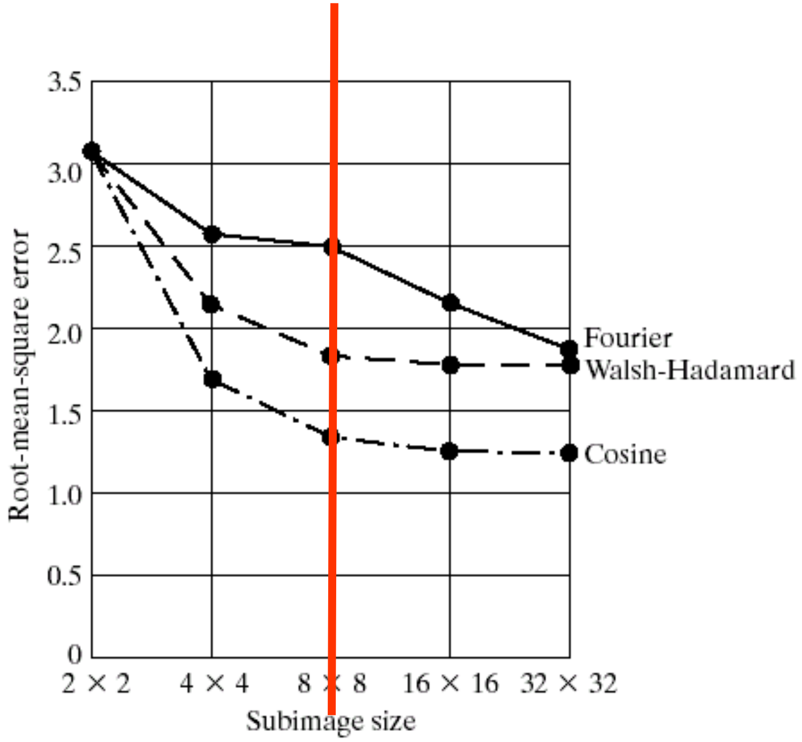
## Transform Coding

Why sub image (nxn)?

- Computational benefit
- Typically 8x8, 16x16
- Error is not very high

Which transform?

- Low error for the same number of coefficients
- Computationally fast
- DCT is preferred



# Image Compression

## Transform Coding

### Quantization Schemes

- Global thresholding
- Local thresholding
- For each block M out of N coefficients to retain