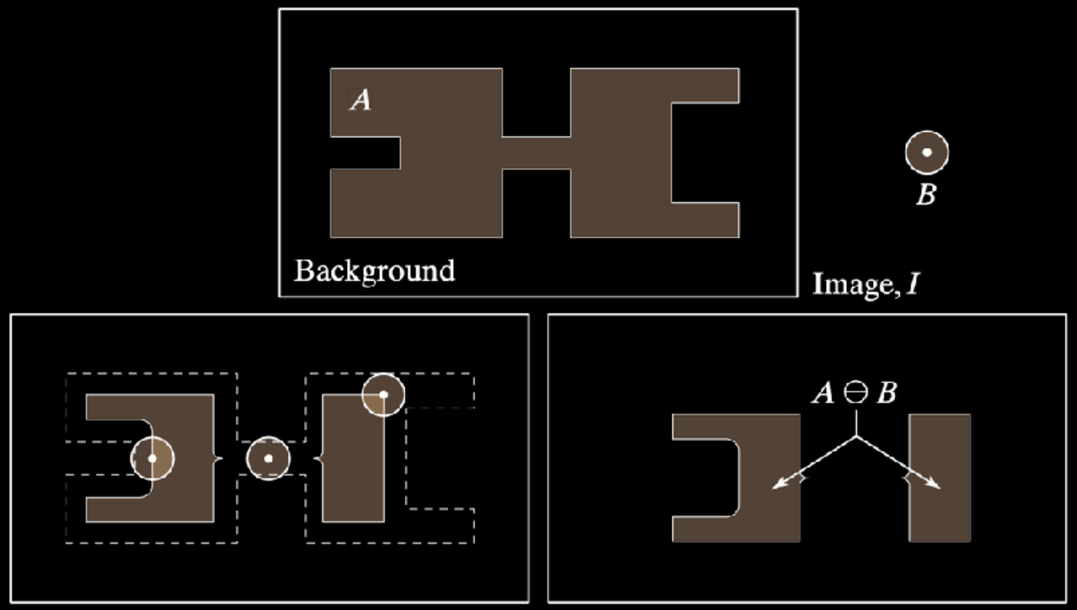
COL783: Digital Image Analysis

21. Morphological Algorithms

Recap: Erosion and dilation

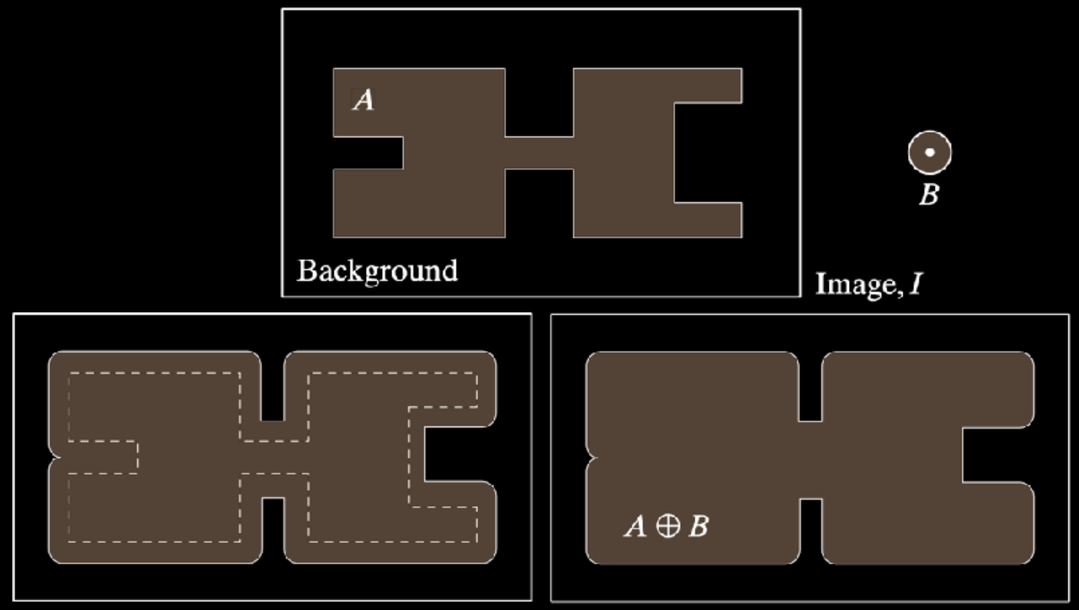
$$A \ominus B = \{z : (B)_z \subseteq A\} = \bigcap_{b \in B} (A)_{-b}$$

$$(A \ominus B)^c = A^c \oplus \hat{B}$$



$$A \oplus B = \{z : (\hat{B})_z \cap A \neq \varnothing\} = \bigcup_{b \in B} (A)_b$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$



Opening and closing

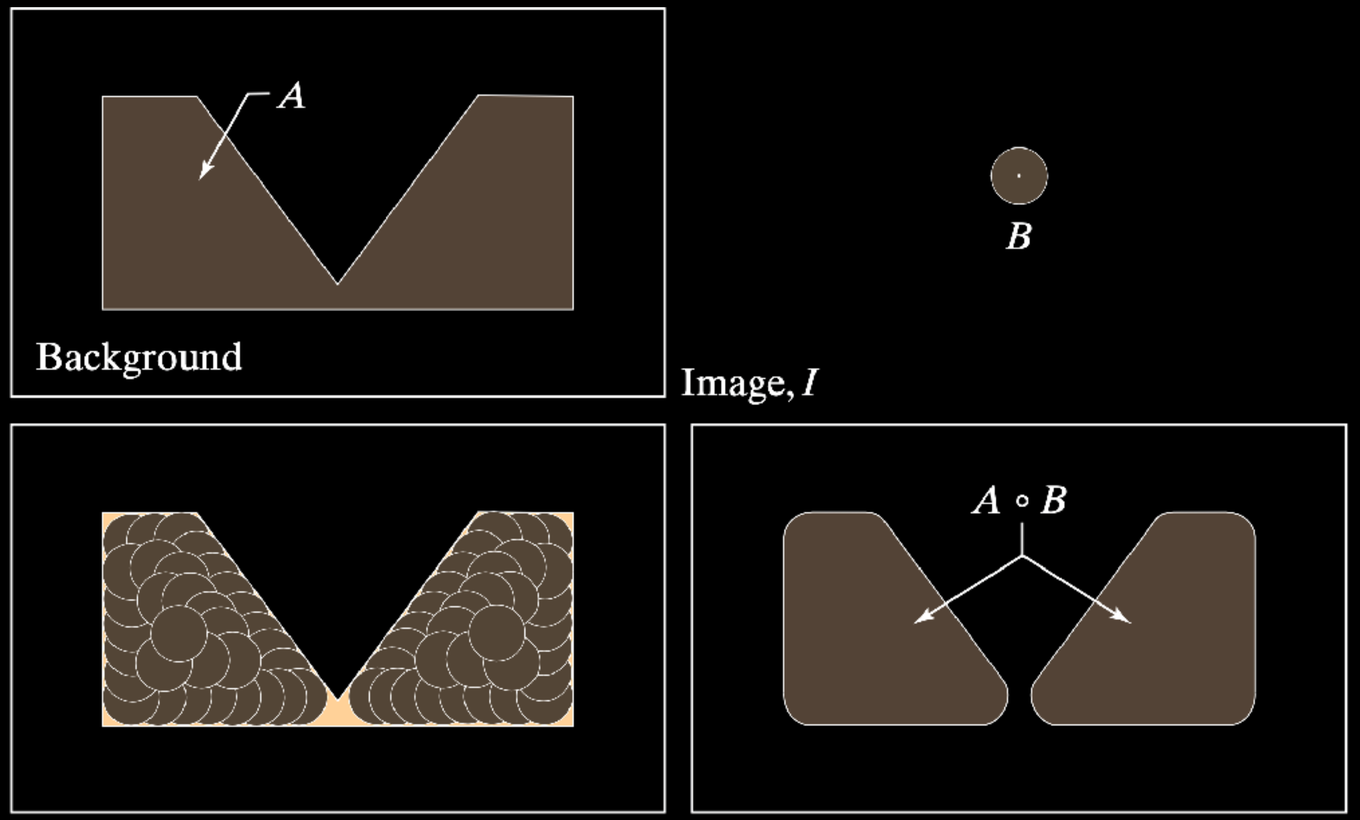
Erosion and dilation are duals but not inverses! $(A \ominus B) \oplus B \neq A \neq (A \oplus B) \ominus B$

Opening: $A \circ B = (A \ominus B) \oplus B$

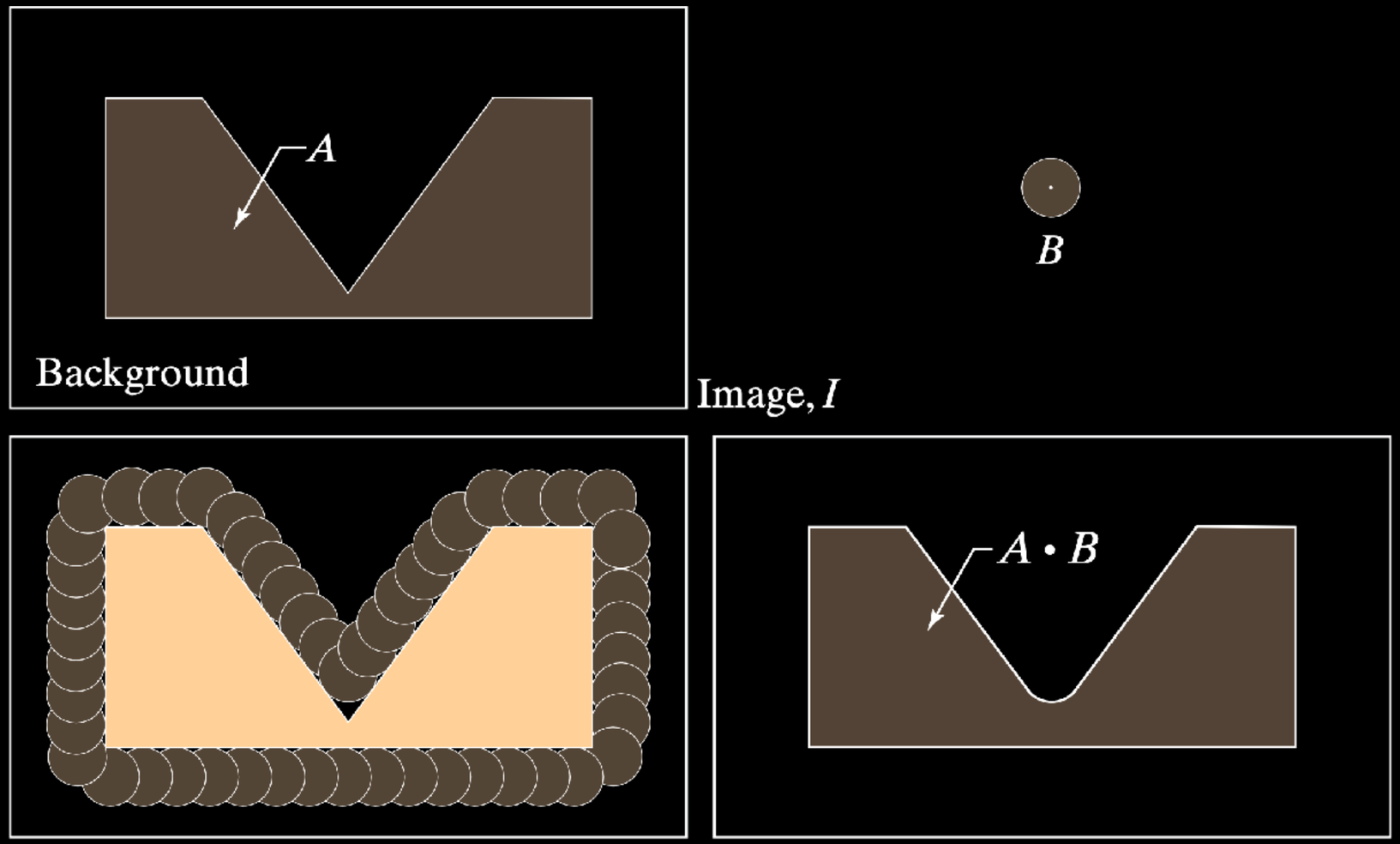
Closing: $A \bullet B = (A \oplus B) \ominus B$

$A \ominus B$ = set of locations where B fits entirely inside A

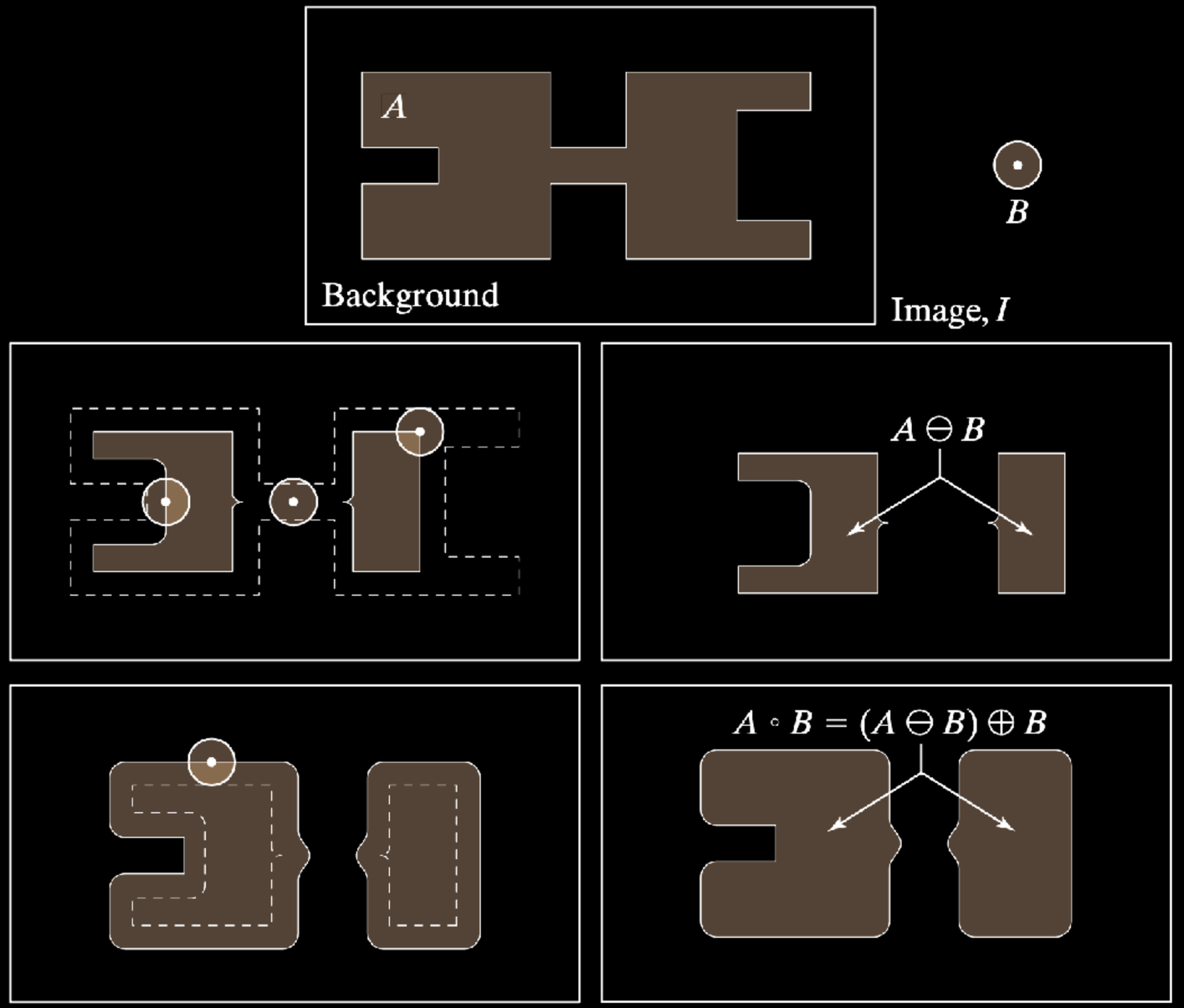
$A \circ B$ = union of B's placed at locations in $A \ominus B$
= union of B's that fit entirely in A



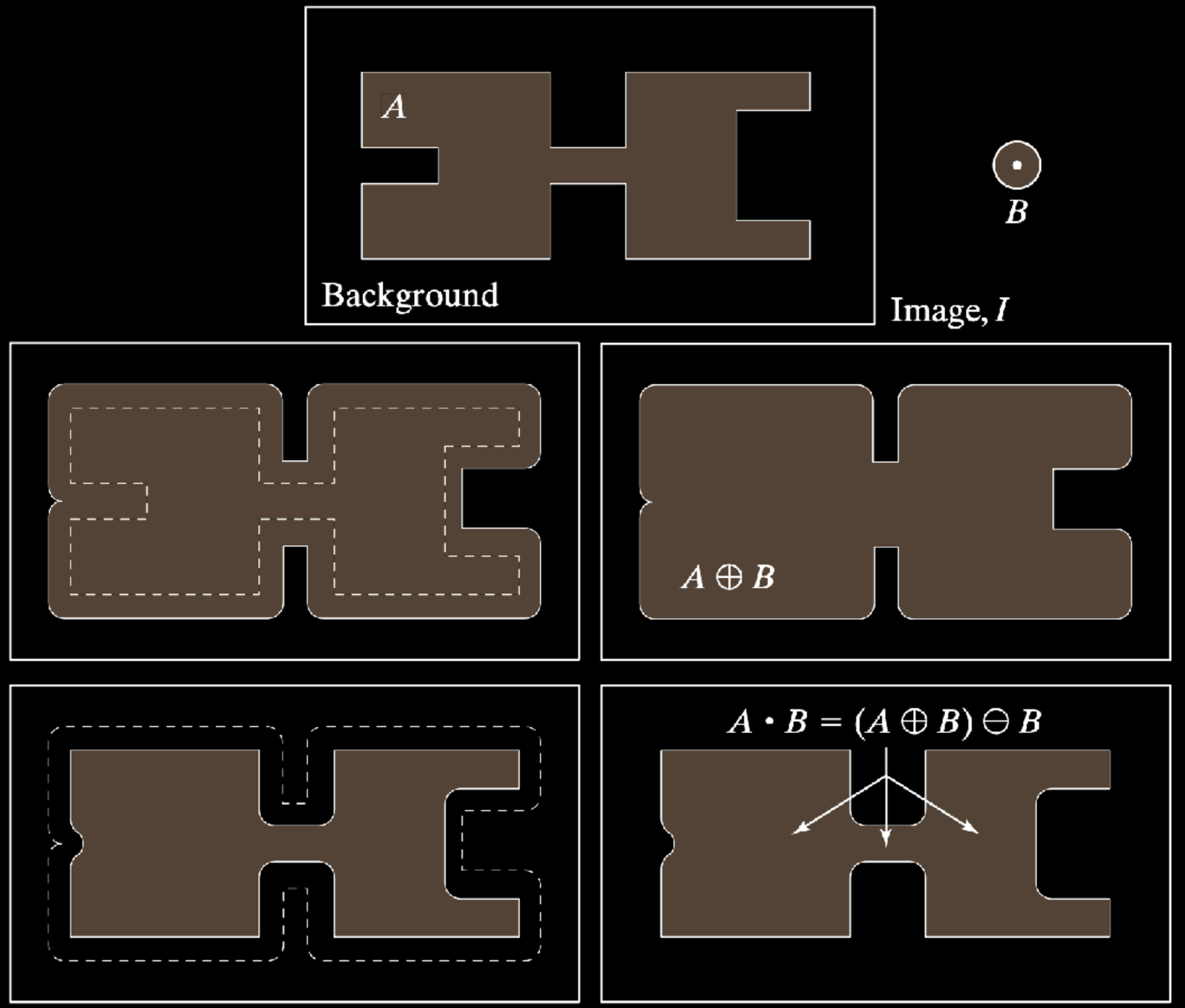
By duality, $A \bullet B$ = *complement* of union of $(B)_z$'s that fit entirely inside A^c i.e. lie entirely *outside* A



Opening removes small components and thin parts



Closing removes small holes and thin gaps



Original image A

$A \ominus B$

$A \circ B$

Example: Noise removal in binary images

$(A \circ B) \oplus B$

$(A \circ B) \bullet B$

Duality: complement of opening = closing of complement

$$(A \circ B)^c = A^c \bullet \hat{B}, \qquad (A \bullet B)^c = A^c \circ \hat{B}$$

(Anti-)extensivity: opening ⊆ original shape ⊆ closing

$$A \circ B \subseteq A, \qquad A \subseteq A \bullet B$$

Monotonicity: opening/closing of subset is subset of opening/closing

$$C \subseteq D \implies C \circ B \subseteq D \circ B, \qquad C \bullet B \subseteq D \bullet B$$

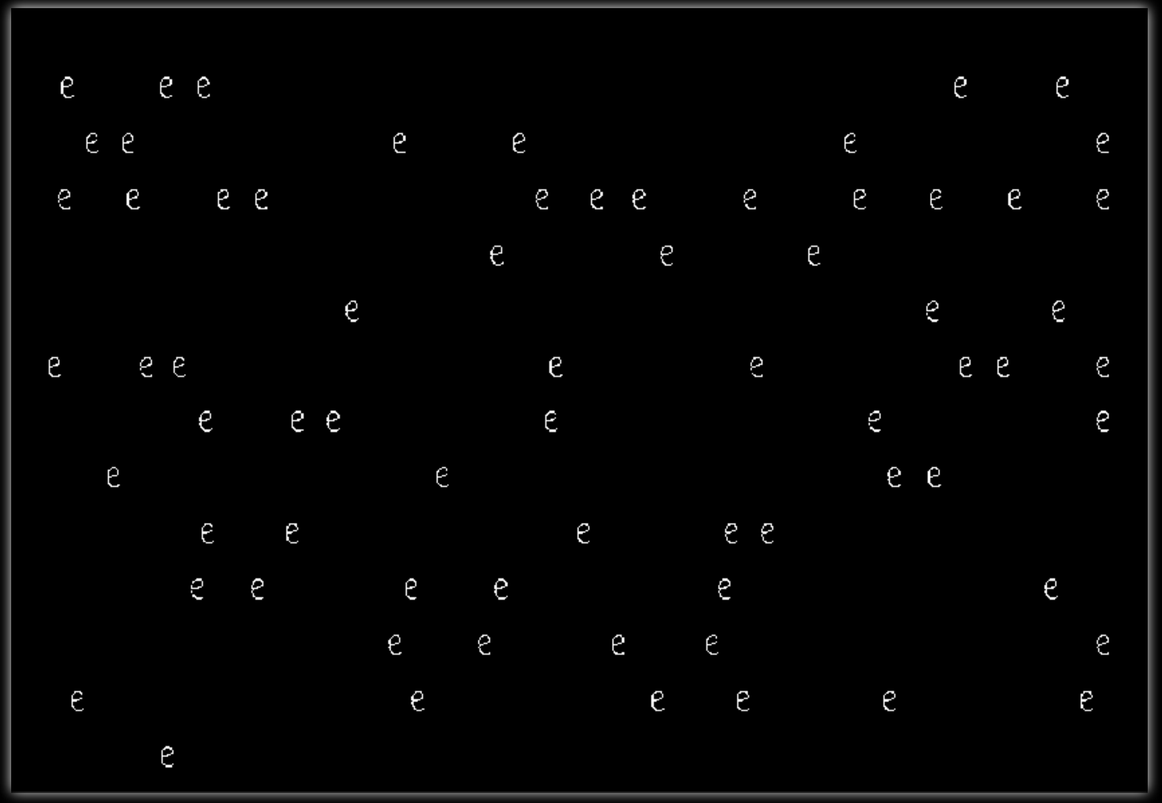
Idempotence: opening/closing again has no effect

$$(A \circ B) \circ B = A \circ B, \qquad (A \bullet B) \bullet B = A \bullet B$$

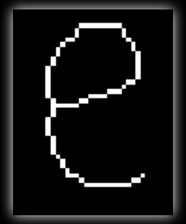
Example: letter detection

INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].



A



$(A \ominus B) \oplus B$

Example: letter detection

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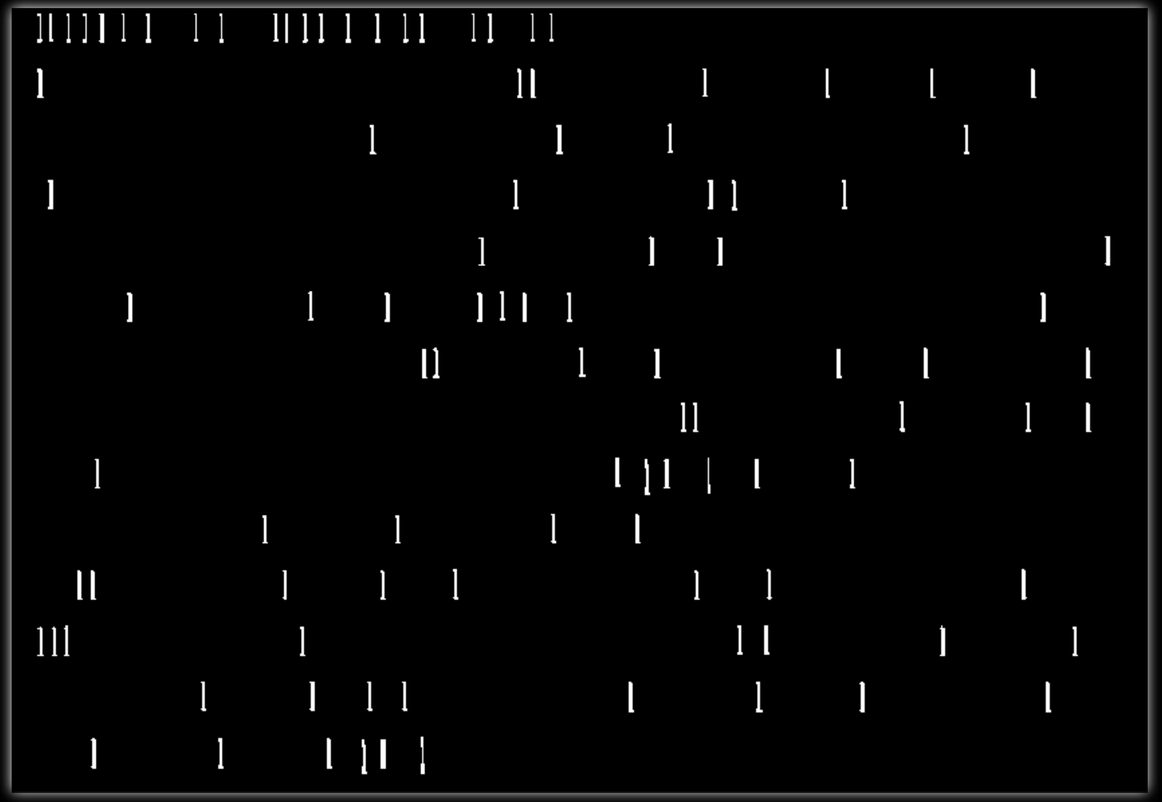

A

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Example: letter detection

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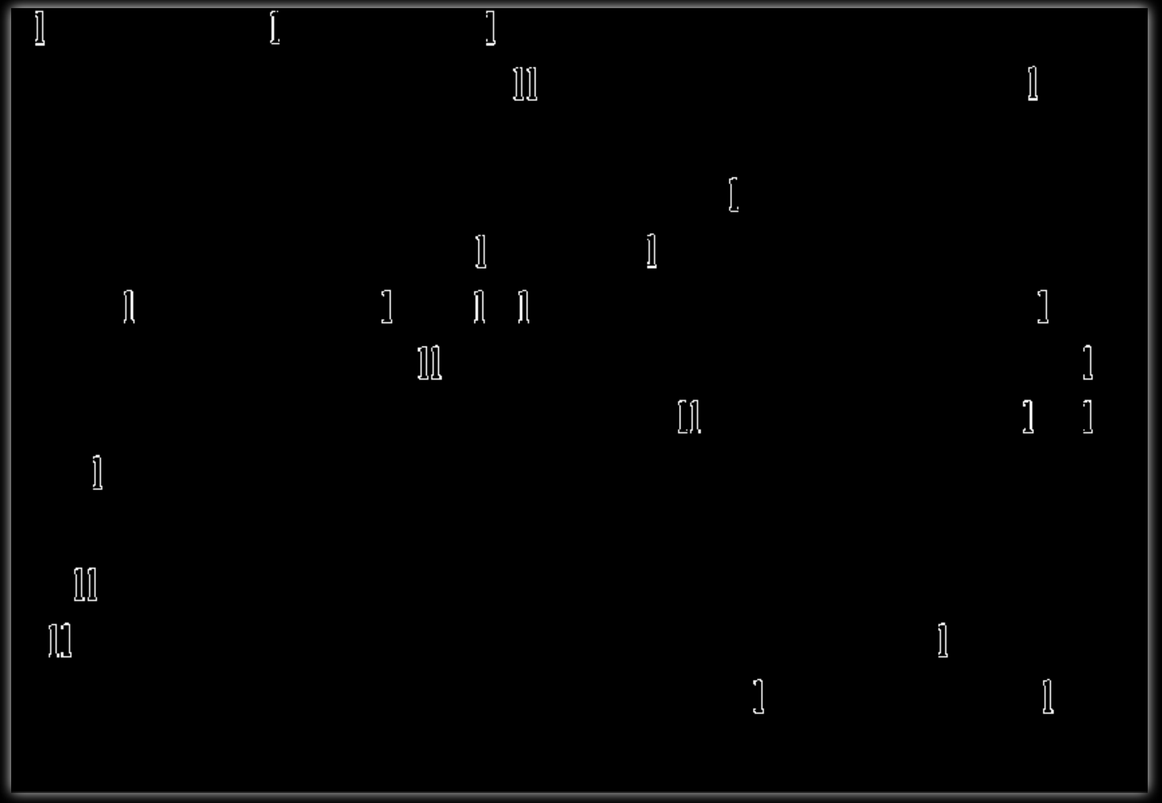


A

$\big((A \ominus B_1) \cap (A^c \ominus B_2)\big) \oplus B_2$

The hit-or-miss transform

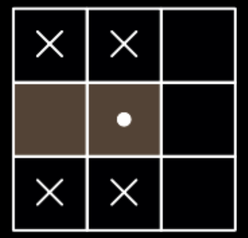
Select pixels with specified foreground *and* background

$$A \circledast B_{1,2} = \{z : (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c\}$$

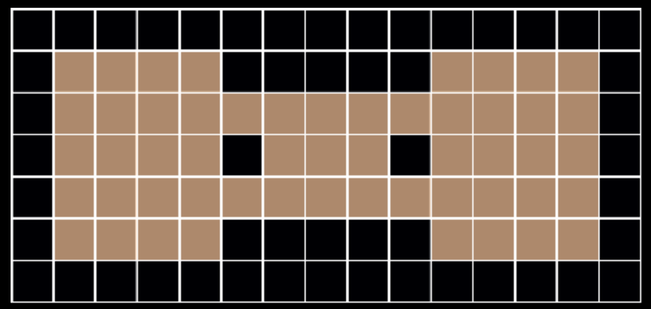
$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

Can rewrite with a single B:

- Foreground pixels in $(B)_z$ should be in A
- Background pixels in $(B)_z$ should be in A^c
- Don’t care pixels in $(B)_z$ can be anything

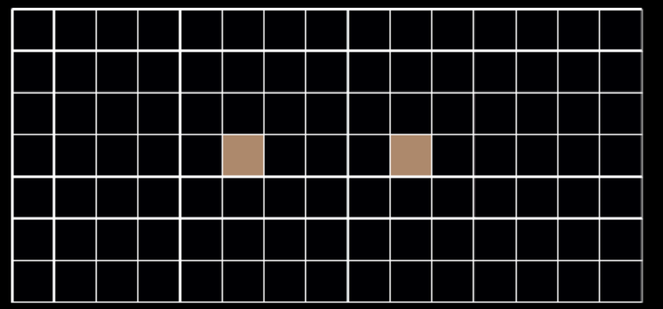


B

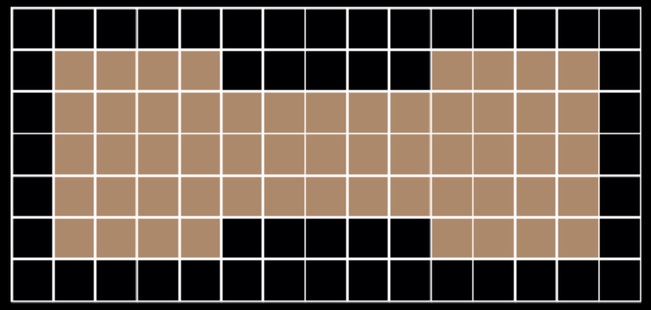


Image, I

B

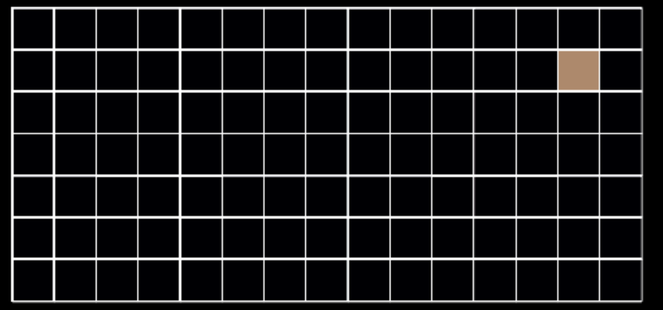


Image, $I \circledast B$

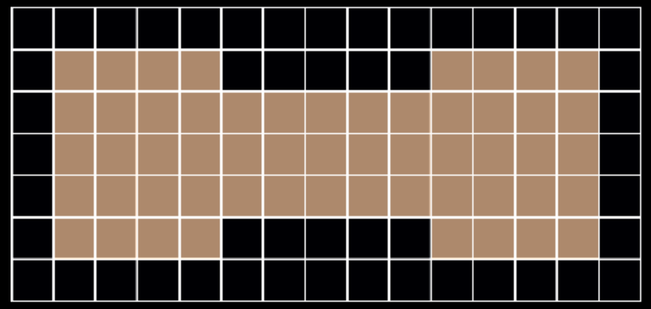


Image, I

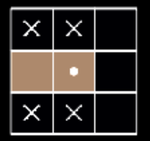
B



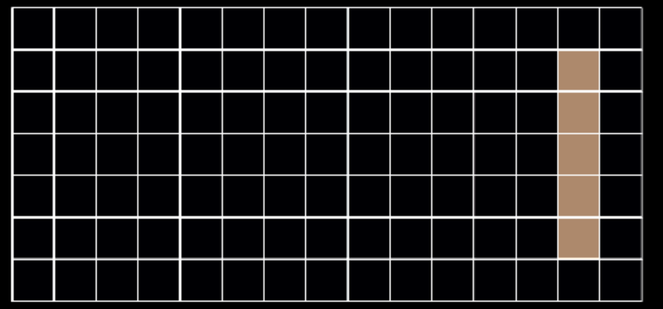
Image, $I \circledast B$



Image, I



B



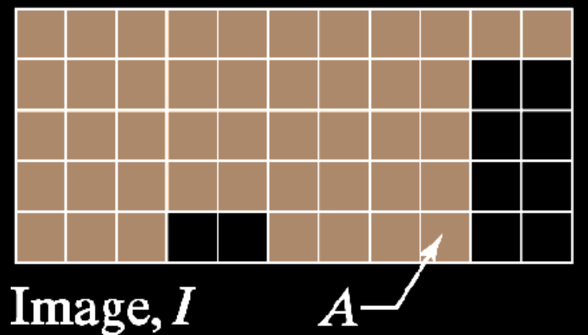
Image, $I \circledast B$

Thinning

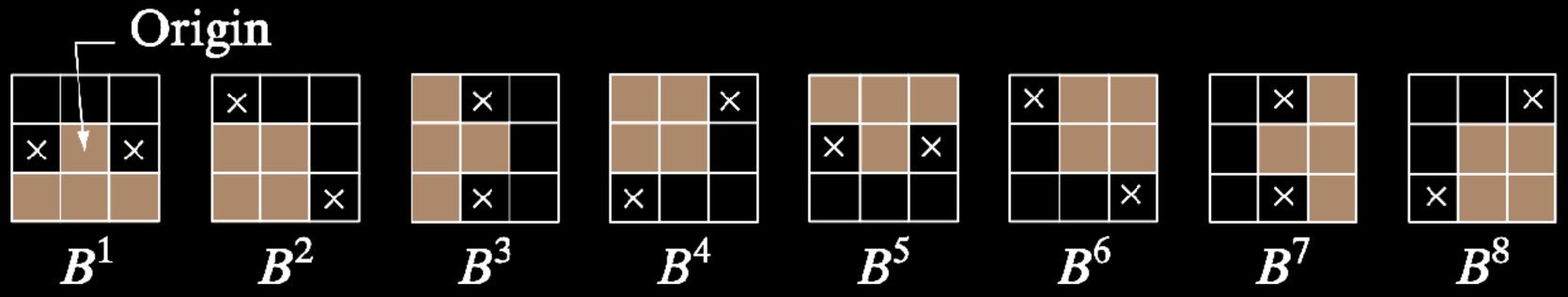
Remove selected foreground pixels: $A \otimes B = A \backslash (A \circledast B)$

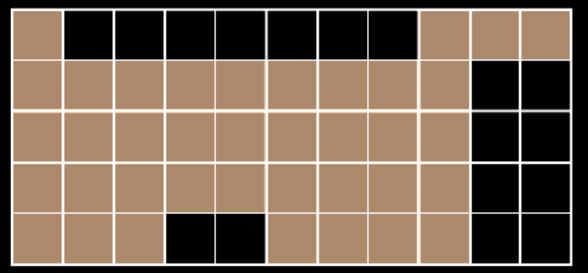
Usually need various structuring elements $\{B^1, B^2, \ldots, B^n\}$

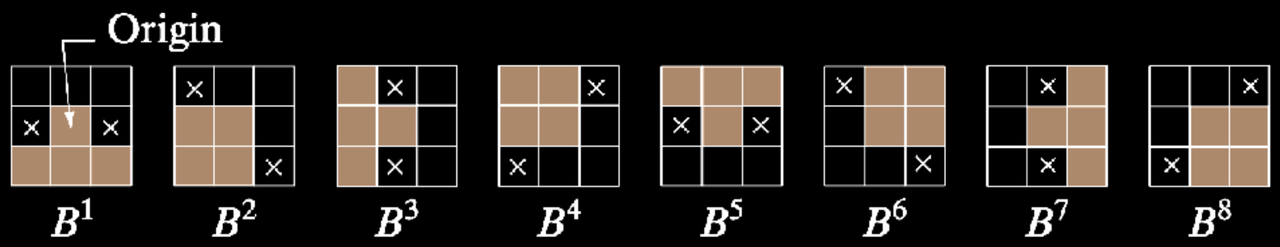
$$A \otimes \{B^1, B^2, \ldots, B^n\} = A \otimes B^1 \otimes B^2 \otimes \cdots \otimes B^n$$

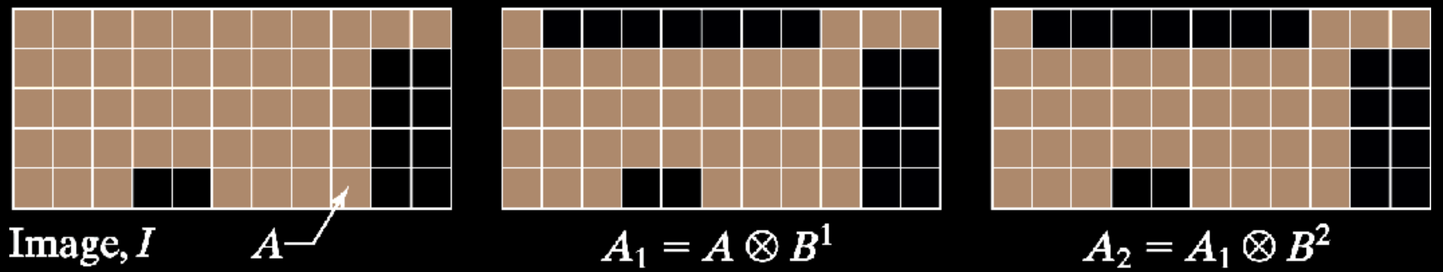


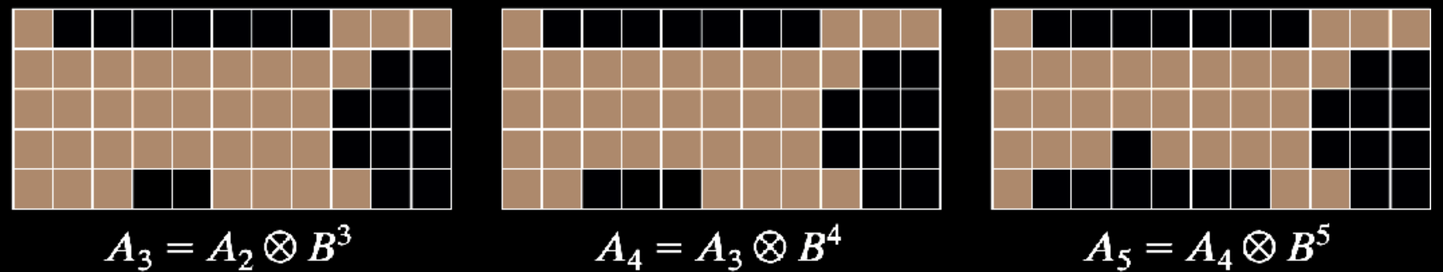
Example:

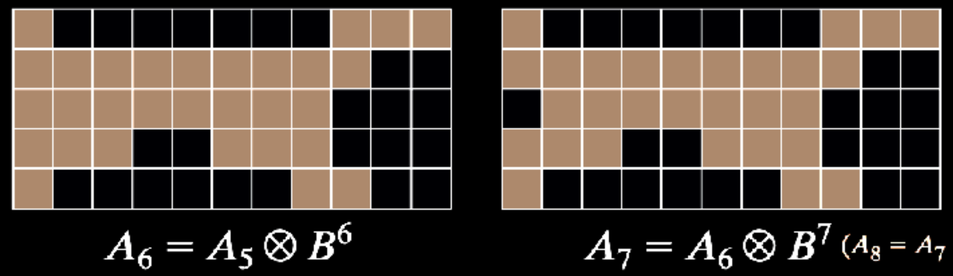




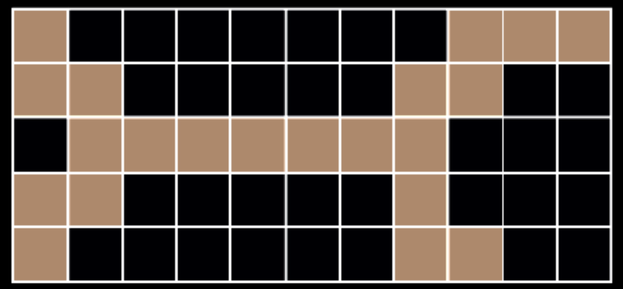








Thinning passes are repeated until convergence:



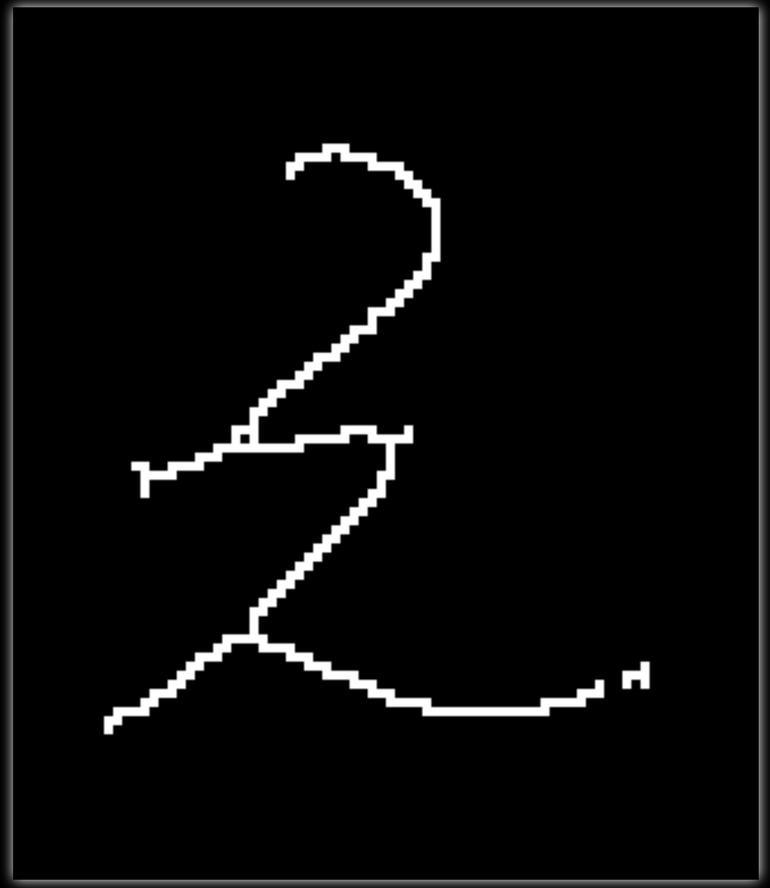
$A_{14} = A_{13} \otimes B^6$

No more changes after this.

Example: Thinning of handwritten stroke

Original image

Thresholding + dilation

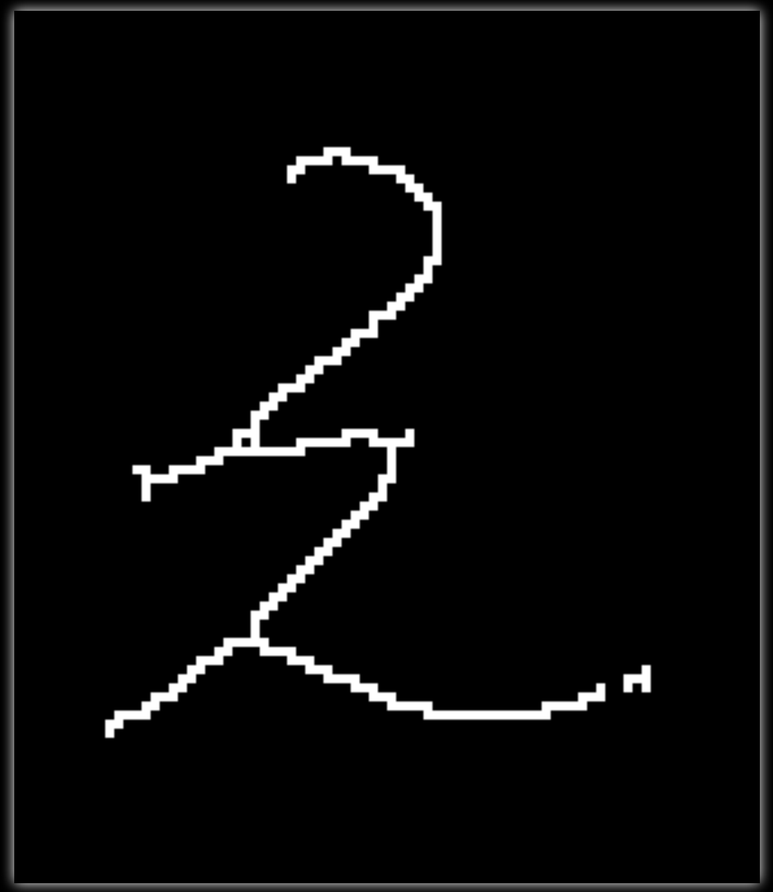


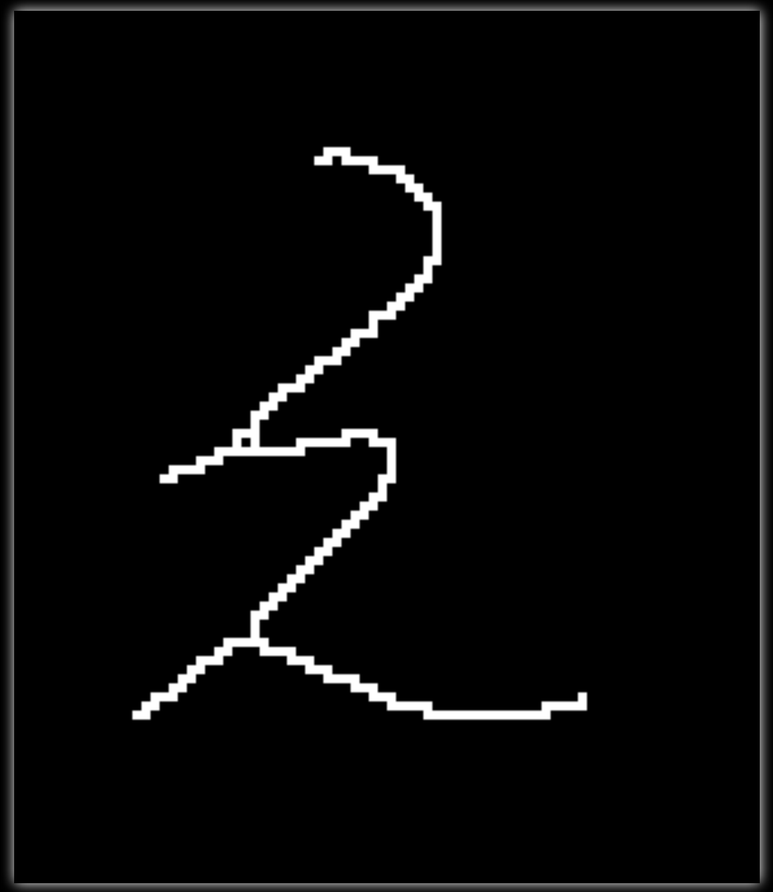
Thinning

Example: Pruning

0	0	0
0	1	0
0		

0	0	0
0	1	0
		0





Thickening

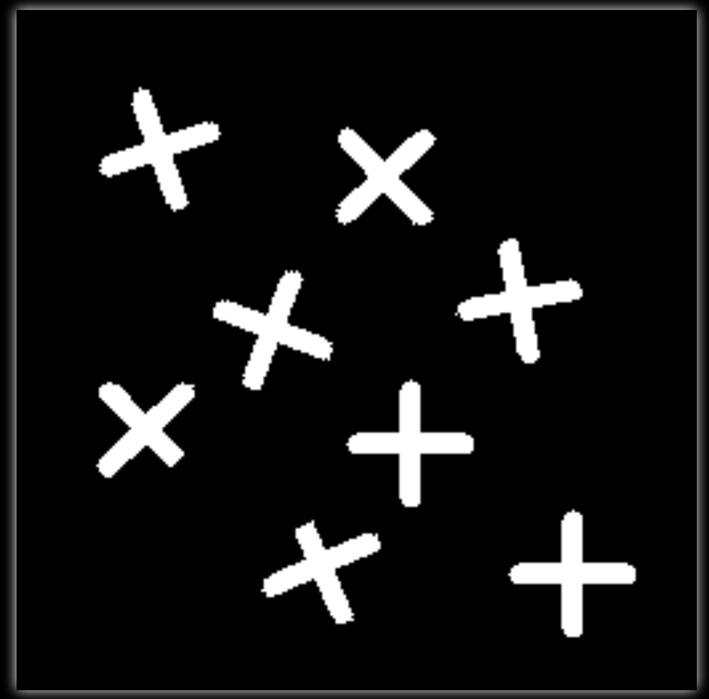
Dual of thinning: *add* selected *background* pixels

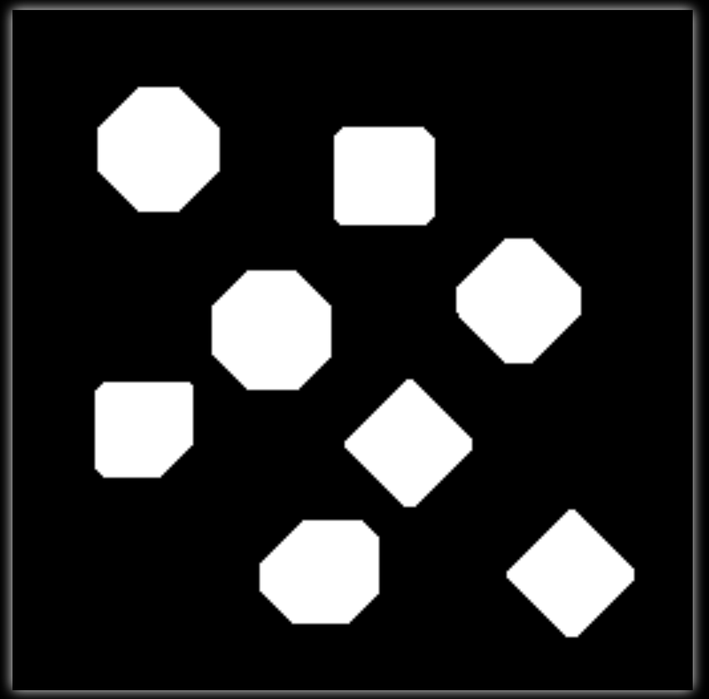
$$A \odot B = A \cup (A \circledast B)$$

Example: convex hull (45° approximation)

1	1	
1	0	
1		0

	1	1
	0	1
0		1

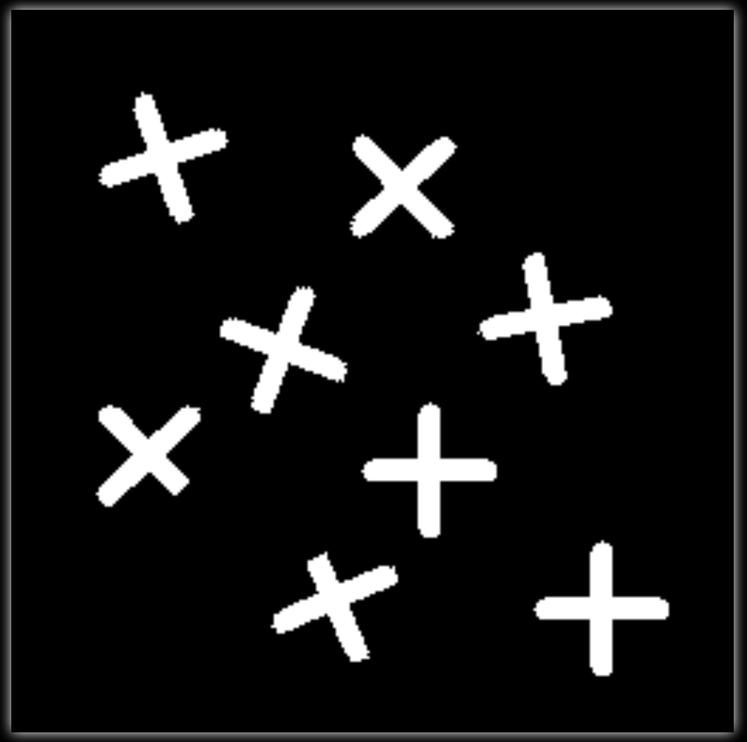




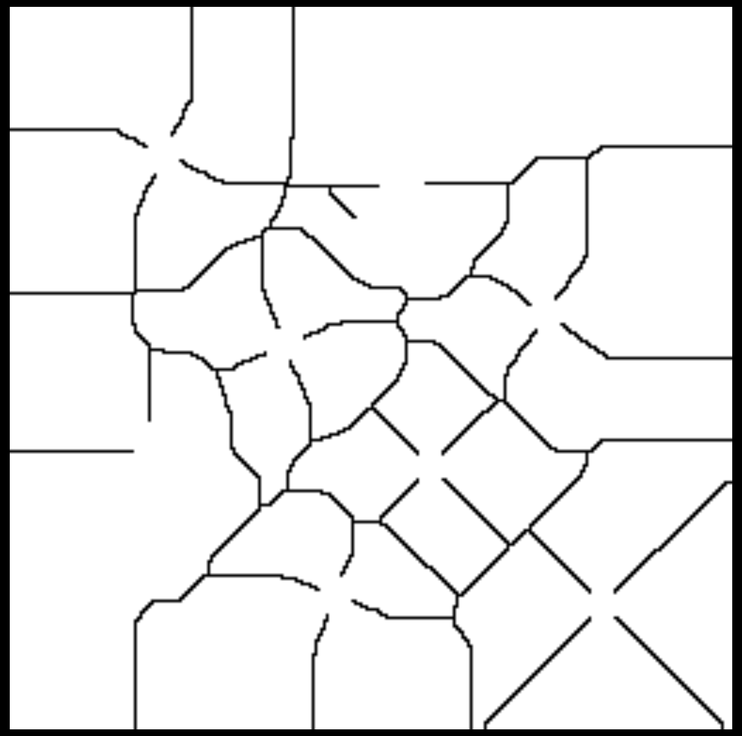
Example: 45° Voronoi diagram

0	0	0
	0	
1	1	1

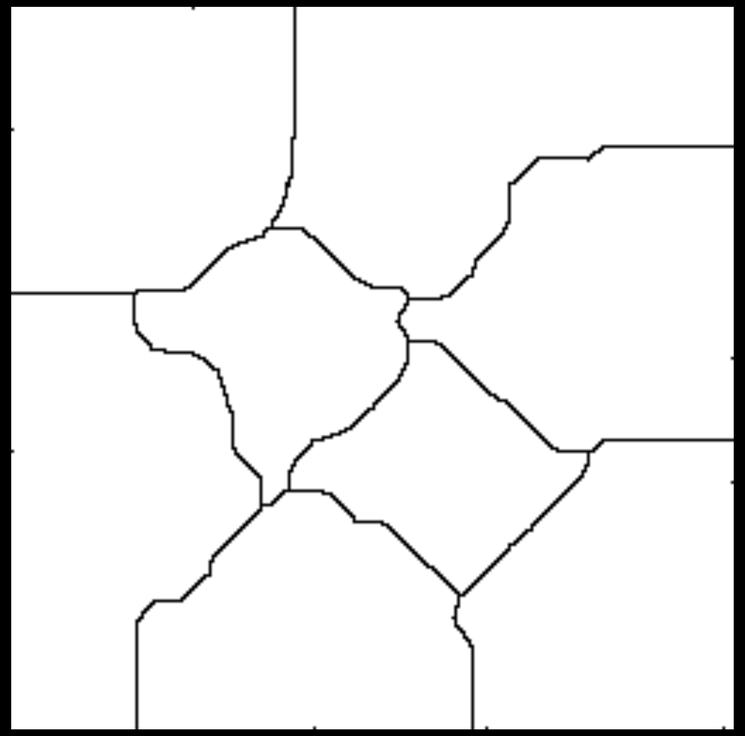
	0	
1	0	0
1	1	



Original



After thickening



After pruning background

Homework

1. Read DIP book Sec. 9.3–9.5 except “Morphological Reconstruction”
2. Start working on finishing Assignment 3
   (Only one question in second part, but its subquestions are plenty of work!)