



# Special Topics in Multimedia System

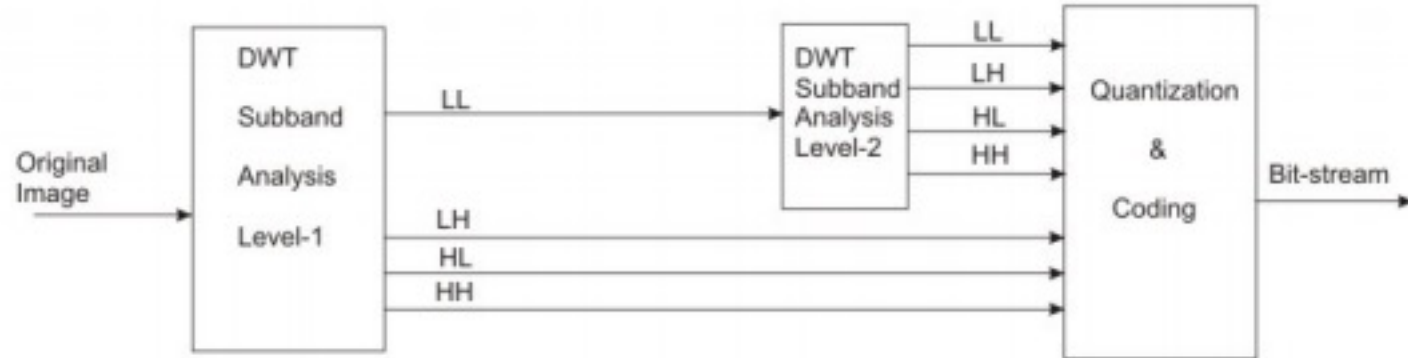
Indian Institute of Technology Delhi  
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New Delhi

SIL801



# Wavelet

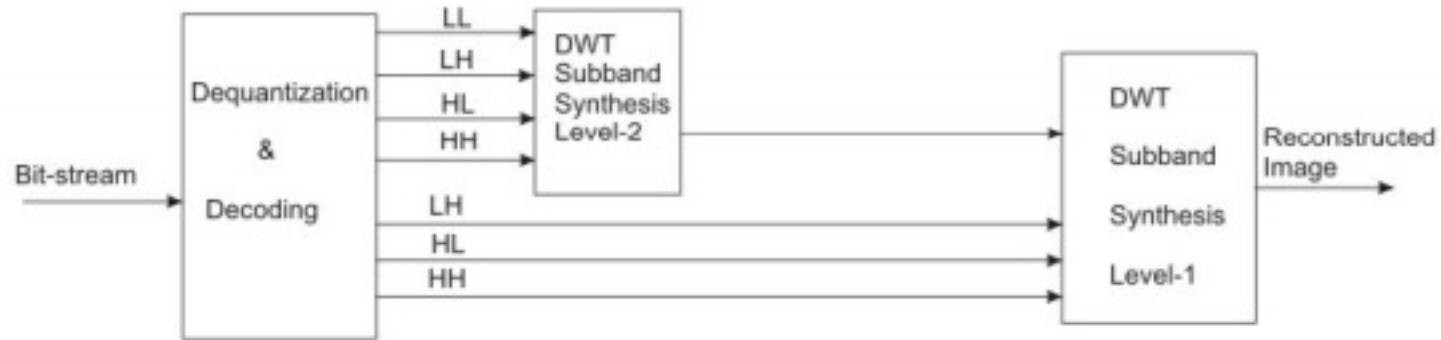
## Compression





# Wavelet

## Decompression





# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

- Introduced by Shapiro in 1993.

**J. M. Shapiro**, “Embedded Image Coding Using Zerotrees of Wavelet Coefficients,” *IEEE Trans. on Signal Processing*, Vol. 41, No. 12, pp. 3445 – 3462, Dec. 1993.



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

- Effectively exploits the self-similarity between subbands and the fact that the high-frequency subbands mostly contain insignificant coefficients.
- Defines the relationship between the subbands, based on the spatial locations.
- Defines a data structure in the form of a hierarchical tree that includes spatially related coefficients across different subbands



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

- **Zerotree** identifies the parts of a tree that have all the DWT coefficients insignificant starting with a root.
- The EZW algorithm is based on successive approximation quantization and this facilitates the embedding algorithm.
- Embedded implies progressive - coded bits are ordered in accordance with their importance - every bit added increases the accuracy, e.g.,  $\pi$ , can stop at a desirable accuracy. Bit stream with MSB first and LSB last.

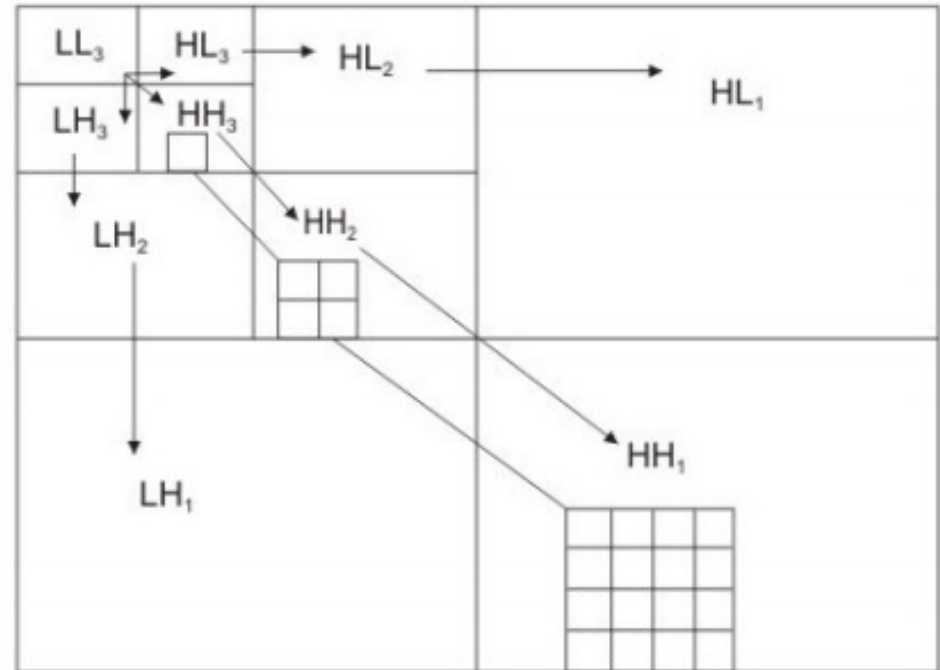


# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Relationship between subbands

For each stage (except the first one), every coefficient is calculated as a weighted average of several neighbor coefficients from the previous stage. Parent-child dependencies of subbands: every coefficient is a “parent” for the related coefficients in the previous scale.





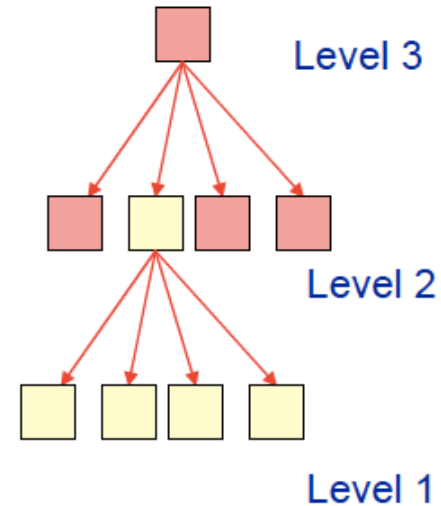
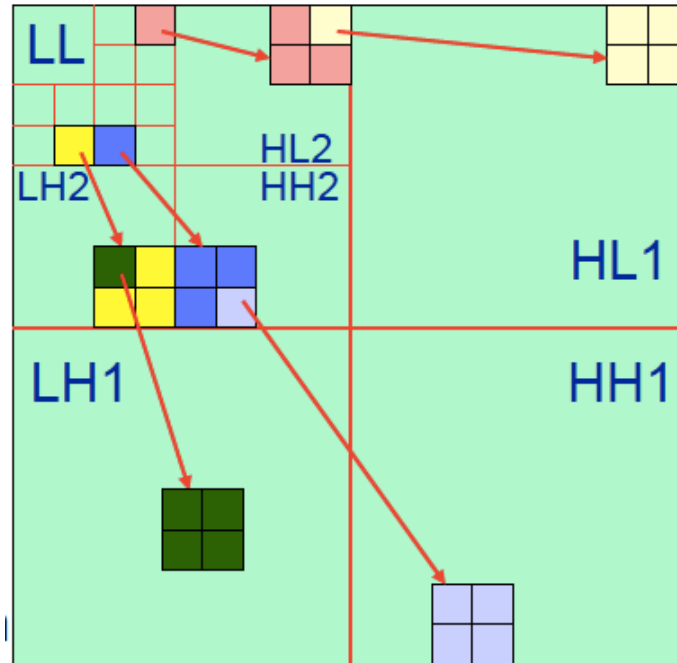


# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Zero Tree

A quad-tree of which all nodes are equal to or smaller than the root. The 'Tree' can be coded with a single symbol.





# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Zero Tree

A quad-tree of which all nodes are equal to or smaller than the root. The 'Tree' can be coded with a single symbol.

### Main Idea

Assumption: the wavelet coefficients decrease with scale. All the coefficients in a quad-tree will be smaller than a threshold if the root is smaller than it. This assumption can be violated but in practice its probability is very high.



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Significance of Coefficients

DWT coefficient of magnitude  $|X|$  is **significant** with respect to a given threshold  $T$  if  $|X| > T$  and is **insignificant** otherwise. The significance of DWT coefficients are first examined with the highest value of threshold in the first pass and then progressively, the threshold is decreased by a factor of 2 in subsequent passes.

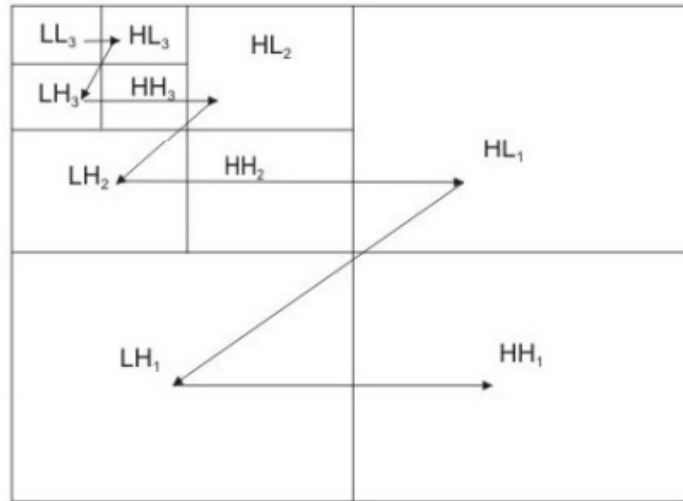


# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Significance of Coefficients

Zig Zag Scan

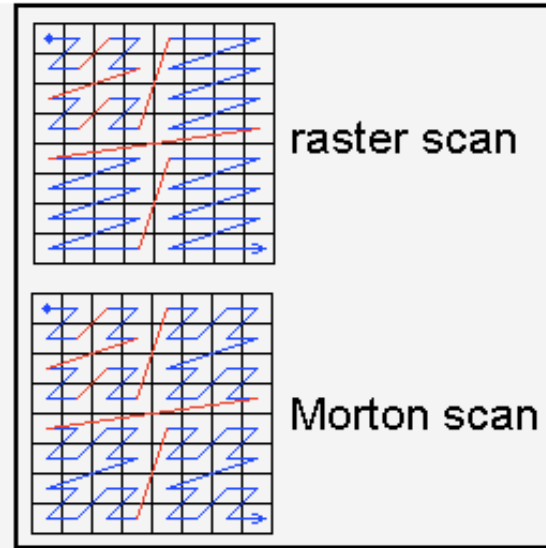


# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Significance of Coefficients

63	-34	49	10	7	13	-12	7
-31	23	14	-13	3	4	6	-1
15	14	3	-12	5	-7	3	9
-9	-7	-14	8	4	-2	3	2
-5	9	-1	47	4	6	-2	2
3	0	-3	2	3	-2	0	4
2	-3	6	-4	3	6	3	6
5	11	5	6	0	3	-4	4



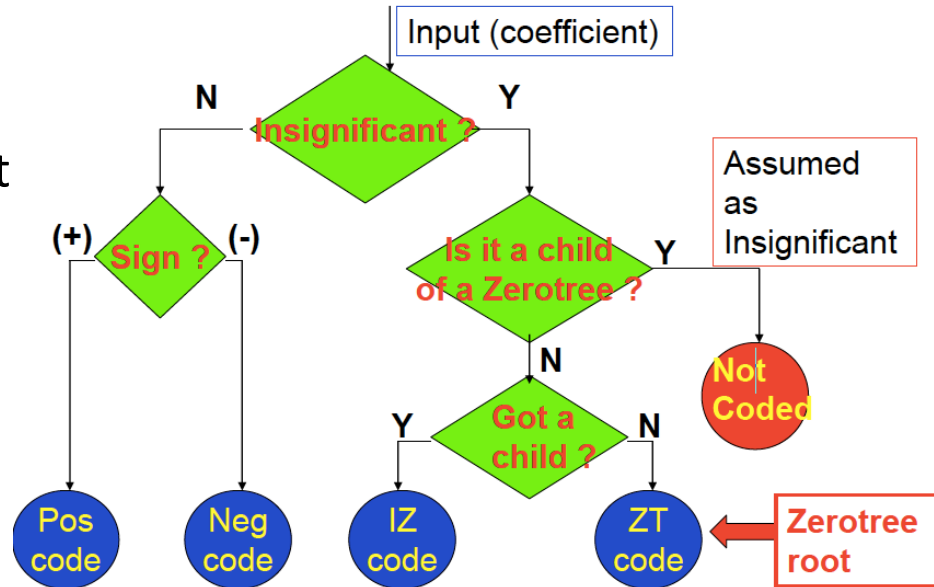


# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Significance Map

- PS (POS): Positive Significant
- NS (NEG): Negative Significant
- IZ: Isolated Zero
- ZTR : Zero Tree Root



Source: Nimrod Peleg Slides



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Successive Approximation Quantization

Successive Approximation Quantization (SAQ) performs encoding of magnitudes of DWT coefficients in successive stages. An initial threshold to examine the significance is first set up such that  $T_0 > |X_{\max}| / 2$ , where  $X_{\max}$  is the maximum of all DWT coefficients.

In each stage of encoding, it reduces the threshold by half and examines the significance once more ( $T_i = T_{i-1}/2$ ).



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Successive Approximation Quantization

Each stage has two passes **Dominant** pass and **Subordinate** pass.

A dominant pass is used to encode those coefficients that have not yet (that is, till the previous stage of encoding) been found to be significant with respect to a threshold.

A dominant pass is followed by a subordinate pass in which the coefficients found to be significant in the subordinate list are scanned and their magnitudes are refined with an added bit of precision.





# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Successive Approximation Quantization

Each stage has two passes **Dominant** pass and **Subordinate** pass.

The encoding process alternates between dominant pass and subordinate pass and the threshold is halved after each dominant pass. The encoding stops when some target bit rate is achieved.

```
threshold = initial_threshold;
do
{
    dominant_pass(image);
    subordinate_pass(image);
    threshold = threshold/2;
}
while
(threshold > minimum_threshold);
```



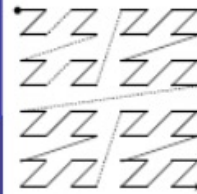
# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Example

Example of 3-level WT of an 8x8 image

63	-34	49	10	7	13	-12	7
-31	23	14	-13	3	4	6	-1
15	14	3	-12	5	-7	3	9
-9	-7	14	8	4	-2	3	2
-5	9	-1	47	4	6	-2	2
3	0	-3	2	3	-2	0	4
2	-3	6	-4	3	6	3	6
5	-11	5	6	0	3	-4	4



Largest coefficient magn = 63  $\rightarrow T_0 = 32$

... So after thresholding we have:

63	-34	49	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	47	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

Example

63	-34	49	10	7	13	-12	7
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-5	9	-1	47	4	6	-2	2
3	0	-3	2	3	-2	0	4
2	-3	6	-4	3	6	3	6
5	11	5	6	0	3	-4	4

63	-34	49	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	47	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Example

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3	0	-3	2	3	-2	0	4
2	-3	6	-4	3	6	3	6
5	11	5	6	0	3	-4	4

63	-34	49	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	47	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Comment	Subband	Coefficient Value	Symbol	Reconstruction Value
(1)	LL3	63	POS	48
	HL3	-34	NEG	-48
(2)	LH3	-31	IZ	0
(3)	HH3	23	ZTR	0
	HL2	49	POS	48
(4)	HL2	10	ZTR	0
	HL2	14	ZTR	0
	HL2	-13	ZTR	0
	LH2	15	ZTR	0
(5)	LH2	14	IZ	0
	LH2	-9	ZTR	0
	LH2	-7	ZTR	0
(6)	HL1	7	Z	0
	HL1	13	Z	0
	HL1	3	Z	0
	HL1	4	Z	0
	LH1	-1	Z	0
(7)	LH1	47	POS	48
	LH1	-3	Z	0
	LH1	-2	Z	0



# Wavelet Encoding

## Embedded Zero Tree Wavelet Encoding (EZW)

### Example

Dominant List Contains  
Pointers to all these zeros

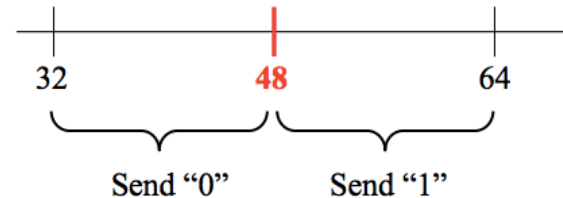
			0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0		0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Subordinate List

63  
34  
49  
47

### First Subordinate Pass

Now refines magnitude of each element on Subordinate List... Right now we know that each element's magnitude lies in (32,64]



Stream due to 1<sup>st</sup> Sub. Pass: 1 0 1 0



# Wavelet Encoding

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## Embedded Zero Tree Wavelet Encoding (EZW)

### Limitation

- Works for Non Standard decomposition.
- Exploits similarity of coefficients across bands.
- Does not make grouping of insignificant coefficients to improve the coding efficiency.



# Wavelet Encoding

## Set Partitioning in Hierarchical Trees (SPIHT)

It is a modified form of embedded coding of wavelet coefficients that carries the major strengths of EZW, namely ordered coefficient transmission and self-similarity across subbands of similar orientation. In addition, it partitions the set of coefficients into subsets of insignificant coefficients and identifies each significant coefficient.

The approach is proposed by Said and Pearlman

Said, A.; Pearlman, W. A. (1996). "A new, fast, and efficient image codec based on set partitioning in hierarchical trees". *IEEE Transactions on Circuits and Systems for Video Technology*. 6 (3): 243–250



# Wavelet Encoding

## Embedded Block Coding with Optimized Truncation (EBCOT)

Both EZW and SPIHT can only be applied to **dyadic** partitioning of coefficients, in which only the LL subband at a decomposition level are further analyzed. These cannot be applied to wavelet packets in general. Wavelet packets allow more flexible decomposition of subbands and the subbands at higher frequencies can also be decomposed into narrower bands.

Moreover, these two approaches only offer SNR scalability by encoding all the subbands at a given precision in an iteration of passes (dominant –subordinate passes in EZW). These algorithms do not offer any resolution scalability.





# Wavelet Encoding

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## Embedded Block Coding with Optimized Truncation (EBCOT)

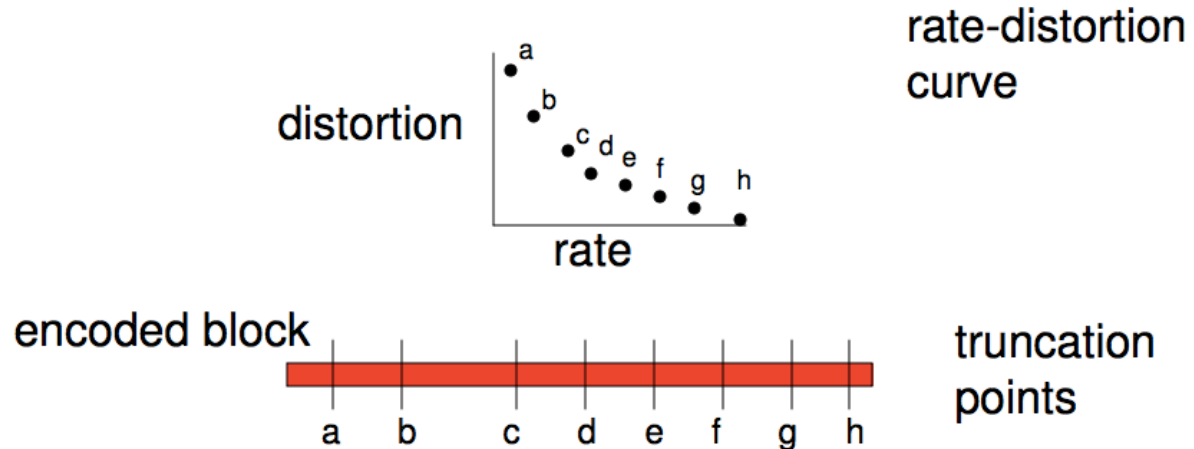
Embedded Block Coding with Optimized Truncation of bit-stream, which can be applied to wavelet packets and which offers both resolution scalability and SNR scalability. Because of its advantages, the EBCOT algorithm has been accepted incorporated within the most recent still image compression standard JPEG-2000.

D. S. Taubman, “High performance scalable image compression with EBCOT,” *IEEE Trans. Image Proc.*, vol. 9, pp. 1158-1170, July 2000



# Wavelet Encoding

## Embedded Block Coding with Optimized Truncation (EBCOT)

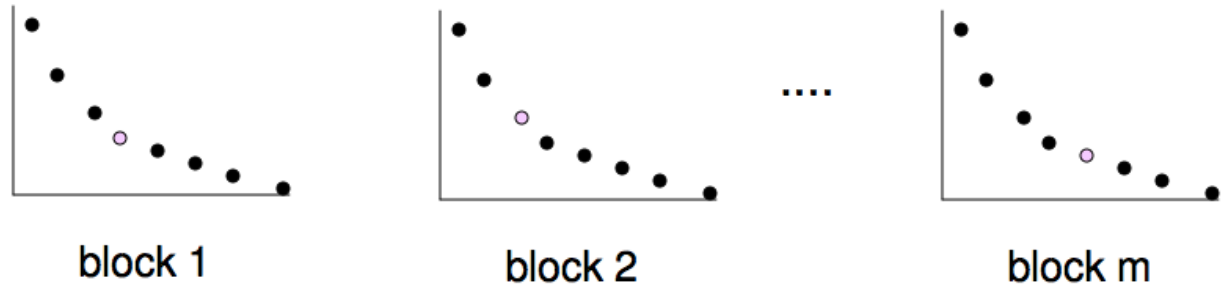


Source <https://courses.cs.washington.edu/courses/csep590a/07au/lectures/lecture12large.pdf>



# Wavelet Encoding

## Embedded Block Coding with Optimized Truncation (EBCOT)



Pick one point from each curve so that the sum of the x values is bounded by  $R$  and the sum of the y values is minimized.

Source <https://courses.cs.washington.edu/courses/csep590a/07au/lectures/lecture12large.pdf>