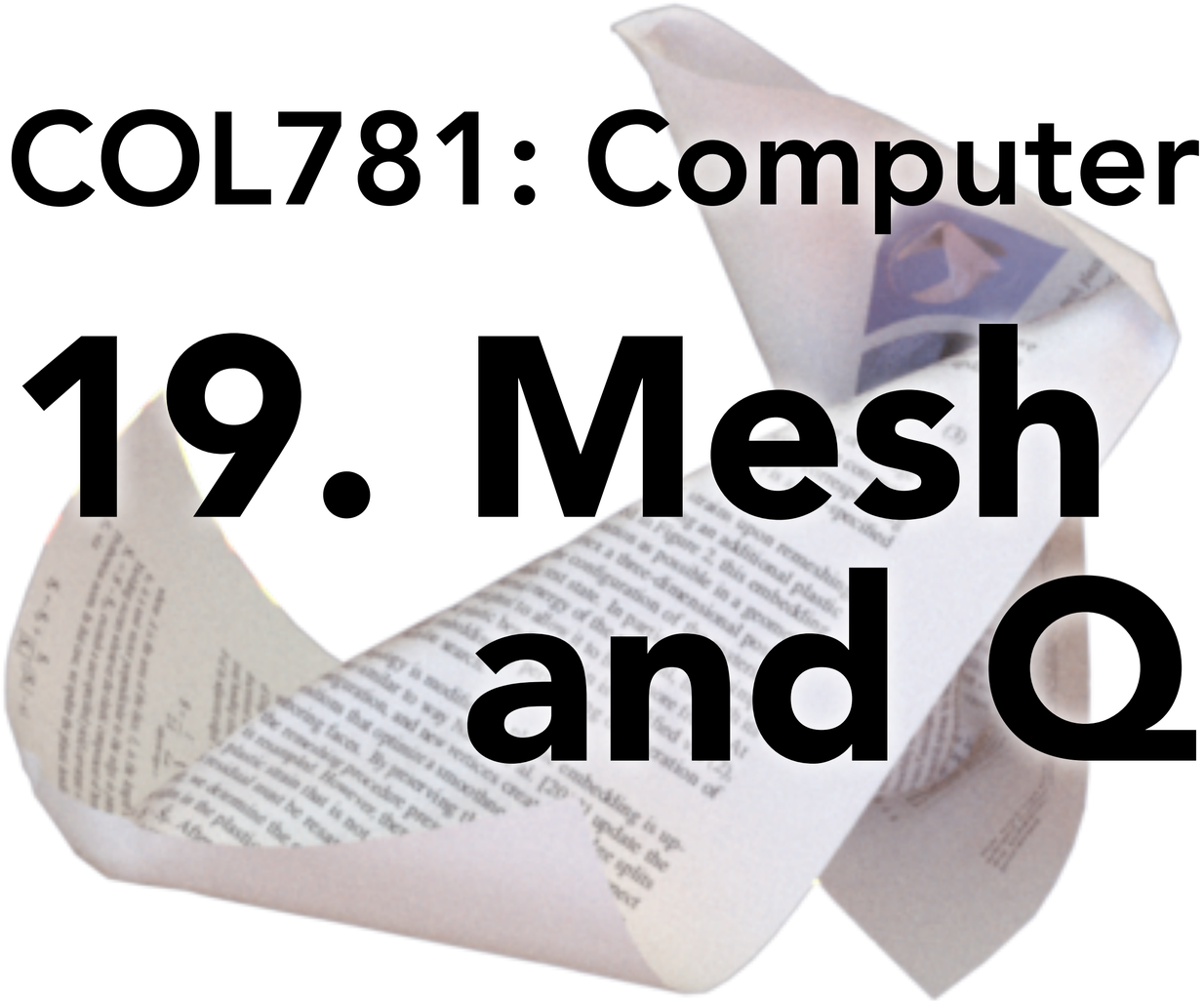
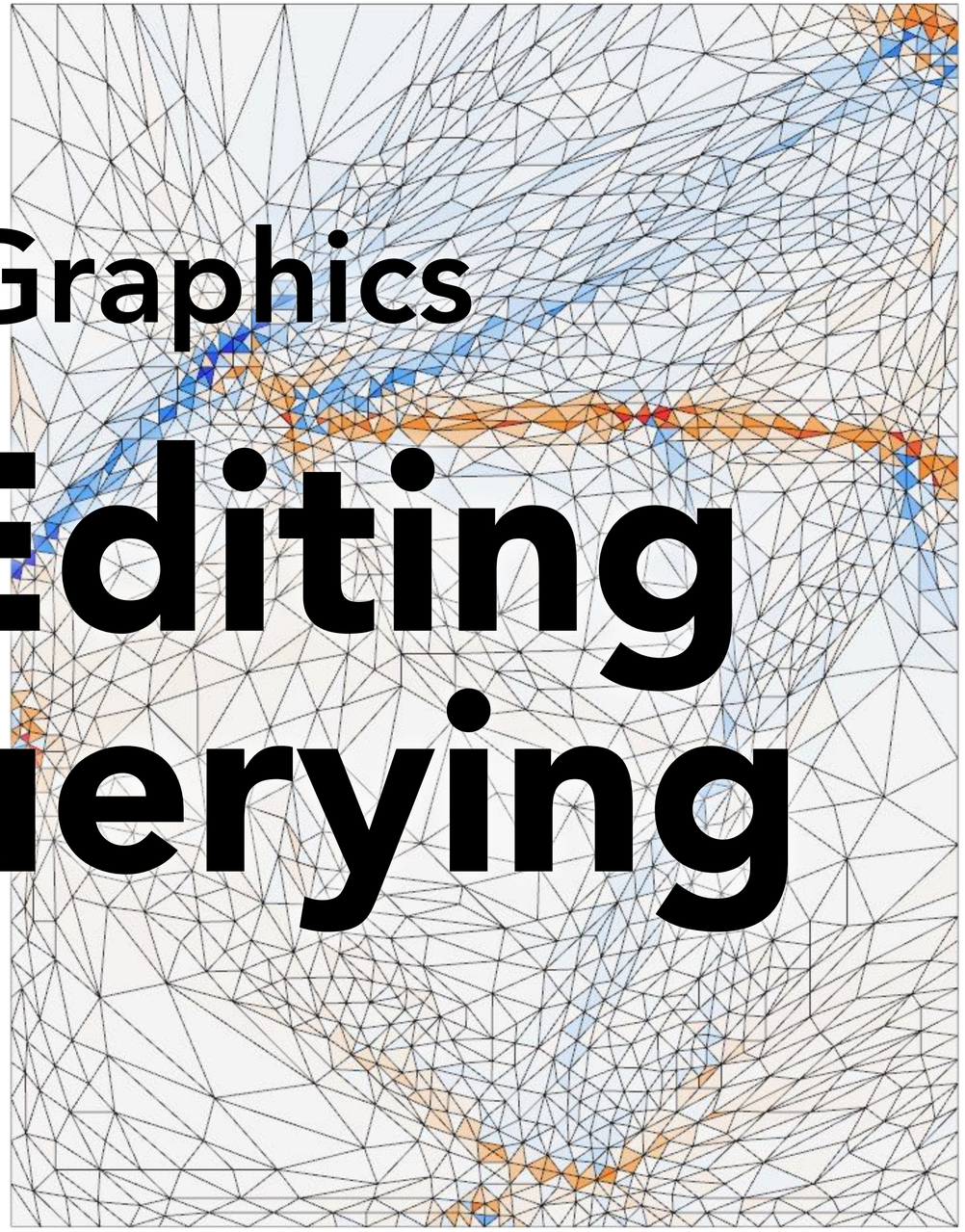
COL781: Computer Graphics

19. Mesh Editing and Querying

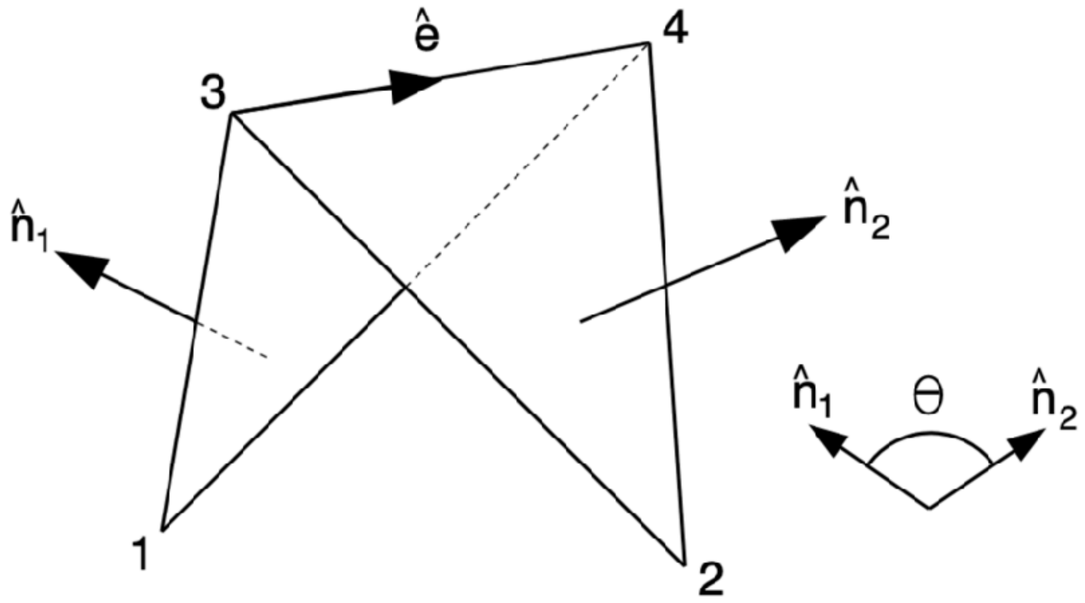


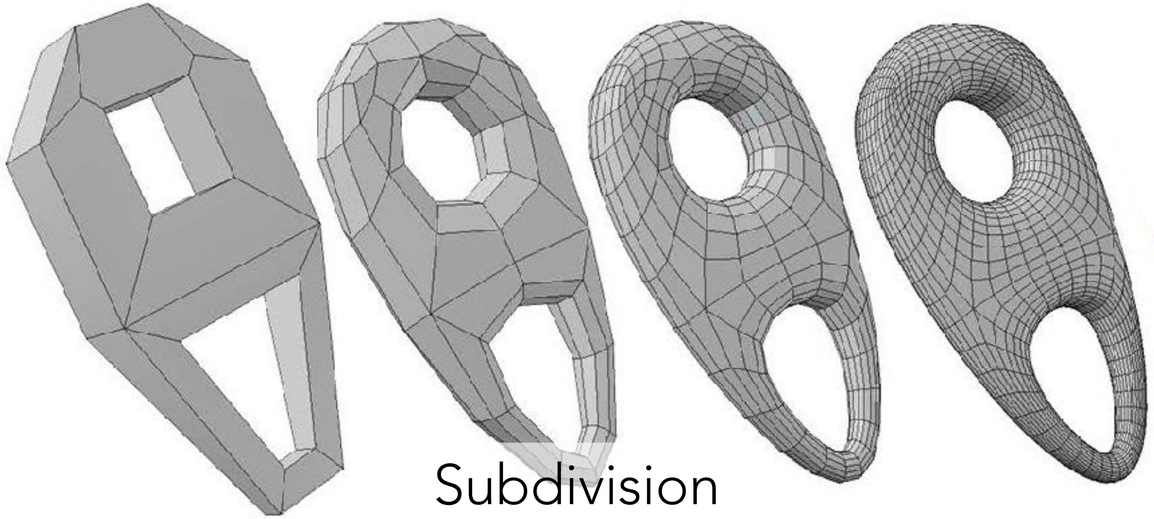


Practice problem

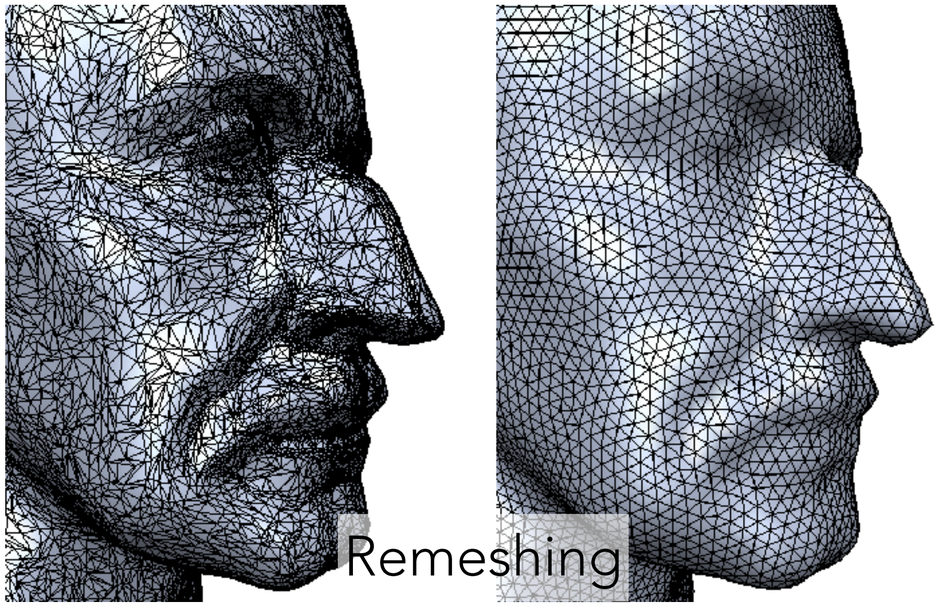
Using a half-edge representation of a triangle mesh, write (pseudo)code to find the "bending angle" at a given edge, i.e. the angle between the normals of the adjacent faces.

This is 180° minus the better-known **dihedral angle**.

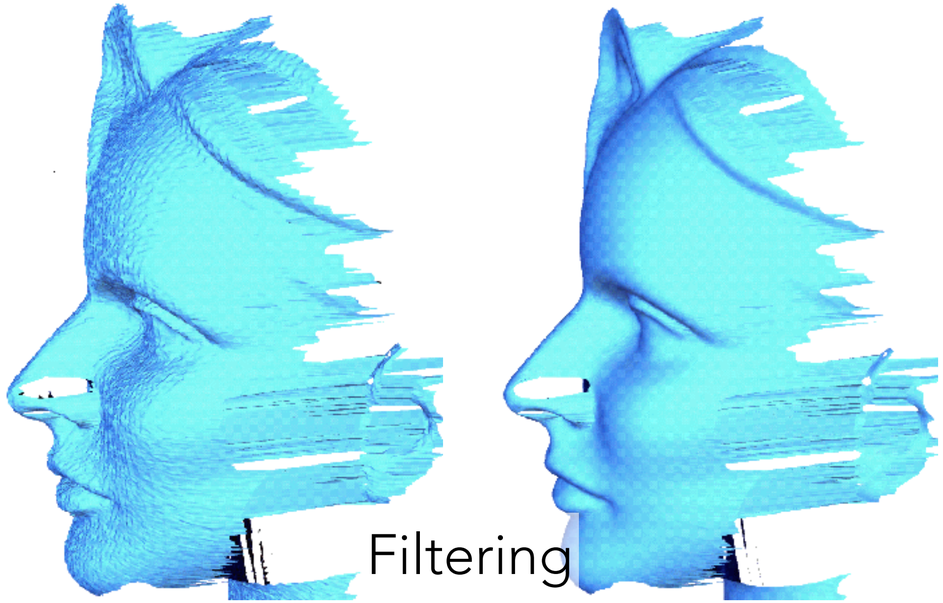




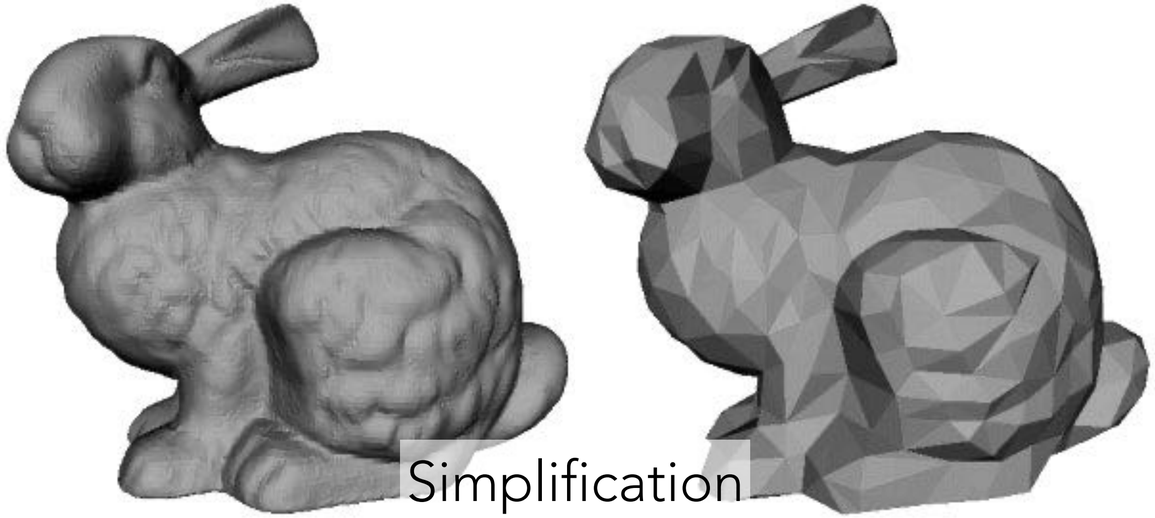
Subdivision



Remeshing



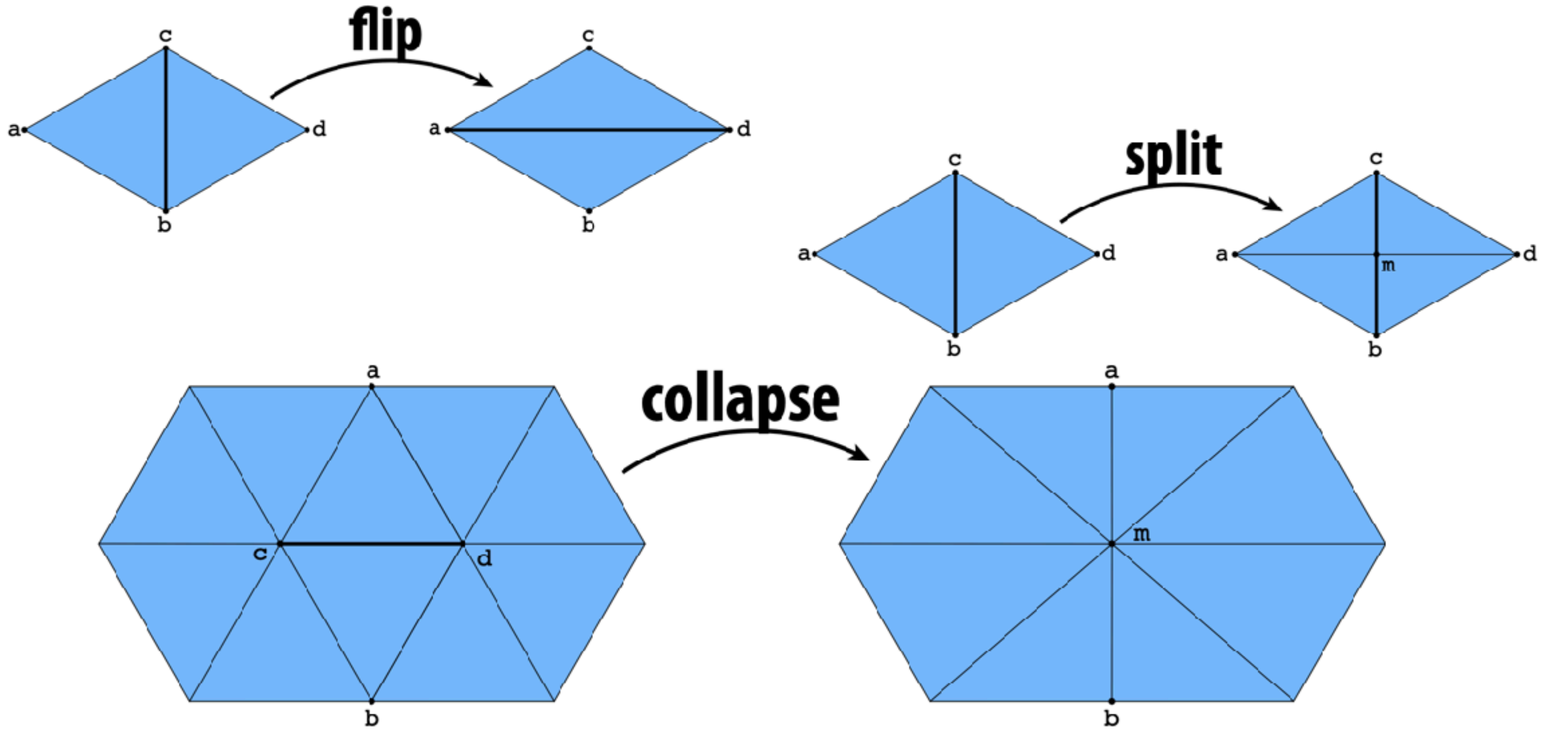
Filtering



Simplification

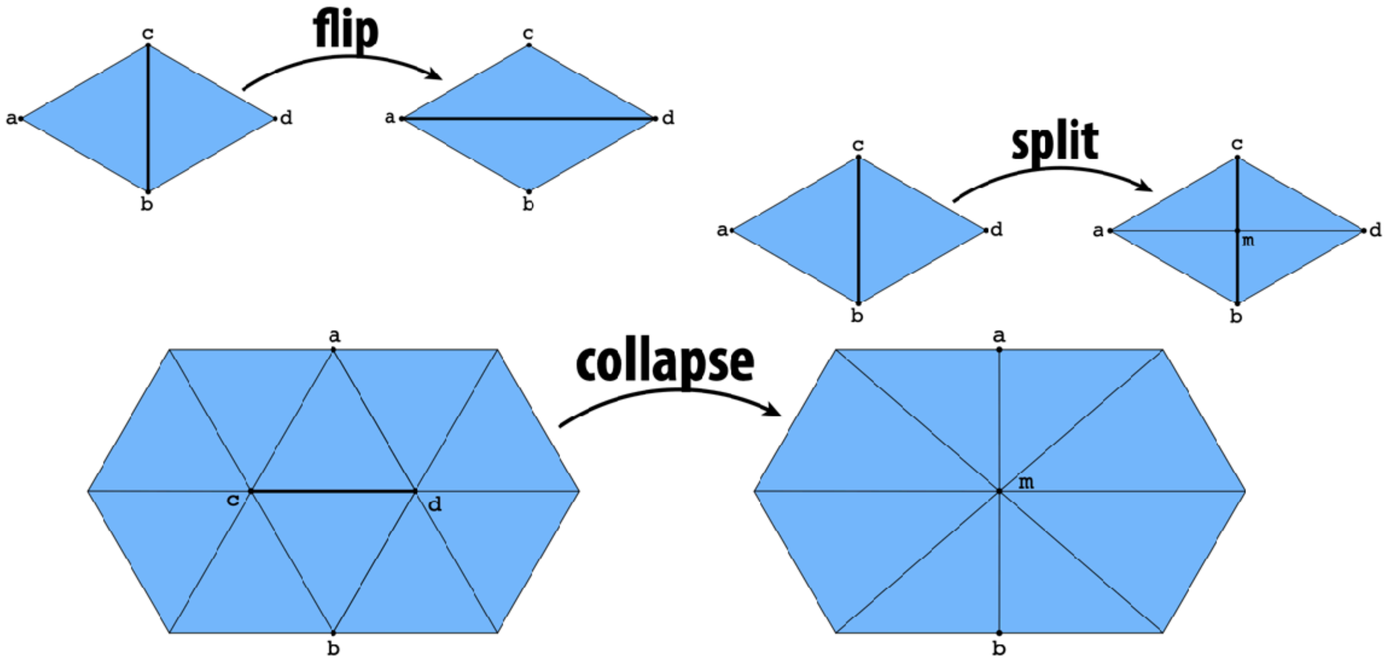
Mesh editing

Atomic mesh operations that preserve surface topology (i.e. don’t create holes or new connections)



How to implement:

- Insert/delete elements
- Reassign pointers (carefully!) to maintain connectivity data

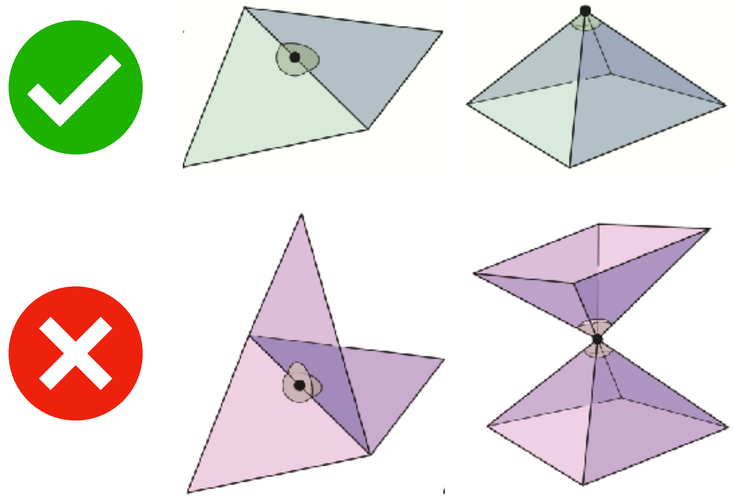


Need to assign positions, texture coordinates, etc. of new vertices

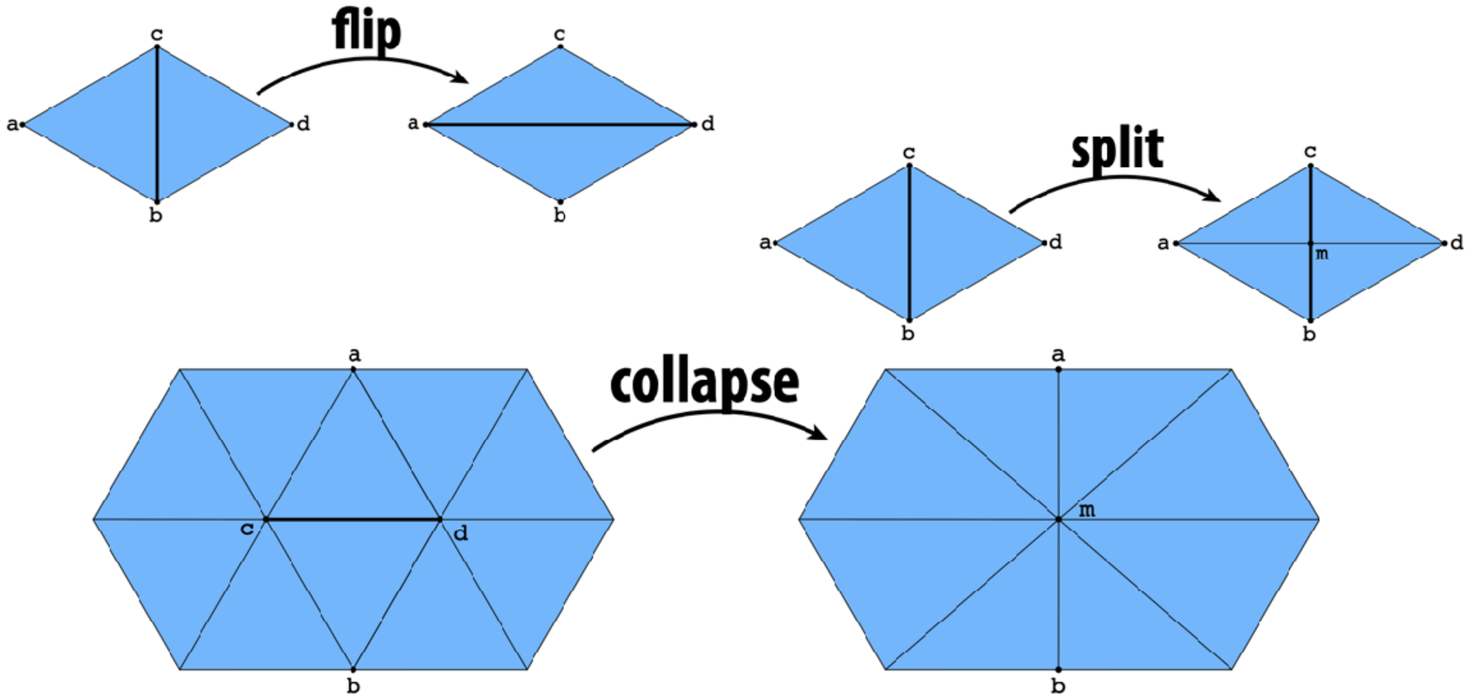
Other data, e.g. normals, of all neighbours may need to be recomputed

Recall the manifold-ness criteria:

- Every edge has exactly 2 adjacent faces
- Every vertex has adjacent faces and edges in a single ring



Do these operations always maintain them?

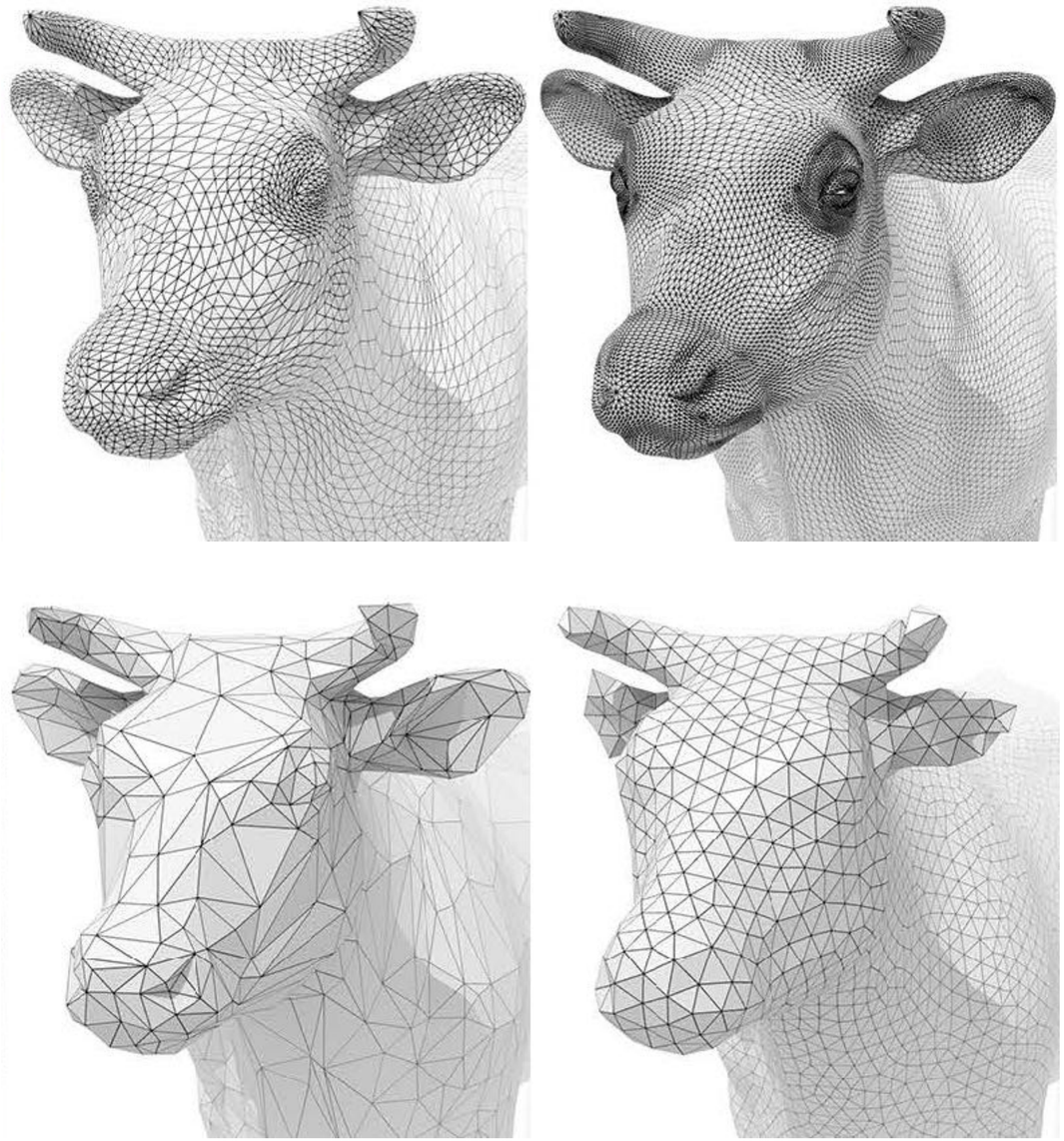


Most high-level mesh editing operations are implemented in terms of these!

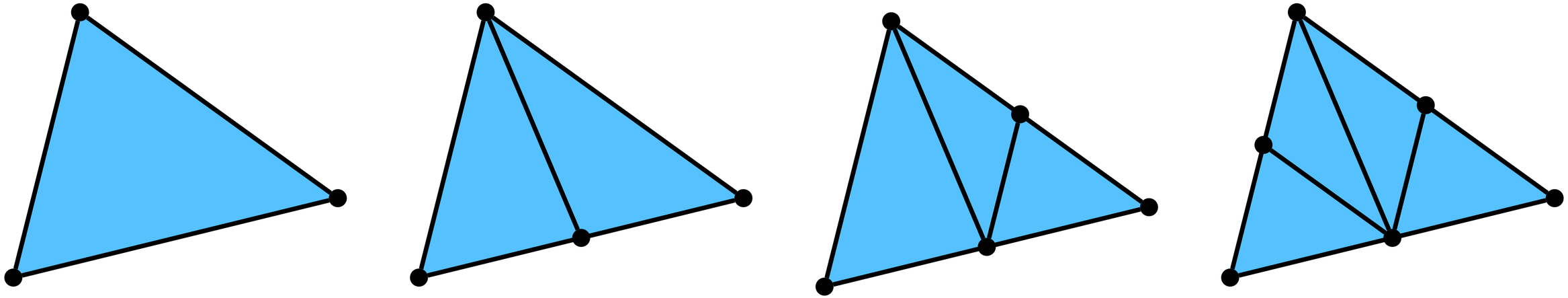
Oversimplified summary:

- **Subdivision:** split all edges
- **Simplification:** collapse unimportant edges
- **Regularization:** split long edges, collapse short edges

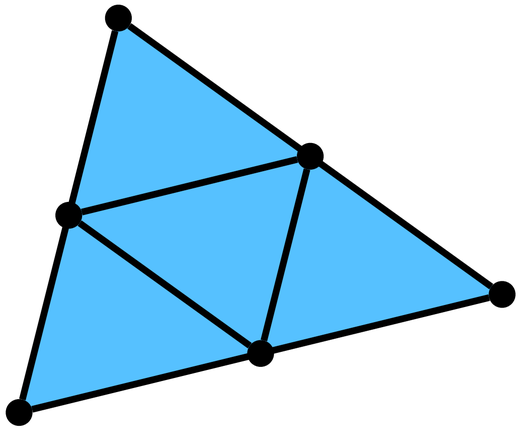
…So why do we need edge flips?



Let’s subdivide this triangle by splitting all three edges…



Resulting mesh doesn’t look so good until we flip one edge:

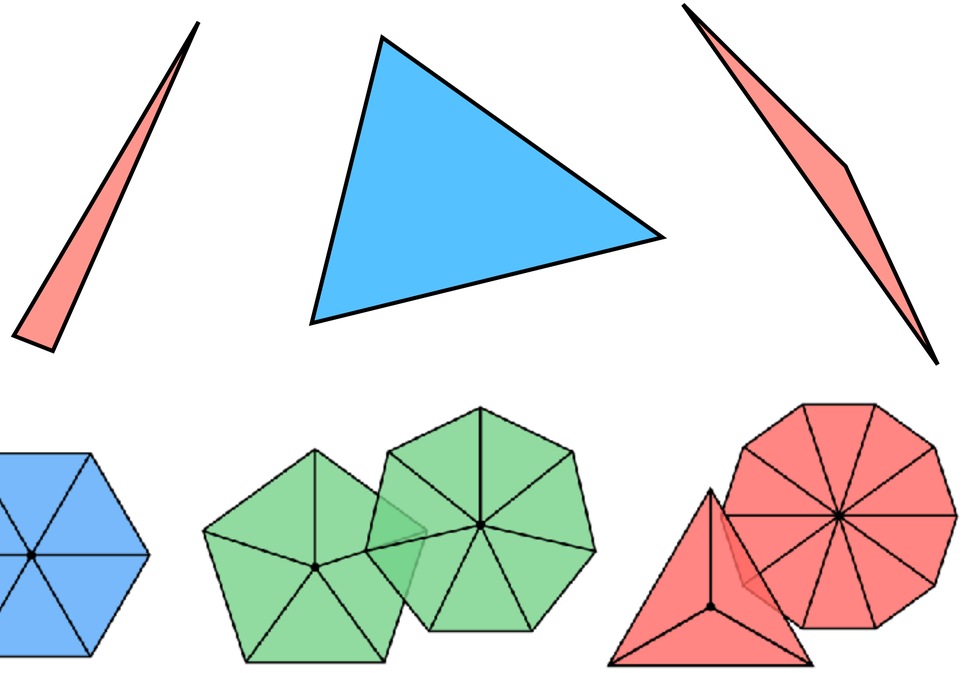


In general, edge flips are almost always necessary to obtain a “nice” triangulation.

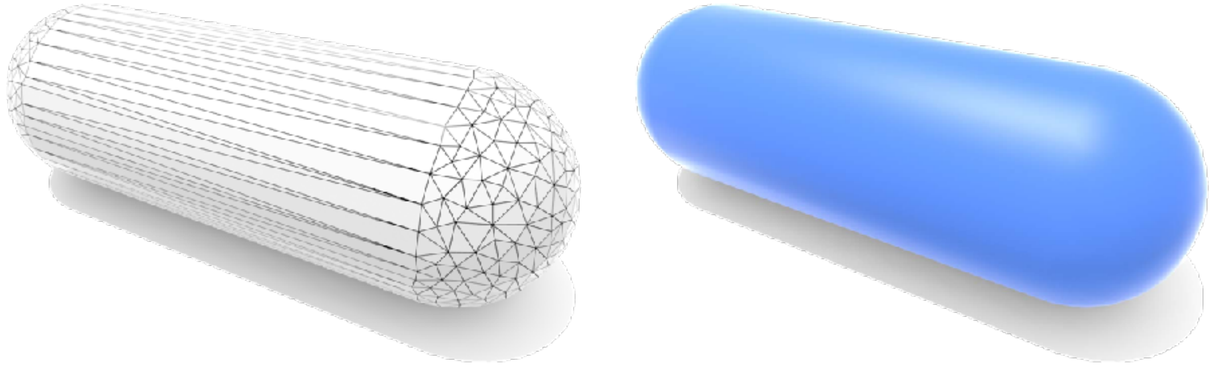
When is a mesh “nice”?

Various desirable properties:

- Well-shaped polygons: triangles close to equilateral, not long and skinny
- Vertex degrees close to regular: 6 for triangle meshes, 4 for quad meshes
- Surface close to original (smooth) shape

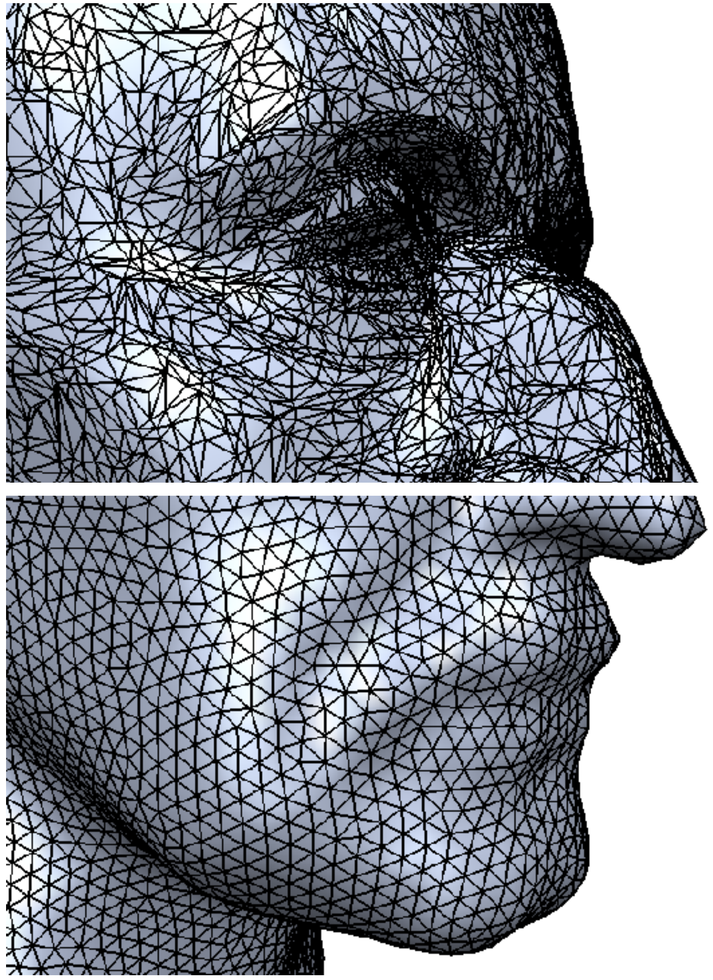


Often this is a tradeoff!



Example: Isotropic remeshing [Botsch & Kobbelt 2004]

1. Split all edges longer than 4/3× desired length
2. Collapse all edges shorter than 4/5× desired length (unless this will create a long edge)
3. Flip edges to bring vertex degrees closer to 6
4. Move vertices tangentially towards average of neighbours
5. Repeat steps 1-4 several times



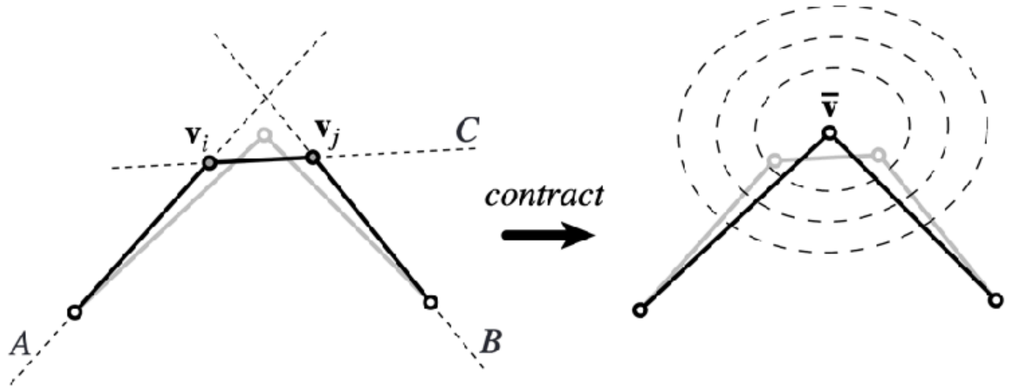
Example: Mesh simplification [Garland & Heckbert 1997]

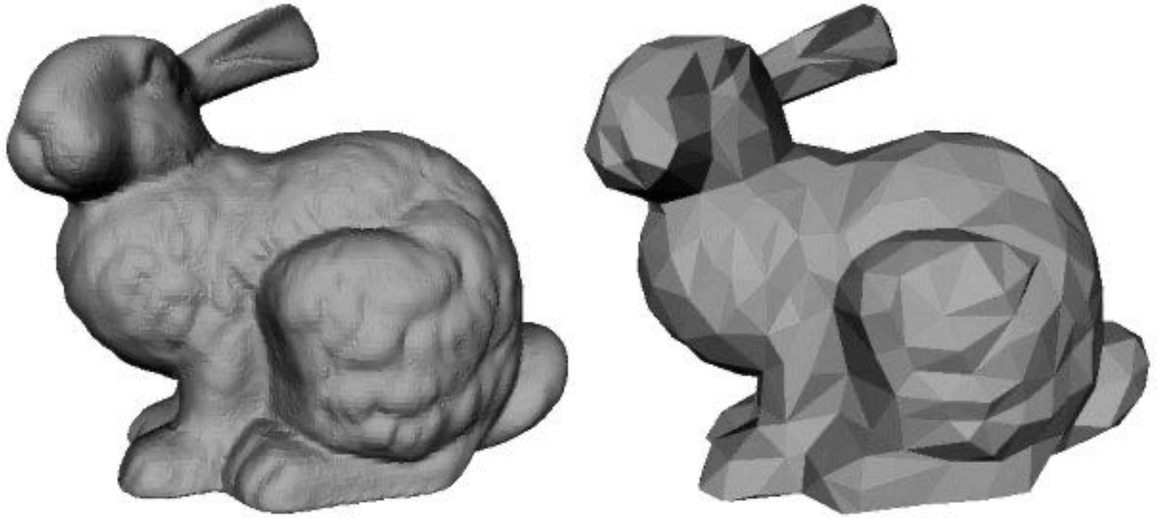
Define cost of collapsing an edge ≈
deviation of new vertex from original surface

1. Collapse edge with lowest cost
2. Repeat until desired number of vertices remain

Simple, greedy, works really well!

The trick: how to estimate deviation from surface efficiently?

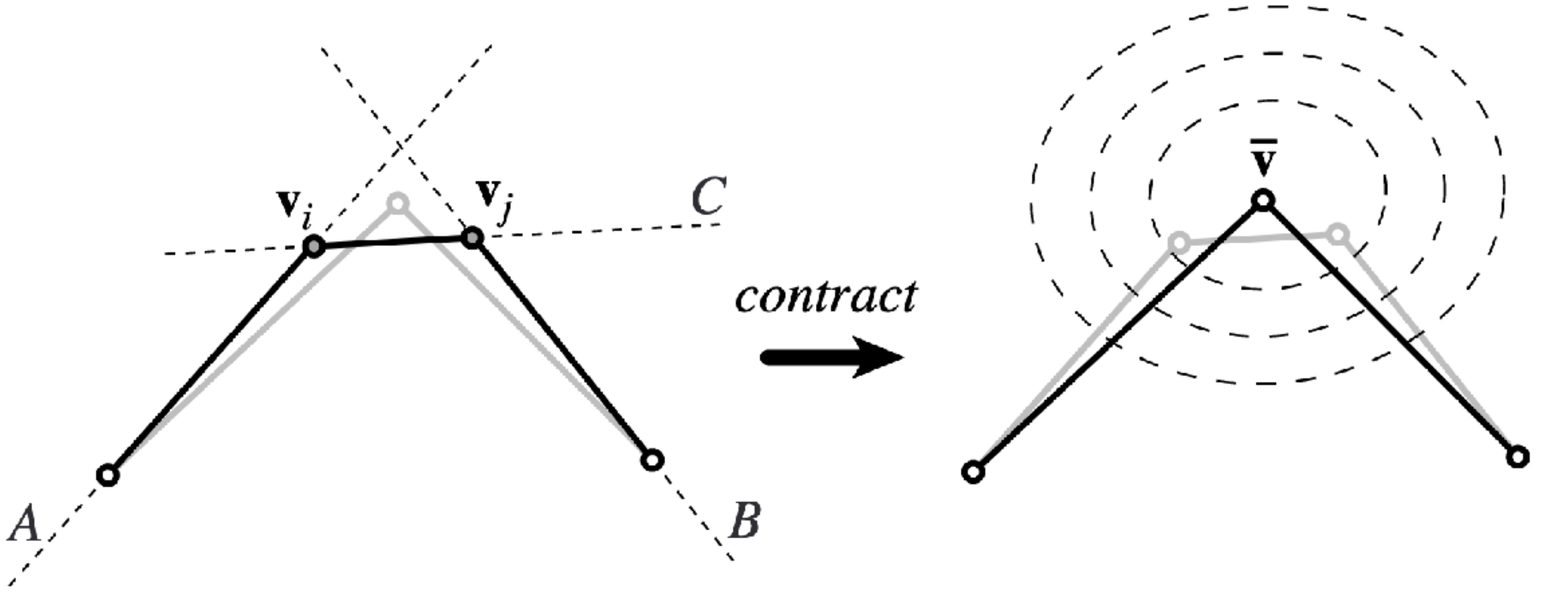




For each face **f**, error of a point **p** = dist(**p**, plane(**f**))^2 = (**n** · (**p** − $\mathbf{p}_0$))^2

For each vertex, "quadric error" $Q(\mathbf{p}) = \sum_{\text{adj. face } \mathbf{f}_i}$ dist(**p**, plane($\mathbf{f}_i$))^2 is quadratic in **p**

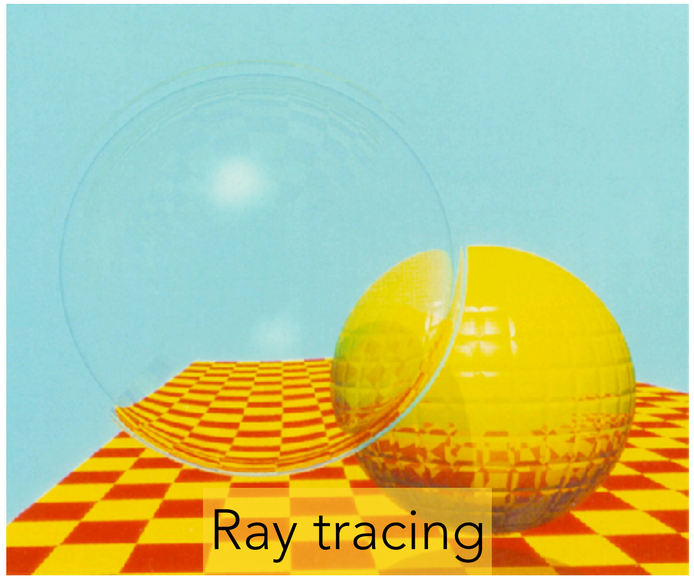
Cost of collapsing edge ($\mathbf{v}_i$, $\mathbf{v}_j$) to new vertex $\bar{\mathbf{v}}$: $Q_i(\bar{\mathbf{v}}) + Q_j(\bar{\mathbf{v}})$
where $\bar{\mathbf{v}}$ is chosen to give the minimum cost.



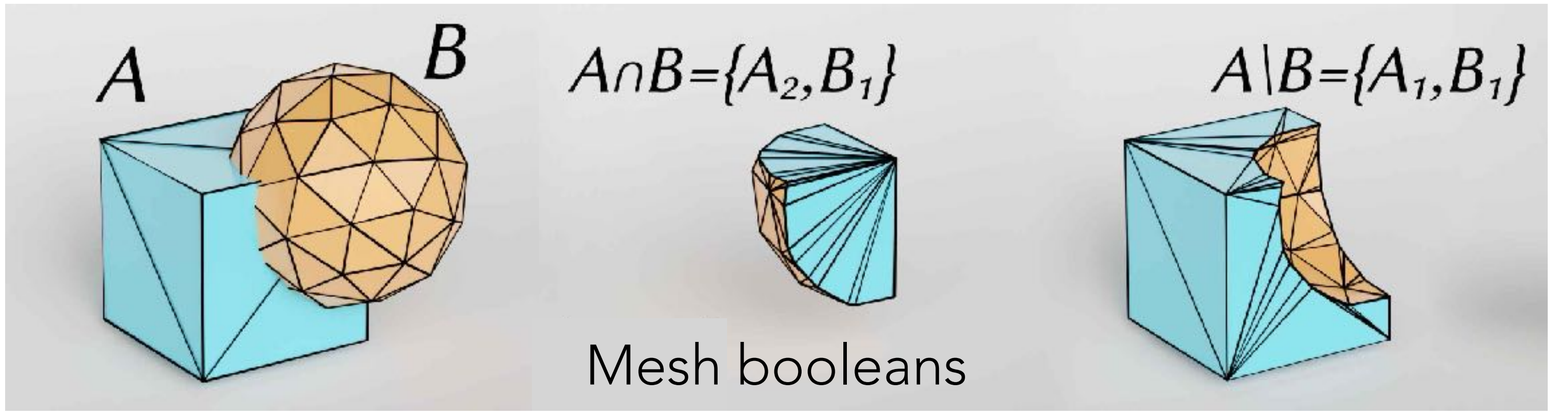
Geometric queries

Once we have modeled a shape, we will still want to answer questions like:

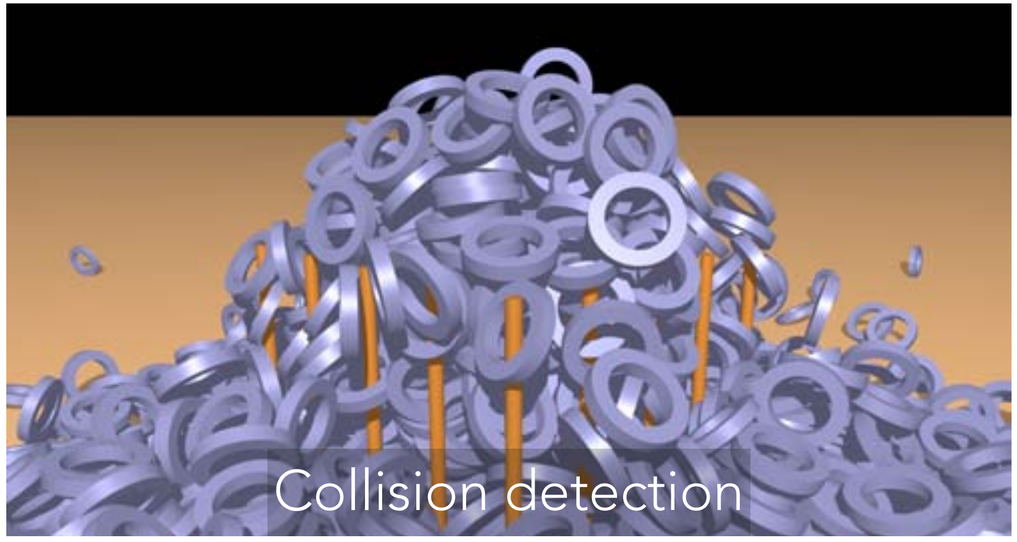
- Given a point **p**, what is the closest point on the shape?
- Is the point **p** inside or outside the shape?
- Given a ray **o** + *t***d**, where does the ray intersect the shape?
- Given two shapes S_1 and S_2, what are the pair of points in closest proximity? Do the two shapes intersect each other?



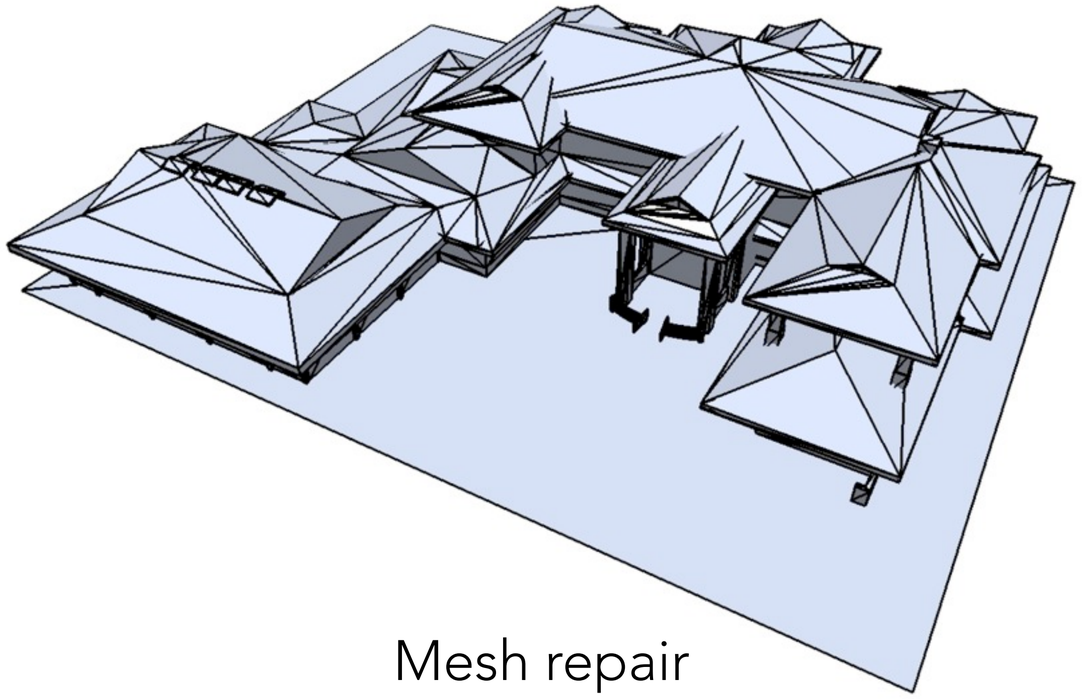
Ray tracing



Mesh booleans



Collision detection



Mesh repair

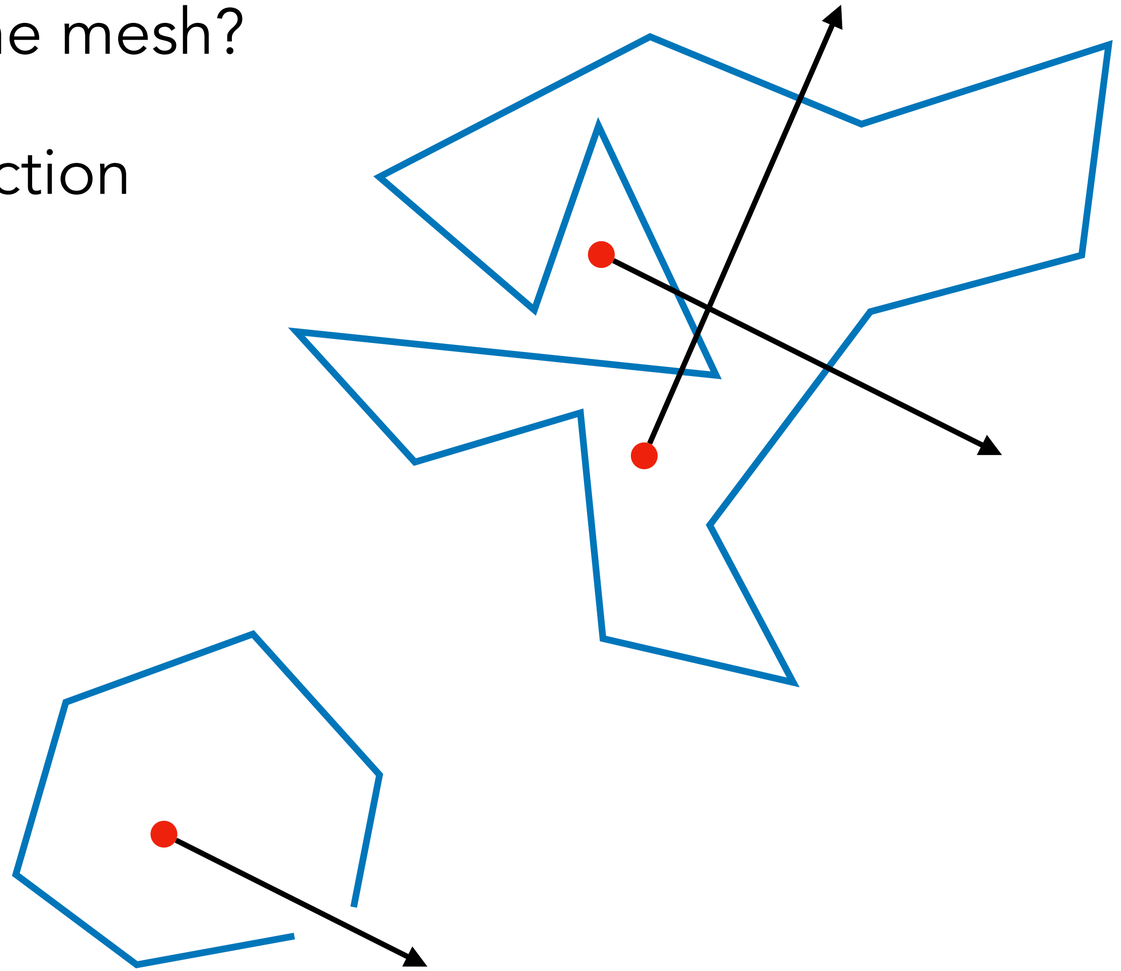
Inside/outside test

Given a point **p**, is it inside or outside the mesh?

Trace a ray to infinity in an arbitrary direction
and count the number of intersections.

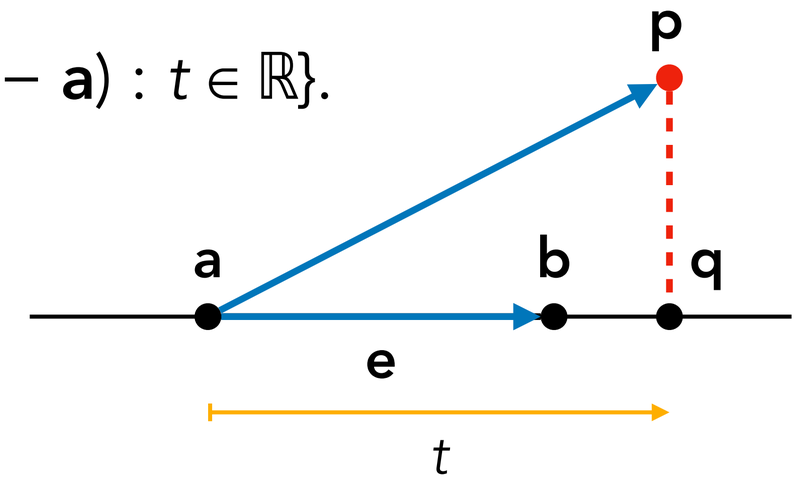
Odd = inside, even = outside.

Of course, the mesh has to be closed
for this to make sense…



Closest point on a line

The line between **a** and **b** is the set of points $\{\mathbf{a} + t(\mathbf{b} - \mathbf{a}) : t \in \mathbb{R}\}$.



Let $\mathbf{e} = \mathbf{b} - \mathbf{a}$, $\hat{\mathbf{e}} = \mathbf{e}/\|\mathbf{e}\|$.

Closest point to **p** on the line:

$$\mathbf{q} = \mathbf{a} + (\hat{\mathbf{e}} \cdot (\mathbf{p} - \mathbf{a}))\hat{\mathbf{e}}$$

Equivalently… $\mathbf{q} = \mathbf{a} + t\mathbf{e}$ where

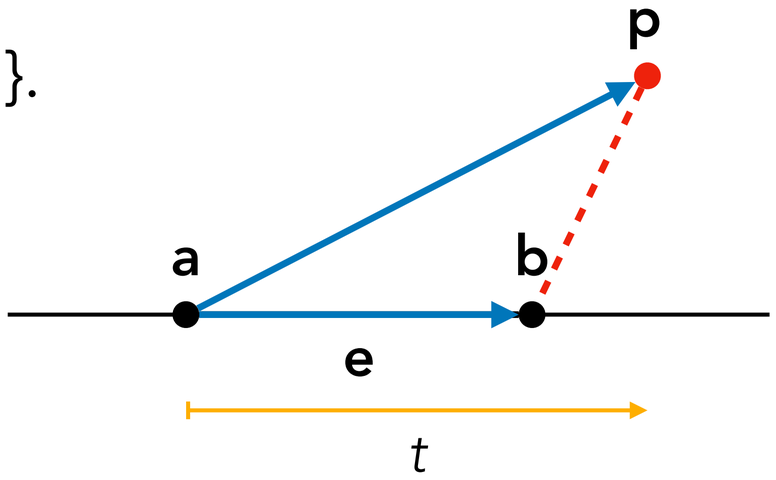
$$t = \frac{\mathbf{e} \cdot (\mathbf{p} - \mathbf{a})}{\|\mathbf{e}\|^2}$$

Closest point on a line segment

The line segment is the set of points $\{\mathbf{a} + t\mathbf{e} : 0 \leq t \leq 1\}$.

Closest point to **p** on line segment:

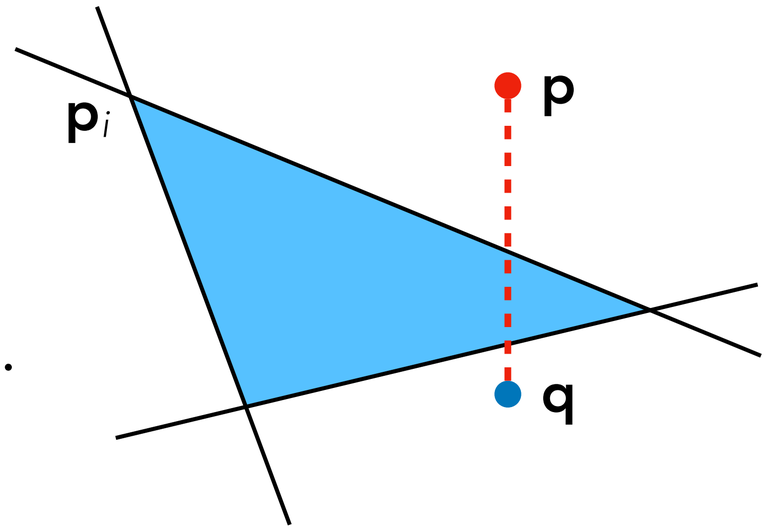
- Project **p** to the line by computing t as before.
- Project **q** again to the line segment:
  - If $t < 0$: set $t = 0$
  - If $t > 1$: set $t = 1$



Closest point on a triangle

A triangle is the set of points $\{\mathbf{p}_0 + b_1\mathbf{e}_1 + b_2\mathbf{e}_2 : 0 \le b_1, b_2, 1-b_1-b_2 \le 1\}$ where $\mathbf{e}_1 = \mathbf{p}_1 - \mathbf{p}_0$, $\mathbf{e}_2 = \mathbf{p}_2 - \mathbf{p}_0$.

Closest point to $\mathbf{p}$ on triangle:



- Solve $(\mathbf{p}_0 + b_1\mathbf{e}_1 + b_2\mathbf{e}_2) + h\mathbf{n} = \mathbf{p}$ for b_1, b_2, h.
  $\mathbf{q} = \mathbf{p}_0 + b_1\mathbf{e}_1 + b_2\mathbf{e}_2$ is the closest point on the plane.
  Let $b_3 = 1-b_1-b_2$.

- If $0 \le b_1, b_2, b_3 \le 1$: closest point is $\mathbf{q}$.

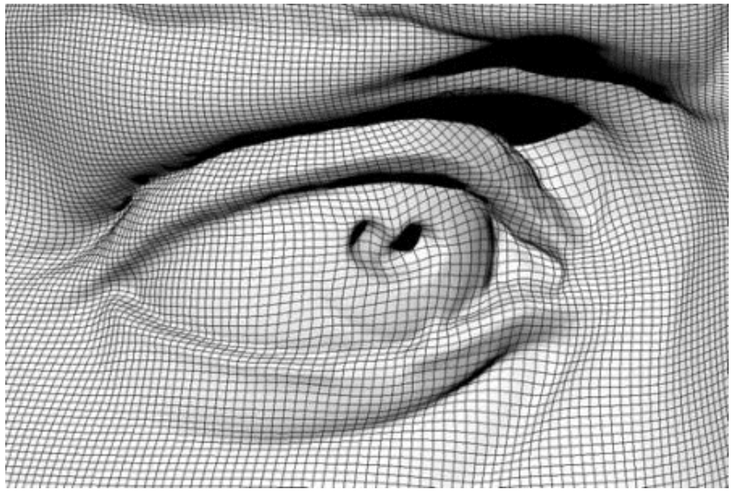
- If $b_i < 0$: find closest point on opposite edge.

Closest point on a triangle mesh

Naïve algorithm:

- For each triangle, find the closest point $\mathbf{q}_i$ and its distance $\|\mathbf{p} - \mathbf{q}_i\|$
- Return the $\mathbf{q}_i$ with the lowest distance

Not scalable to high-resolution meshes!



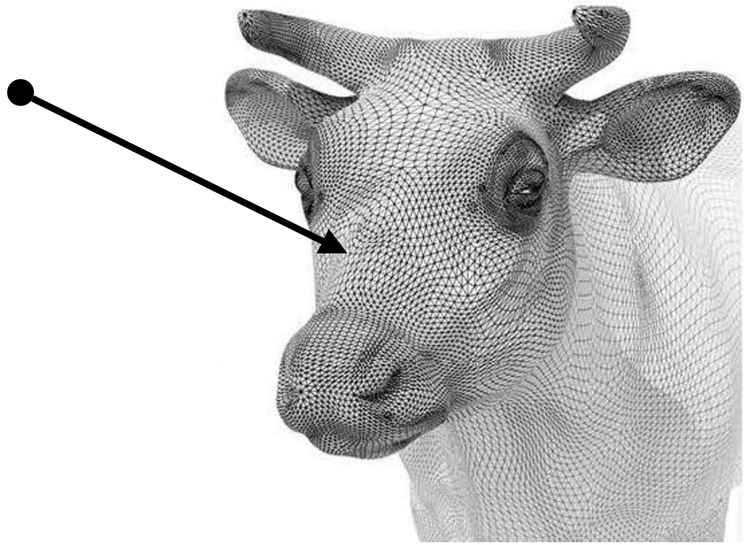
We have the same problem in ray tracing:

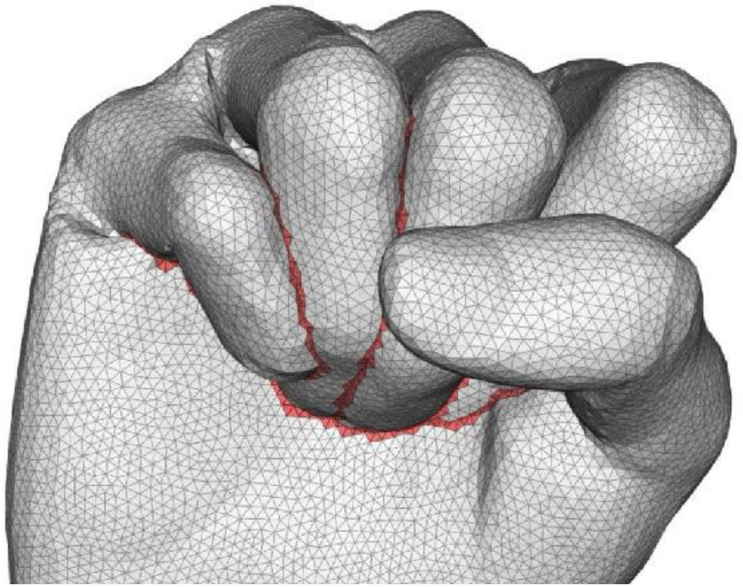
- Intersect each triangle with the ray and return the earliest?

and mesh-mesh intersection:

- Test each pair of triangles with each other and see if they intersect?

Next class: **spatial data structures** for accelerating such queries.





Other announcements

Rest of Assignment 2 will be posted soon (today or tomorrow)

I am on leave from 15th to 27th

- Thursday lecture: Probably online or recorded
- Friday lecture: No class
- Minor exam: Substitute invigilator