

# Domain Aware Markov Logic Networks

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Combining logic and probability has been a long standing goal of AI research. Markov Logic Networks (MLNs) achieve this by attaching weights to formulas in first-order logic, and can be seen as templates for constructing features for ground Markov networks. Most techniques for learning weights of MLNs are domain-size agnostic, i.e., the size of the domain is not explicitly taken into account while learning the parameters of the model. This often results in extreme probabilities when testing on domain sizes different from those seen during training. In this paper, we propose *Domain Aware Markov Logic Networks* (DA-MLNs) which present a principled solution to this problem. While defining the ground network distribution, DA-MLNs divide the ground feature weight by a scaling factor which is a function of the number of connections the ground atoms appearing in the feature are involved in. We show that standard MLNs fall out as a special case of our formalism when this function evaluates to a constant equal to 1. Experiments on the benchmark Friends & Smokers domain show that our approach results in significantly higher accuracies compared to existing methods when testing on domains whose sizes different from those seen during training.

## Introduction

Markov Logic (Domingos and Lowd 2009) is a powerful Statistical Relational Learning (SRL) formalism, which represents the underlying domain using weighted first-order logic formulas. Markov Logic has been successfully applied to a number of problems including those in information extraction, NLP, social network analysis, robot mapping and computational biology (Domingos and Lowd 2009). Most existing applications of Markov logic implicitly assume that test data is similar in size to the training data, and hence, weights learned on the training data can naturally be used for prediction on the test data. But for many real world settings, this assumption may not hold true any more, simply because the high costs of annotation may require us to learn the model on a relatively small sized training data, while still requiring prediction on large test sizes.

To illustrate the problem, suppose we have an MLN with a single formula  $w : P(x, y) \Rightarrow Q(x)$ , where  $w$  is the weight

learned using some training data. If domain size of  $y$  increases in the test data, then for a particular grounding  $q$  of predicate  $Q$ , number of its neighbors (i.e., groundings of  $P$ ), also increase, and it can be shown that the combined effect of all the neighbors results in extreme marginal probability being assigned to  $q$ . Poole et al. (2014) characterized this behavior for some classes of MLNs, but they did not provide any solution to the problem. So the question is how to *transfer* the weights learned from training data of a certain size to the weights suitable for test data of a different size. Jain et al. (2010) proposed Adaptive Markov Logic Networks (AMLNs) in which weights are learned over multiple databases of different sizes, and these weights are then approximated using a linear combination of pre-defined set of basis functions. Their approach has mainly two limitations : (a) The basis functions (hence the weights) depend only on the size of the domain, whereas as described above, we would like to focus on the number of neighbors of ground atoms in a formula (b) They do not provide any strong (theoretical) justification for why their basis function approach should work. Further, in our experiments, we did not find their approach suited to benchmark problems even when compared to standard MLNs.

In this work, we propose a modified MLN formalism, called *Domain Aware Markov Logic Networks (DA-MLNs)*, in which weights are dynamically adapted to different domain sizes based on some of the ideas described above.

## Domain Aware Markov Logic Networks (DA-MLNs)

As described in the previous section, in Markov Logic, marginal probability of a grounding  $q$  of a first-order predicate  $Q$  tends to extreme as the cumulative effect of number of ground formulas in which  $q$  appears increases. A natural solution to this problem is to *scale down* the effect of each ground formula (in which  $q$  appears), so that even in very large domains, cumulative effect would not result in extreme probabilities. Intuitively, this scaling factor should depend on the number of connections the ground atoms in a formula are involved in. To formalize this notion, we first define number of connections of a first-order predicate.

**Definition 1. (NumConnections)** Let  $F$  be a first order formula containing predicates  $[P_1, P_2, \dots P_m]$ . Let  $Vars(P_j)$

be the set of logical variables appearing as arguments of  $P_j$ . Let  $Vars(P_j)^-$  denote the set of all the logical variables in  $F$  not appearing in  $P_j$ . Then, we define  $NumConnections$  of  $P_j$ , denoted by  $c_j$  as,  $\max\left(1, \prod_{x \in Vars(P_j)^-} |\Delta x|\right)$ , where  $\Delta x$  is the domain of variable  $x$ . Intuitively,  $c_j$  is the number of ground formulas which affect marginal probability of any instantiation of  $P_j$ .

Since each predicate in a formula can have different number of connections, we define a connection-vector of a first order formula as :

**Definition 2. (Connection-vector)** Let  $F$  be a first-order formula containing predicates  $[P_1, P_2, \dots, P_m]$ . Then connection-vector  $v$  is defined as  $[c_1, c_2, \dots, c_m]$  where each  $c_j (1 \leq j \leq m)$  denotes  $NumConnections$  for  $P_j$ .

**Example :** Consider again our example formula  $w : P(x, y) \Rightarrow Q(x), \Delta x = \Delta y = \{a, b\}$ . Then its connection vector  $v$  would be  $[1, 2]$ . Therefore, the connection-vector of a formula captures the number of connections for each of its predicates. Higher the number of connections in a formula, higher should be the scaling factor for that formula. We define our scaling factor as :

**Definition 3. (Scaling-down Factor)** Let  $F$  be a first-order logic formula. Let  $v$  be its connection vector. Then its scaling-down factor,  $s = \max(v)$ , where  $\max(v)$  returns maximum element of vector  $v$ .

Though we have chosen max function to capture the effect of number of connections in the definition above, other functions such as  $\sum$  could also be used. Empirically, we found max to perform well.

Next, we describe the probability distribution defined by the DA-MLN model. Given an assignment  $x$  to all the ground atoms  $\mathbf{X}$ , the probability of  $\mathbf{X} = x$  is given by:

$$P(\mathbf{X} = x; w) = \frac{1}{Z} \exp\left(\sum_{i=1}^n \frac{w_i}{s_i} n_i(x)\right)$$

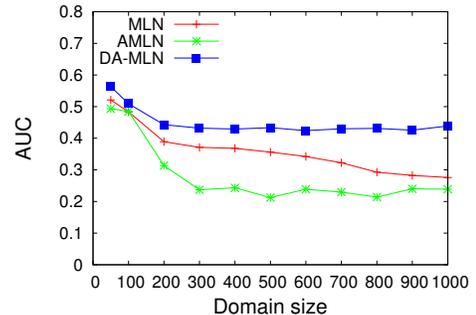
where  $w_i$ ,  $s_i$  and  $n_i$  denote the weight, scaling factor, and the number of true groundings of  $i^{th}$  formula, respectively. DA-MLNs differ in the way their probability distribution is defined. They subsume (become identical to) MLNs if we replace  $s_i$  by 1 for all the formulas. In our analysis, we have been able formally prove that at least for some simple class of formulas, DA-MLNs do not result in extreme marginal probabilities. We plan to publish these results in an extended conference version of the paper. Exploring connections with work on aggregators in relational models (Mehran Kazemi et al. 2017) is a direction for future work.

## Experiments

We implemented DA-MLNs on top of the existing Alchemy (Kok et al. 2008) system. For AMLNs (Jain, Barthels, and Beetz 2010), the weights learned using Alchemy were used to learn the coefficients of the basis functions. For MLNs, we used the default Alchemy implementation. For each of the models, CG was used as the learning algorithm, and Gibbs sampling was used for inference. In each case, default parameter settings were used.

We compared each of the three approaches on the standard Friends & Smokers (FS) domain (Singla and Domingos 2008). The domain has two rules: smoking leads to cancer and friends have similar smoking habits (along with singleton for each predicate). Since we would like to predict on varying domain sizes, we generate the data with domain size of  $n$  as follows: we randomly create  $\sqrt{n}$  number of (equal-sized) groups, such that people in each group are more likely to be friends with each other ( $p_f = 0.8$ ), whereas people across groups are less likely to be friends with each other ( $p_f = 0.1$ ). Each group is randomly decided to be a smoking group with probability  $p_g = 0.3$ . In a smoking group, each person smokes with probability of  $p_s = 0.7$ , and in a non-smoking group, each person smokes with probability  $p_s = 0.1$ . A smoking person has cancer with probability  $p_c = 0.5$ , and a non-smoking person has cancer with probability  $p_c = 0.01$ .

We used a set of randomly generated datasets with domain sizes 20, 40, 60, 80, 100 for learning. For inference, we used randomly generated datasets of sizes varying from 50 to 1000. All the groundings of the *Friends* predicate and randomly chosen 50% groundings of *Smokes* were set as evidence during inference. Figure 1a plots the Area under the Precision-Recall curve (AUC) as we vary the test data sizes. While for smaller domain sizes all the algorithms perform equally well, performance of AMLNs and MLNs drops drastically as the domain size increases. In contrast, DA-MLNs see much less drop in performance with increasing domain size, and perform significantly better than the competitors on larger domains. Test set log-likelihood shows a similar trend (omitted due to lack of space). We have also done some experiments on another real world dataset, and results are quite promising. We plan to publish these results in an extended conference version of the paper.



(a) Domain Size vs AUC (FS)

Figure 1: Results on FS dataset

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