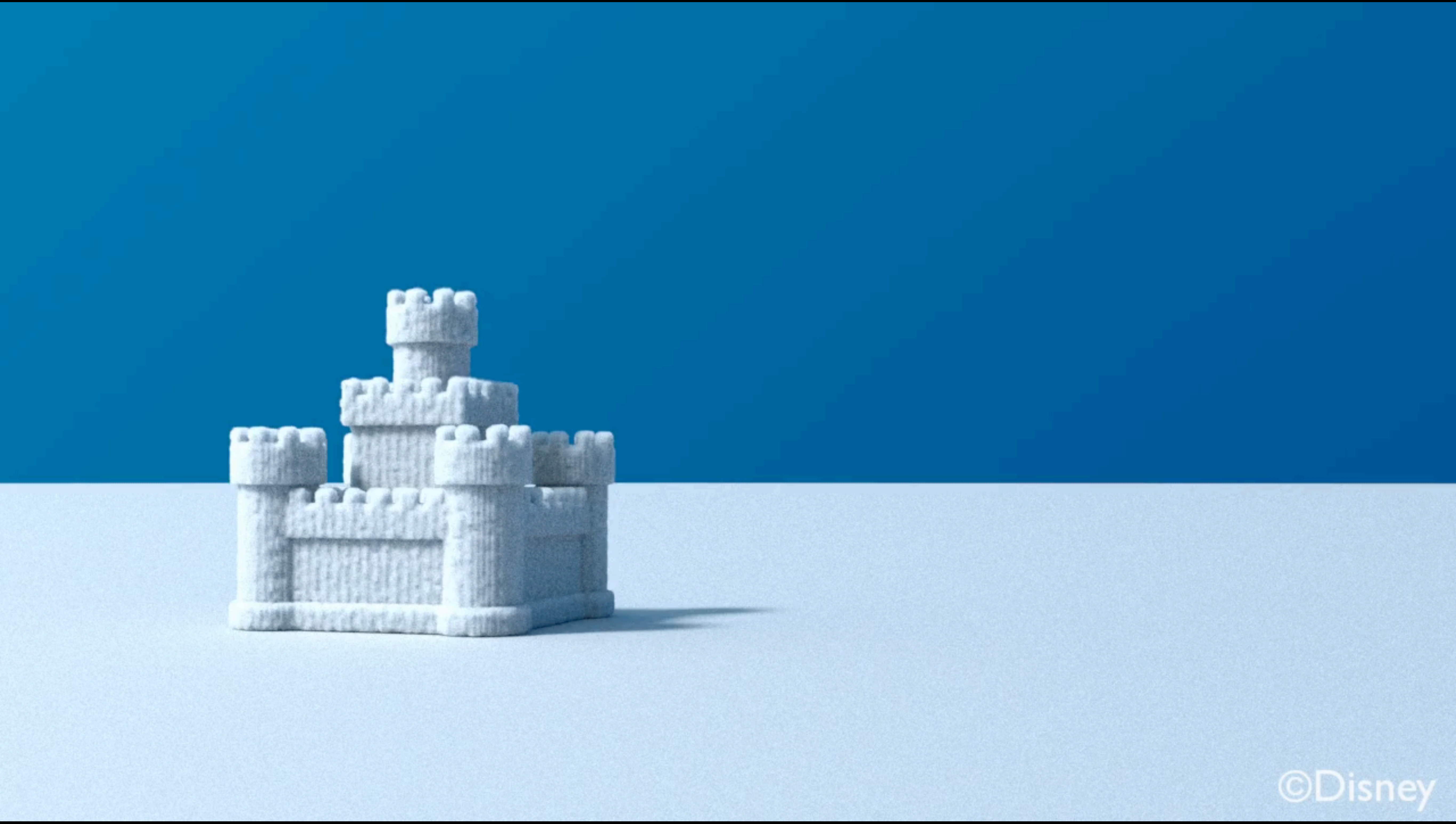
COL865: Special Topics in Computer Applications

Physics-Based Animation

21 — Plasticity and the material point method



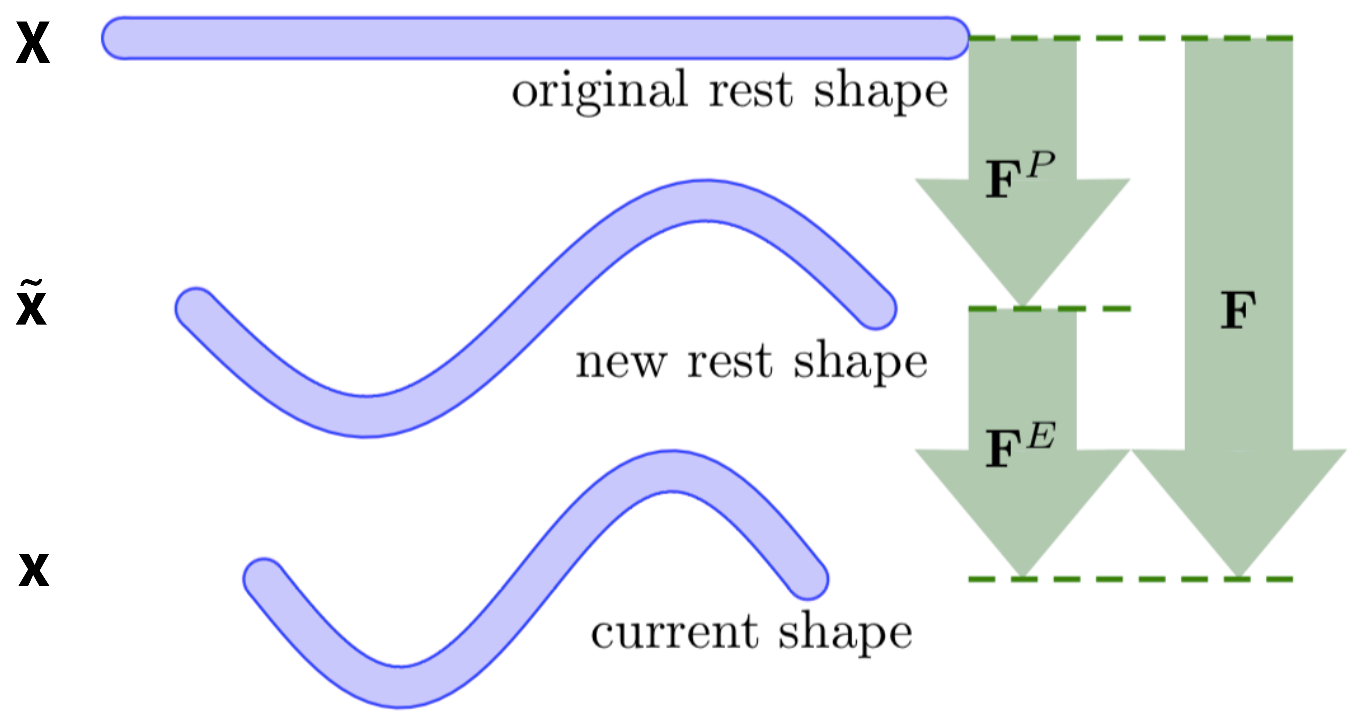
[Stomakhin et al. 2013]

Reading

- Stomakhin et al., “A material point method for snow simulation”, SIGGRAPH 2013
- Jiang et al., *The material point method for simulating continuum materials*, SIGGRAPH 2016 course notes

Plasticity

Multiplicative decomposition



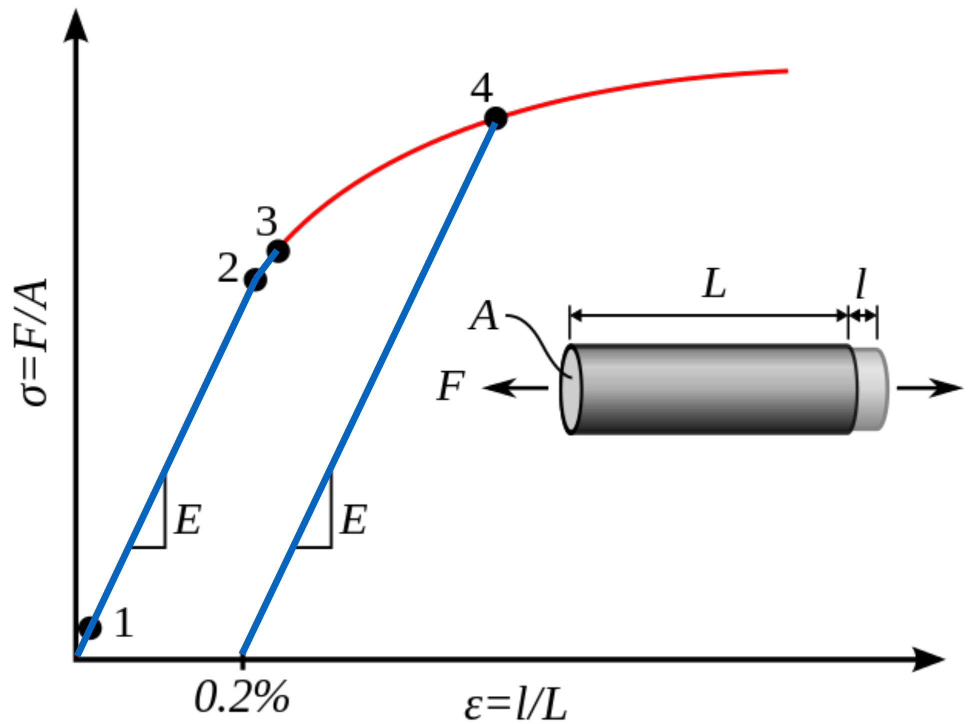
[Jiang et al. 2016]

Plastic deformation = change in rest shape

Elastic forces depend only on $\mathbf{F}^E$

- How does $\mathbf{F}^P$ evolve?

Yield criterion



Plastic state changes only when elastic state violates ***yield criterion***

$$f(\boldsymbol{\sigma}) \leq 0 \quad \text{or} \quad g(\mathbf{F}^E) \leq 0$$

Various models:

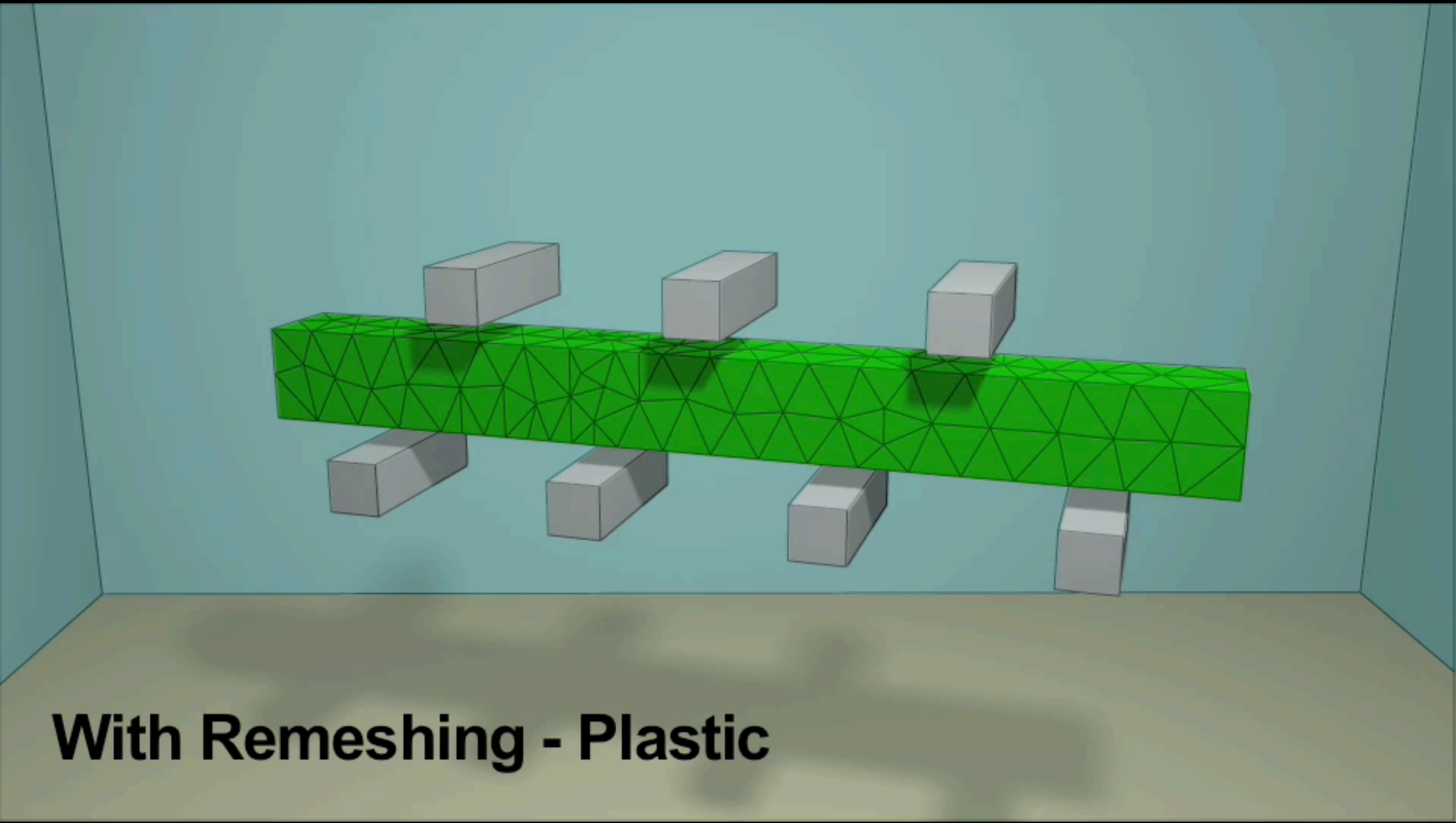
- ***Rate independence*** vs. ***viscoplasticity***:
  Does $\mathbf{F}^P$ update instantaneously or over time?

- ***Perfect plasticity*** vs. ***hardening***:
  Does yield criterion change after plastic deformation?

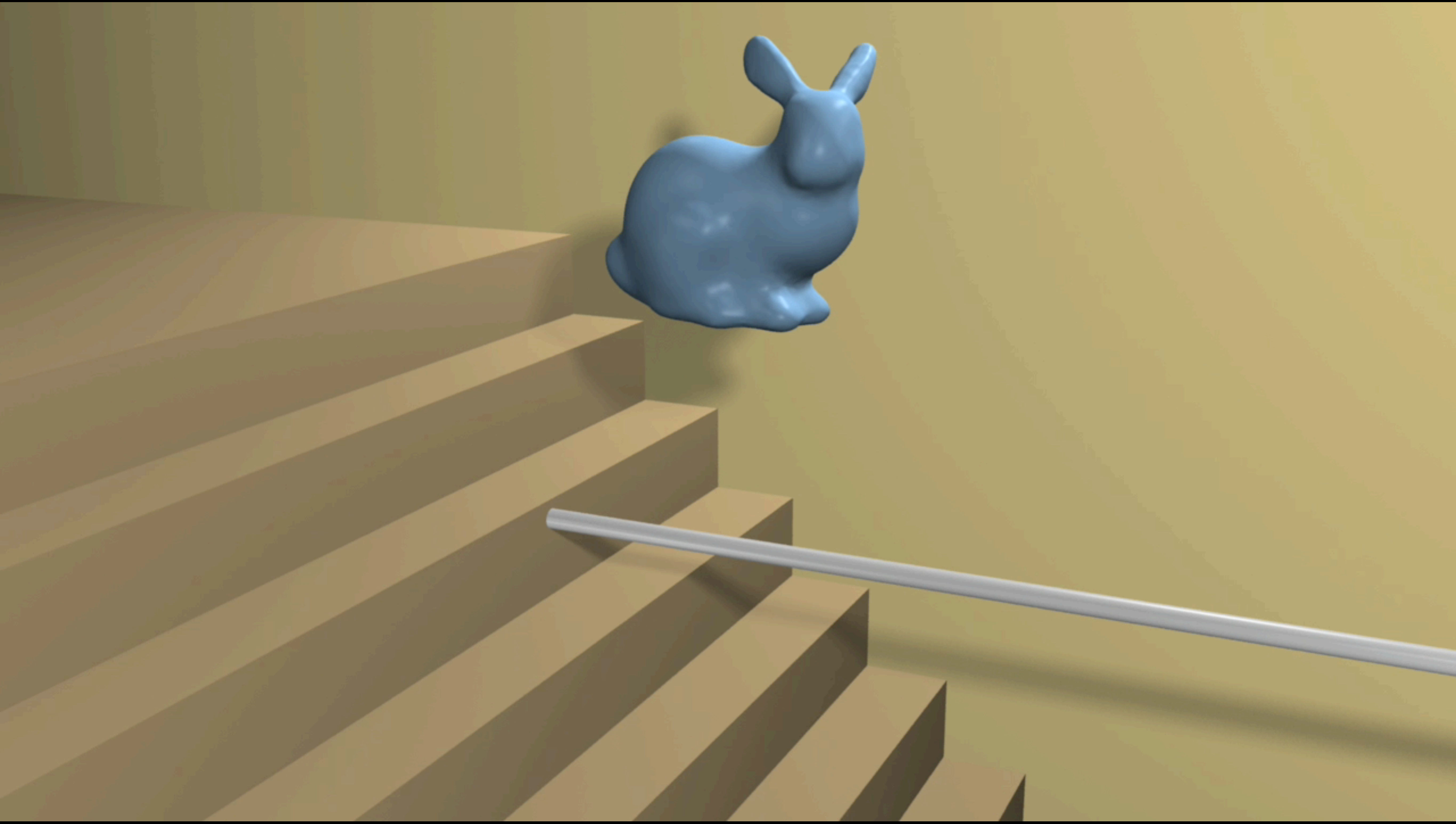
Return mapping

Simplest approach (rate-independent, perfect plasticity):

1. Perform time integration assuming $\mathbf{F}^P$ is constant
2. Compute ***trial elastic deformation***, $\tilde{\mathbf{F}}^E = \mathbf{F}\,(\mathbf{F}^P)^{-1}$
3. ***Return mapping***: project $\tilde{\mathbf{F}}^E$ to yield surface, set $\mathbf{F}^P = (\mathbf{F}^E)^{-1}\,\mathbf{F}$
   [Irving et al. 2004, Jiang et al. 2016]

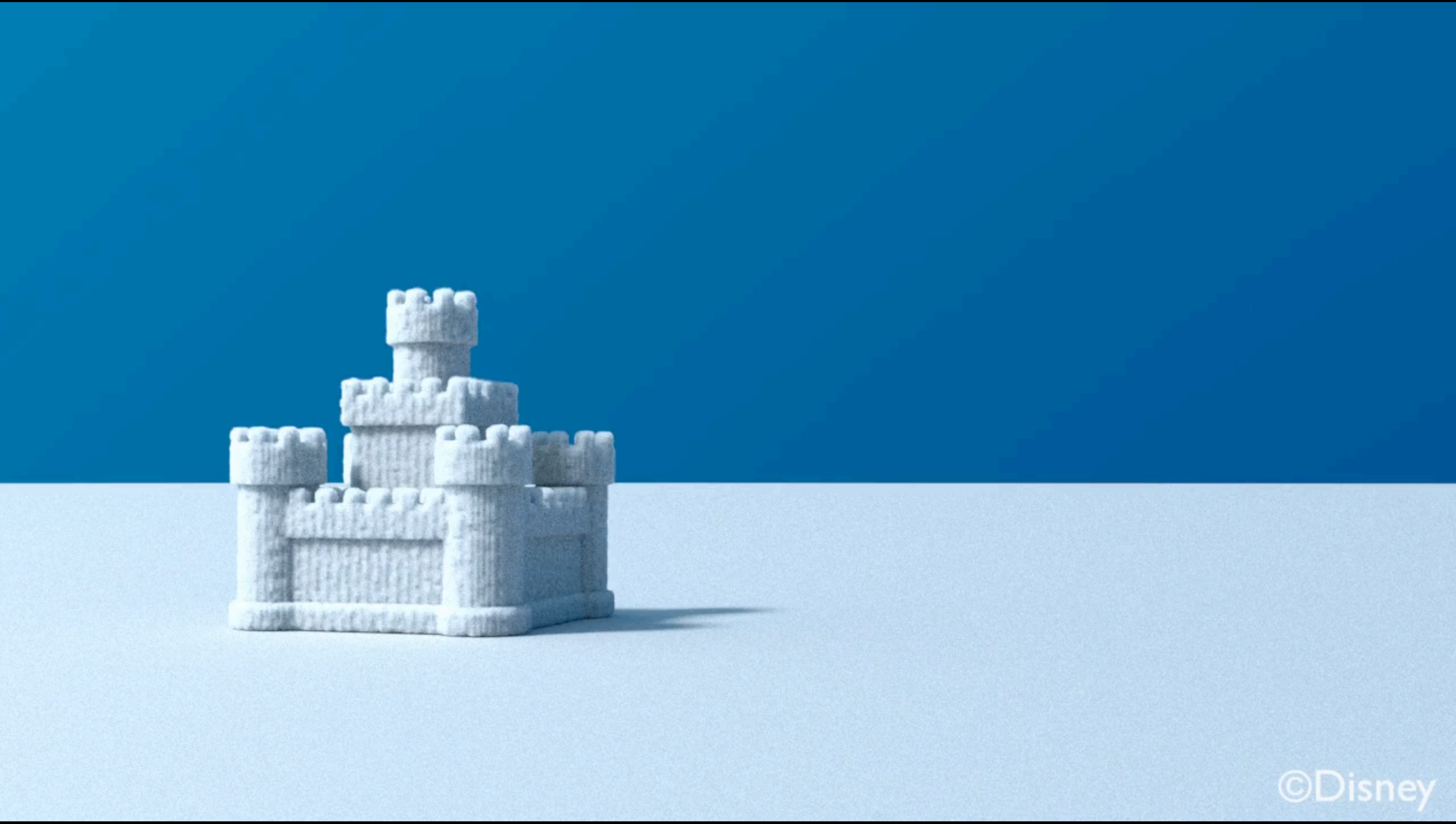


[Wicke et al. 2010]

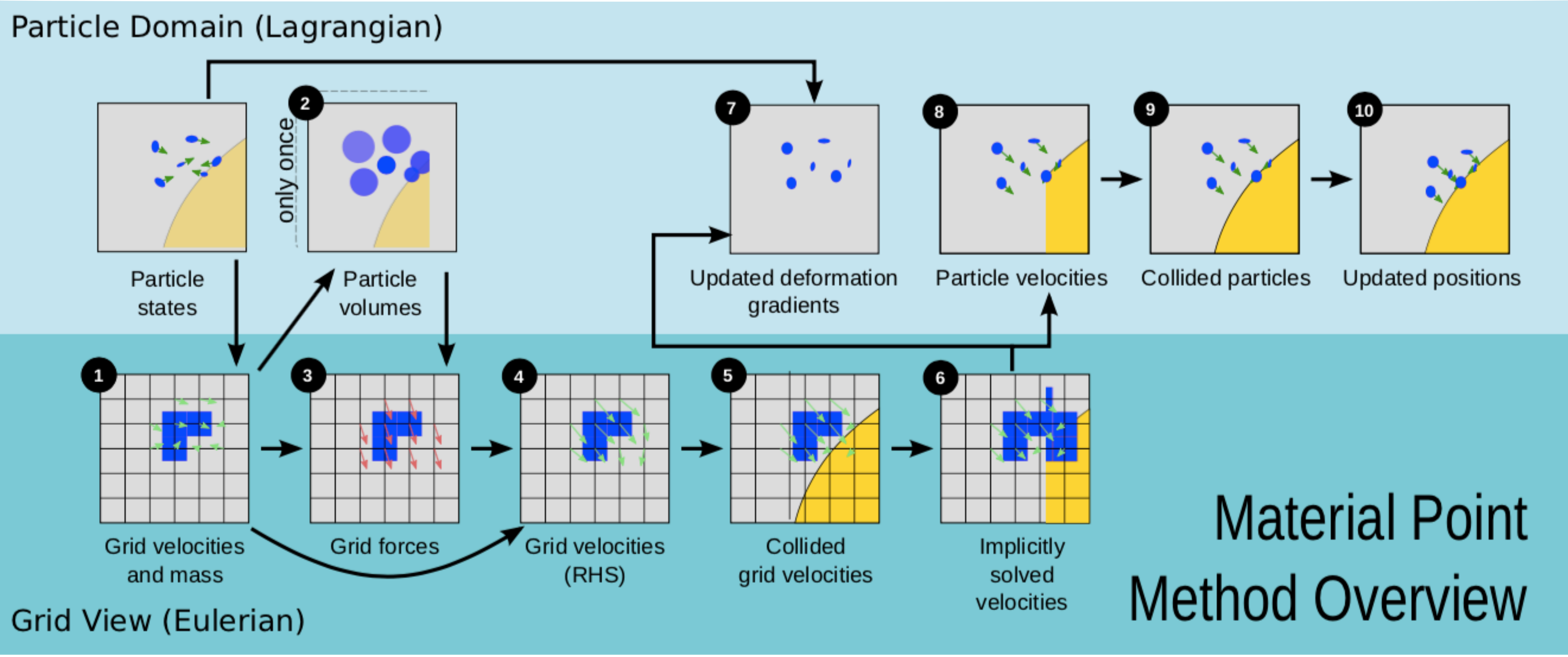


[Bargteil et al. 2007]

The material point method



[Stomakhin et al. 2013]



[Stomakhin et al. 2013]

MPM algorithm

Store data (m, $\mathbf{x}$, $\mathbf{v}$, $\mathbf{F}^E$, $\mathbf{F}^P$) on particles, compute dynamics on grid

1. Particle-to-grid transfer:

$$m_p, \mathbf{x}_p^n, \mathbf{v}_p^n \rightarrow m_i, \mathbf{v}_i^n$$

2. Grid update:

$$\mathbf{F}_p^n, \boldsymbol{\sigma}_p^n \rightarrow \mathbf{f}_i^n, \mathbf{v}_i^{n+1}$$

3. Grid-to-particle transfer:

$$\mathbf{v}_i^{n+1} \rightarrow \mathbf{F}_p^{n+1}, \mathbf{v}_p^{n+1}$$

4. Advect particles:

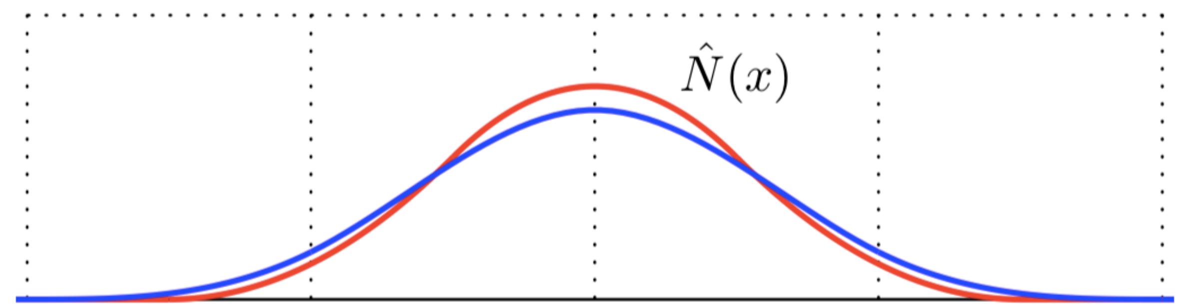
$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t\, \mathbf{v}_p^{n+1}$$

Particle/grid transfers

Particle to grid:

$m_i = \sum m_p N_i(\mathbf{x}_p)$

$(m\mathbf{v})_i = \sum m_p \mathbf{v}_p N_i(\mathbf{x}_p)$
$\mathbf{v}_i = (m\mathbf{v})_i / m_i$



Grid to particles:

$\mathbf{v}_p = \sum \mathbf{v}_i N_i(\mathbf{x}_p)$

$\mathbf{F}_p^{n+1} = (\mathbf{I} + \mathrm{d}\mathbf{v}/\mathrm{d}\mathbf{x}\, \Delta t)\, \mathbf{F}_p^{n}$
then update plastic decomposition $\mathbf{F}^E{}_p$, $\mathbf{F}^P{}_p$

(Better to use APIC velocity transfers [Jiang et al. 2015] instead)

Lagrangian grid interpretation

Imagine a Lagrangian copy of the grid:

$$\hat{\mathbf{x}}_i^n = \mathbf{x}_i$$

$$\hat{\mathbf{x}}_i^{n+1} = \mathbf{x}_i + \mathbf{v}_i^{n+1}\,\Delta t$$

Particles simply get moved along by the grid deformation:

$$\mathbf{v}_p^{n+1} = \mathbf{v}^{n+1}(\mathbf{x}_p)$$

$$\mathbf{x}_p^{n+1} = \hat{\mathbf{x}}^{n+1}(\mathbf{x}_p)$$

$$\mathbf{F}_p^{n+1} = \mathrm{d}\hat{\mathbf{x}}^{n+1}/\mathrm{d}\mathbf{x}\;\mathbf{F}_p^n$$

After updating particles, forget Lagrangian grid

Grid update

Grid forces as negative gradient of energy

$$U = \iiint \boldsymbol{\Psi}(\mathbf{F}^E(\mathbf{X}))\, \mathrm{d}V^0$$
$$\approx \sum \boldsymbol{\Psi}(\mathbf{F}^E{}_p)\, V_p{}^0$$

U depends on $\mathbf{F}^E{}_p$, but $\mathbf{F}^E{}_p$ depends on $\hat{\mathbf{x}}$

$$\mathbf{f}_i = -\nabla_{\hat{\mathbf{x}}i}\, U$$

$$= \sum \mathbf{P}(\mathbf{F}^E{}_p)\, (\mathbf{F}^E{}_p)^T\, \nabla N_i(\mathbf{x}_p)\, V_p{}^0$$

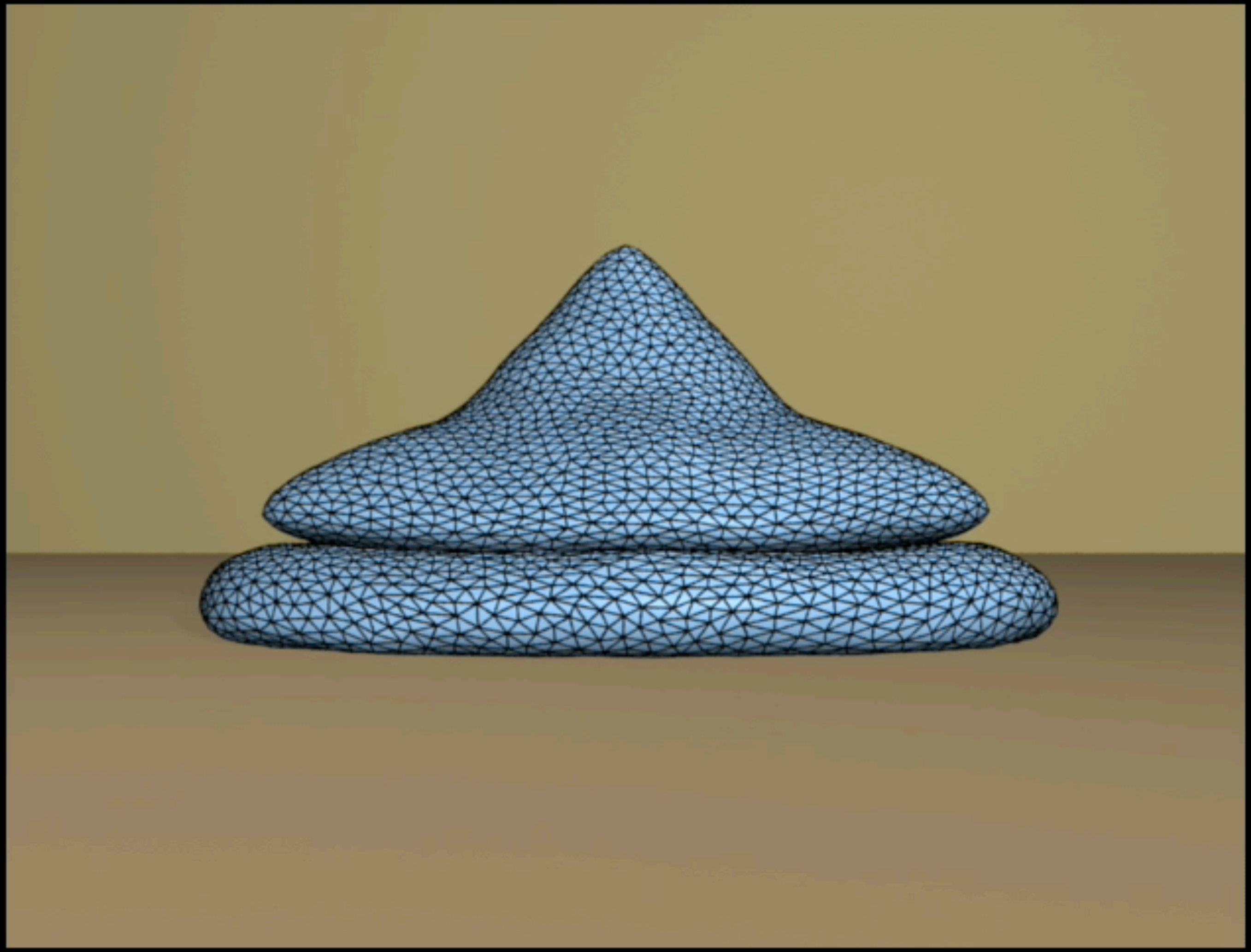
Overview

Method	Volume Preservation	Stiffness	Plasticity	Fracture
Reeve particles	-	-	-	-
Rigid bodies	⋆⋆	⋆⋆	-	⋆
Mesh-based solids	⋆	⋆ ⋆ ⋆	⋆⋆	⋆
Grid-based fluids	⋆ ⋆ ⋆	⋆	⋆⋆	⋆ ⋆ ⋆
SPH	⋆	⋆	⋆	⋆ ⋆ ⋆
MPM	⋆⋆	⋆⋆	⋆ ⋆ ⋆	⋆ ⋆ ⋆

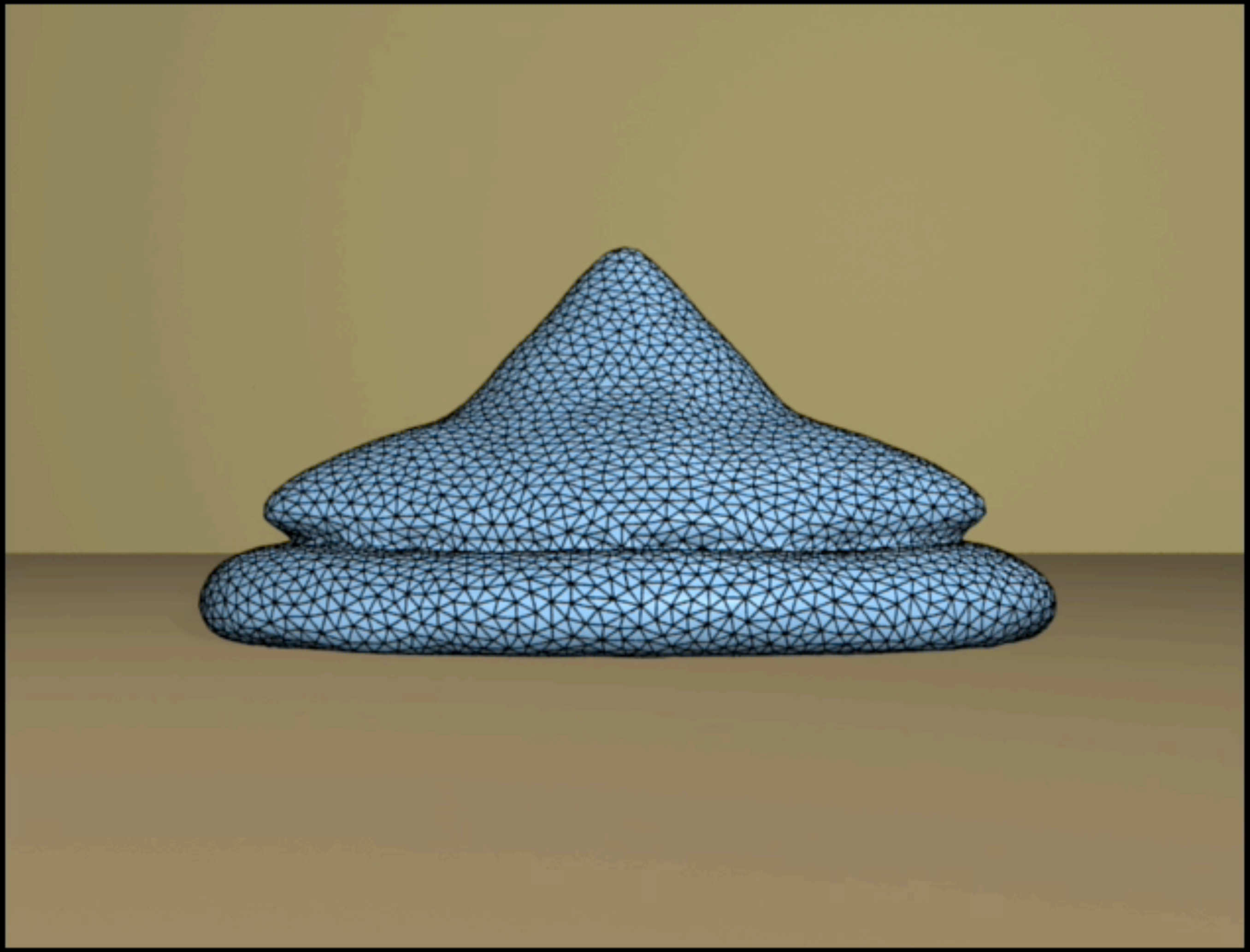
[Stomakhin et al. 2013]

- Large deformations and plasticity are cheap
- Fracture is free

Cross-section moving through the object

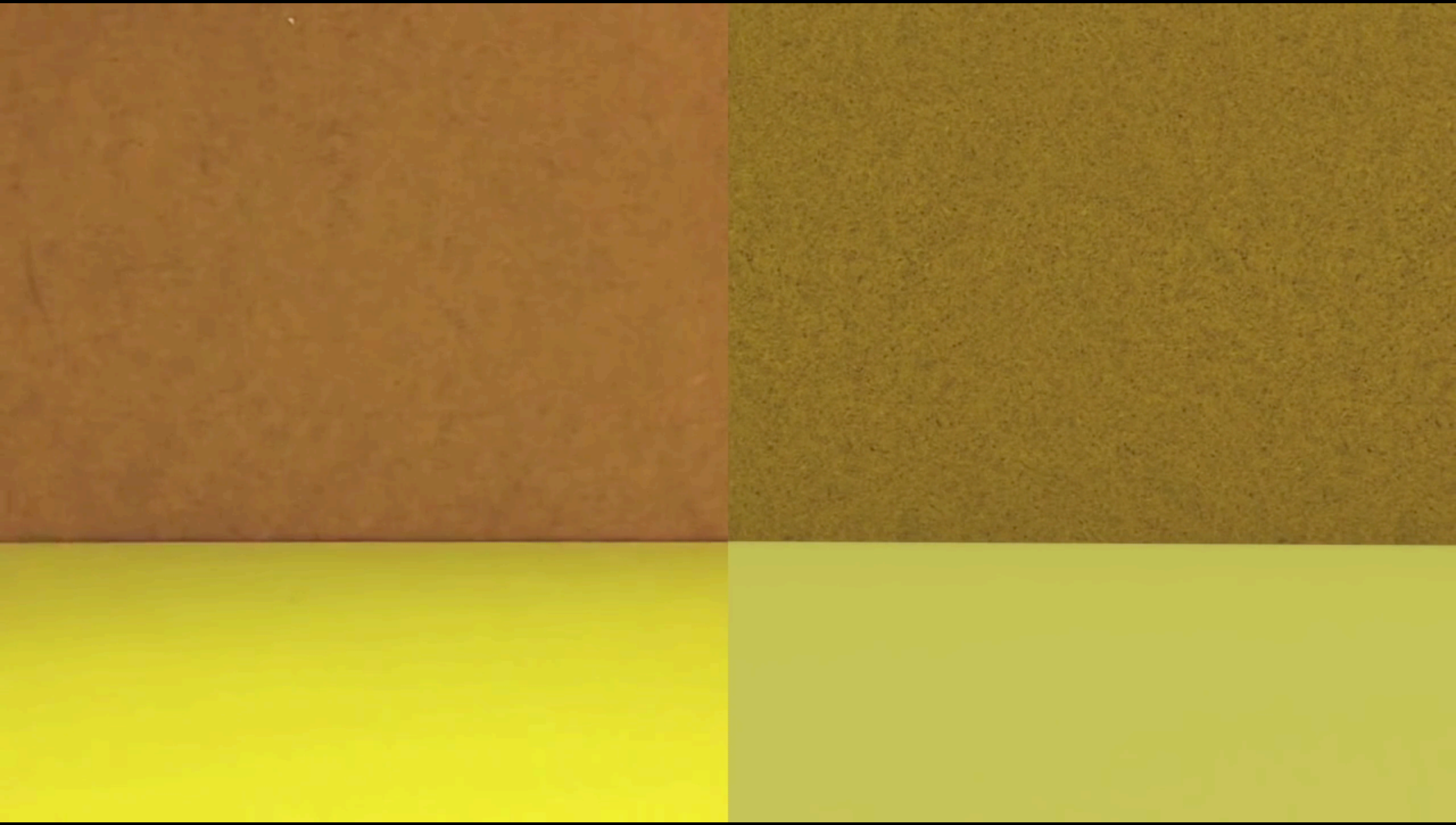


Before Remeshing

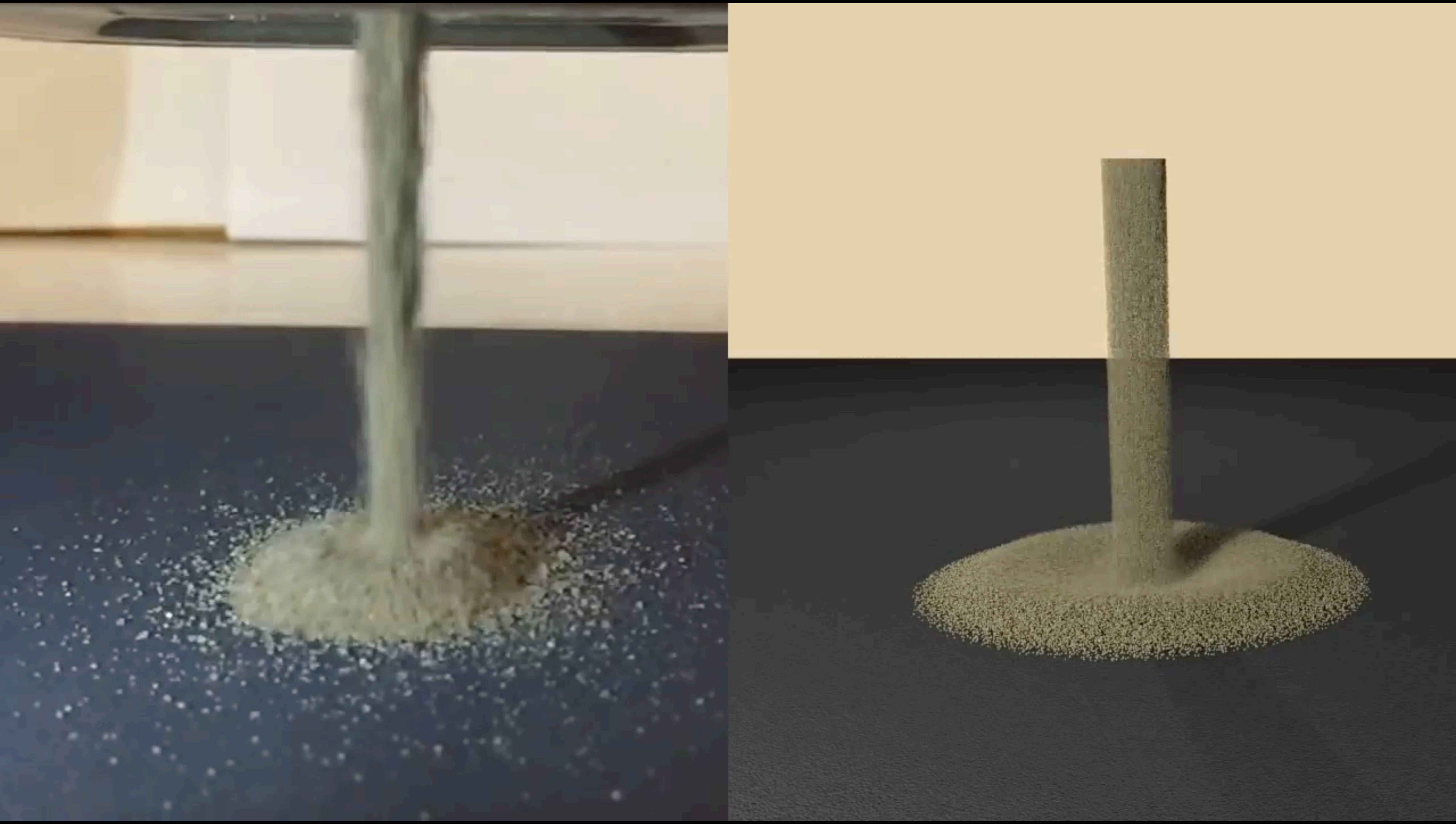


After Remeshing

[Bargteil et al. 2007]



[Ram et al. 2015]



[Klár et al. 2016]