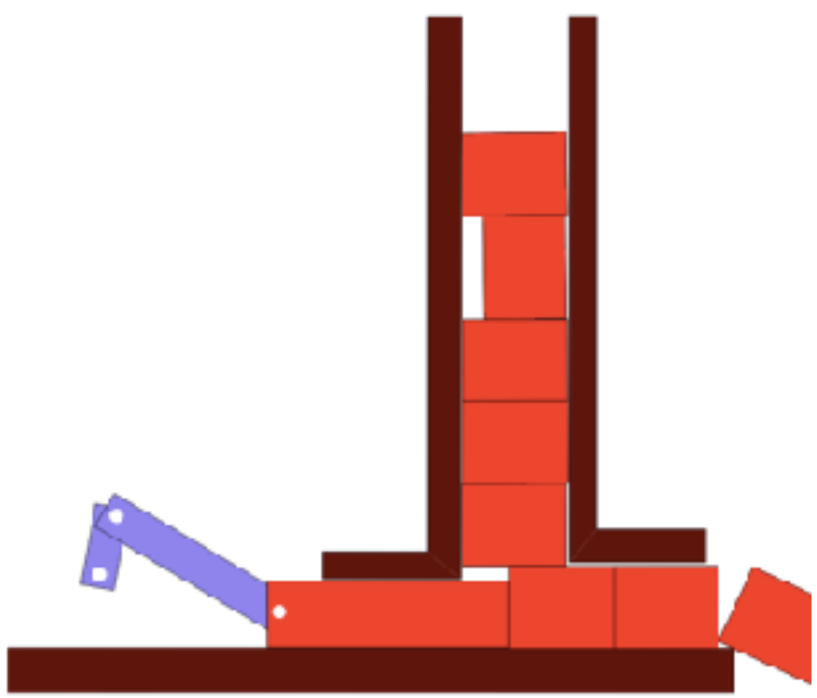
COL865: Special Topics in Computer Applications

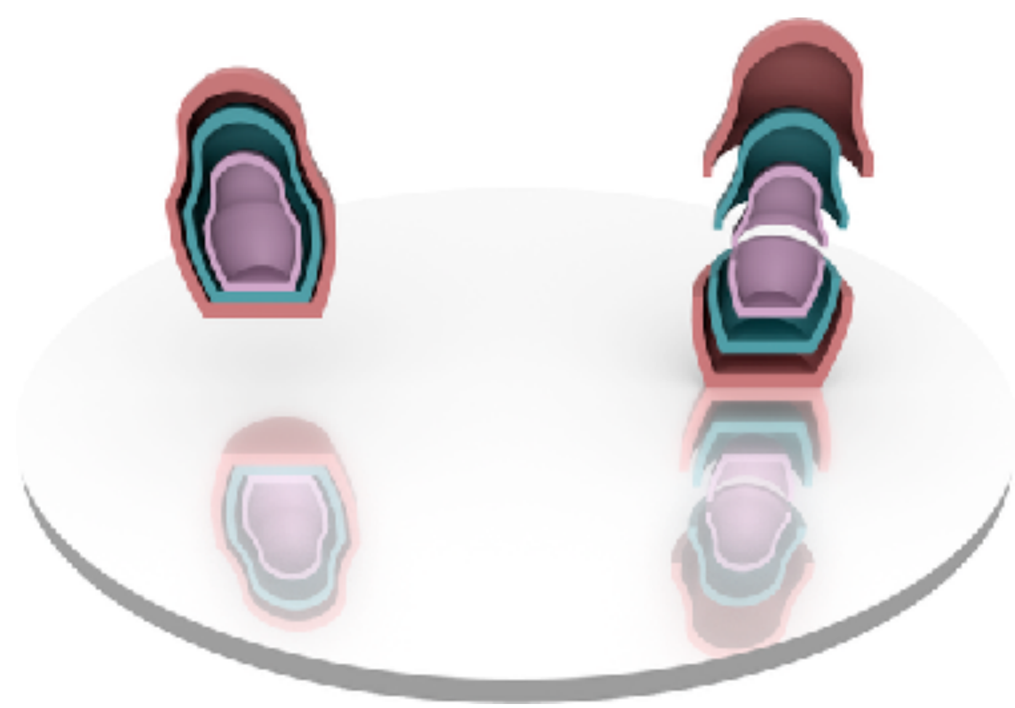
Physics-Based Animation

10 — Collisions and contact

Recommended reading

- Baraff, “Fast Contact Force Computation for Nonpenetrating Rigid Bodies”, 1994
- Smith et al., “Reflections on Simultaneous Impact”, 2012





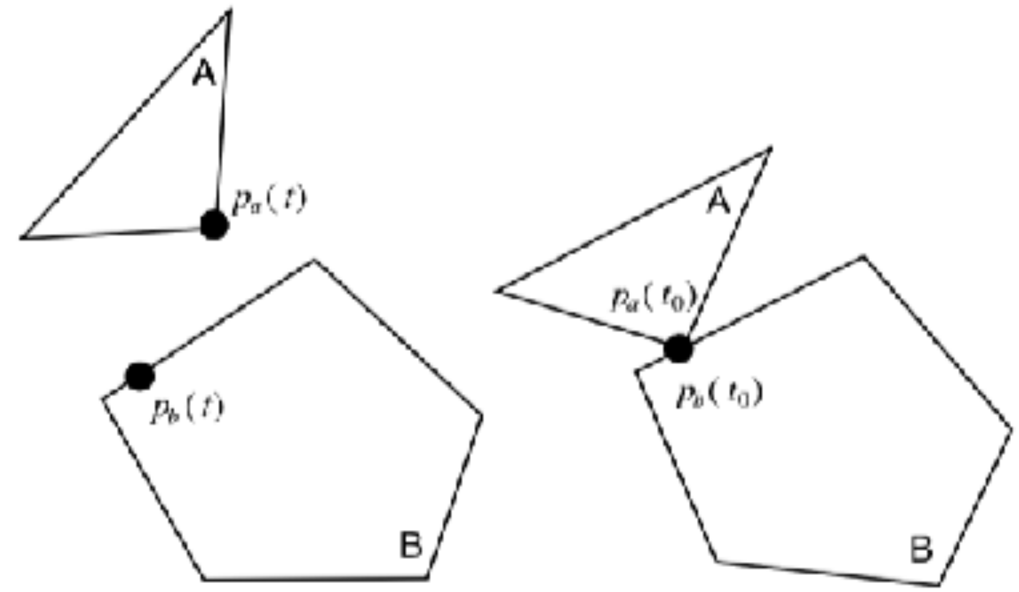
Collisions vs. resting contact

Collision: $v_{\text{rel,n}} < 0$

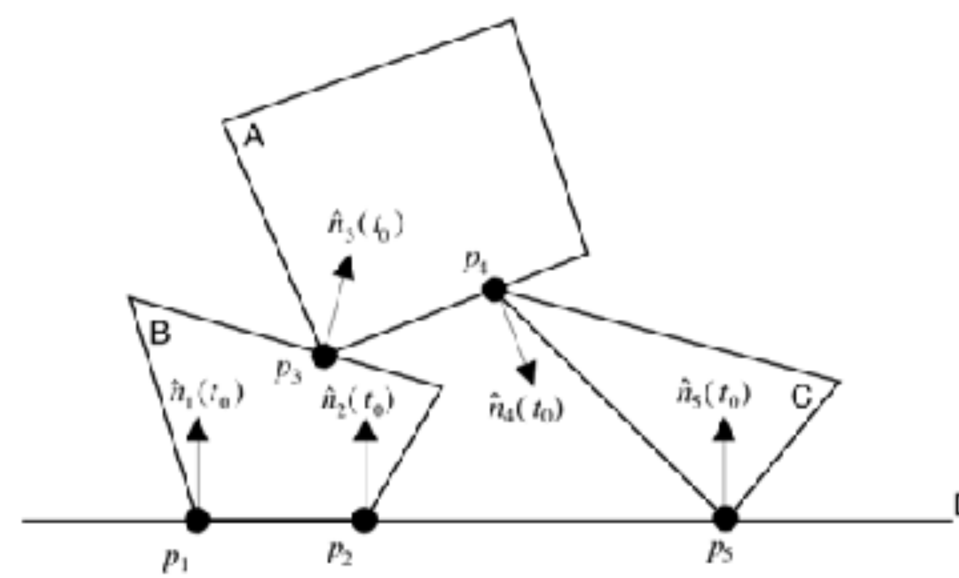
- Impulse causes velocity jump
- Usually only in pairs

Resting contact: $v_{\text{rel,n}} = 0$

- Forces act continuously
- Usually in stacks and piles: must solve interconnected system



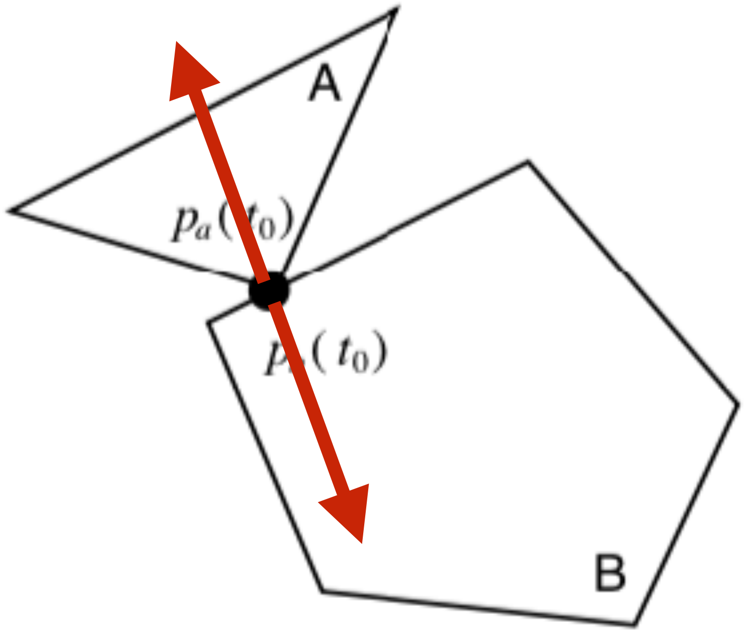
[Witkin & Baraff]



[Witkin & Baraff]

Pairwise collision resolution

Define contact Jacobian $\mathbf{J}$:



$$v_n = \mathbf{J}\,[\mathbf{v}_a;\ \boldsymbol{\omega}_a;\ \mathbf{v}_b;\ \boldsymbol{\omega}_b]$$

Impulsive forces and torques = $\mathbf{J}^T j_n$

New relative normal velocity:

$$v_n^+ = v_n^- + \mathbf{J}\,\mathbf{M}^{-1}\,\mathbf{J}^T j_n$$

Fix $v_n^+ = -\varepsilon\, v_n^-$, solve for j_n

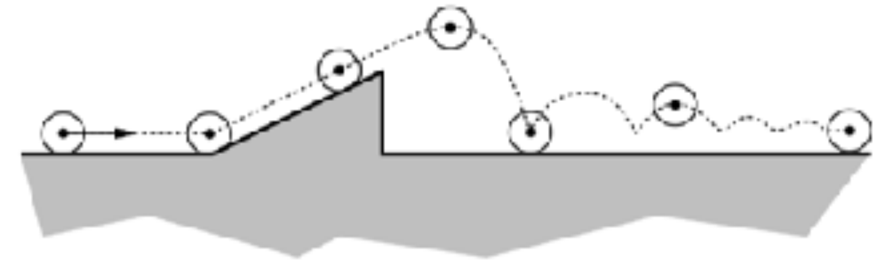
Similar approach for friction (approximates max dissipation):
Fix $\mathbf{v}_t^+ = 0$, solve for $\mathbf{j}_f$.
If $\|\mathbf{j}_f\| > \mu\, j_n$, set $\mathbf{j}_f = -\mu\, j_n\, \mathbf{v}_t^- / \|\mathbf{v}_t^-\|$

Multi-contact algorithms

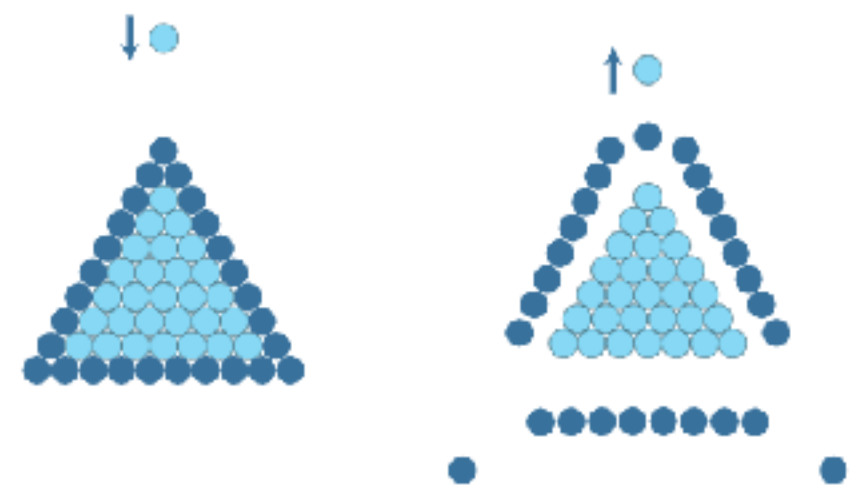
Separating collisions and contact is awkward

- Transition from collisions to rolling
- Multiple collisions in one time step

Often use same algorithm for both

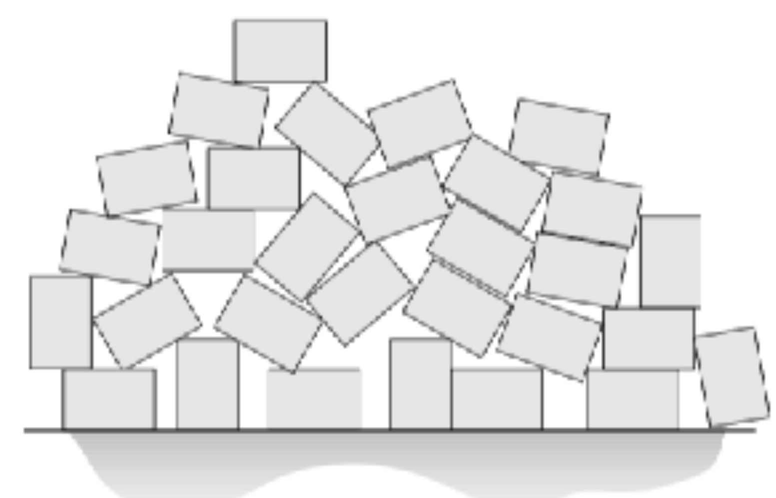


[Mirtich and Canny 1994]



[Smith et al. 2012]

Lots of algorithms in comp. mech, robotics [Mirtich and Canny 1994, Moreau 1994, Stewart and Trinkle 1996, …] but expensive for large stacks and piles



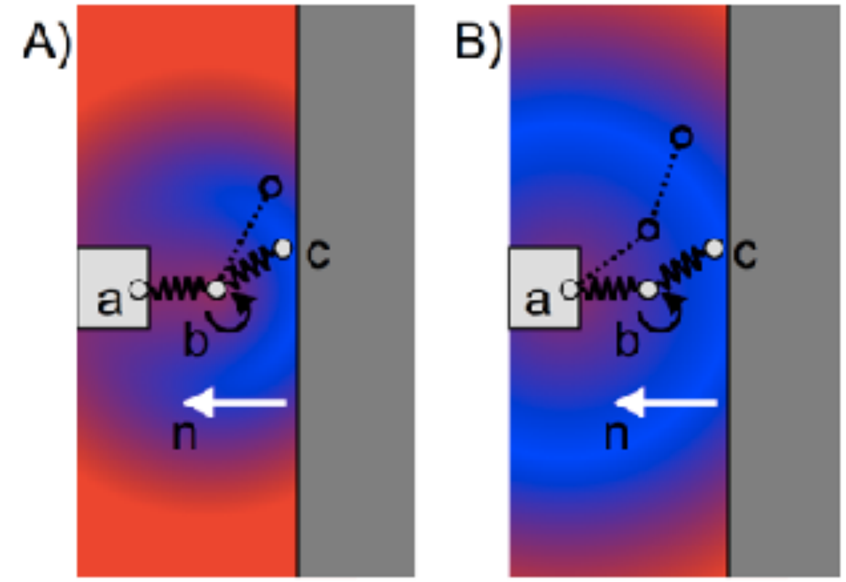
[Erleben 2007]

Time stepping issues

Collisions generally occur in the ***middle*** of a time step!

- Advance only to time of collision?
  "Zeno problem": time steps get smaller and smaller
- Live with it: Only detect interpenetrations, not future collisions
- Apply forces at start to prevent interpenetration at end

Are contact forces coupled with elastic forces? ***Implicit contact***



[Otaduy et al. 2009]

Typical strategy

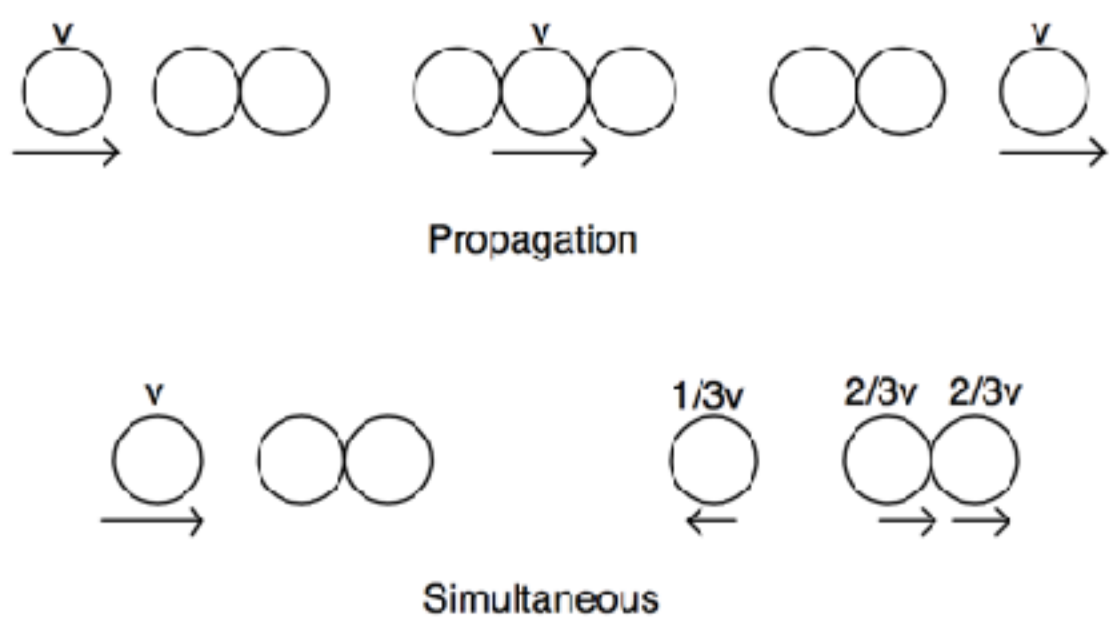
Rigid body state: $\mathbf{q} = (\mathbf{x}, \boldsymbol{q})$, $\mathbf{u} = (\mathbf{v}, \boldsymbol{\omega})$

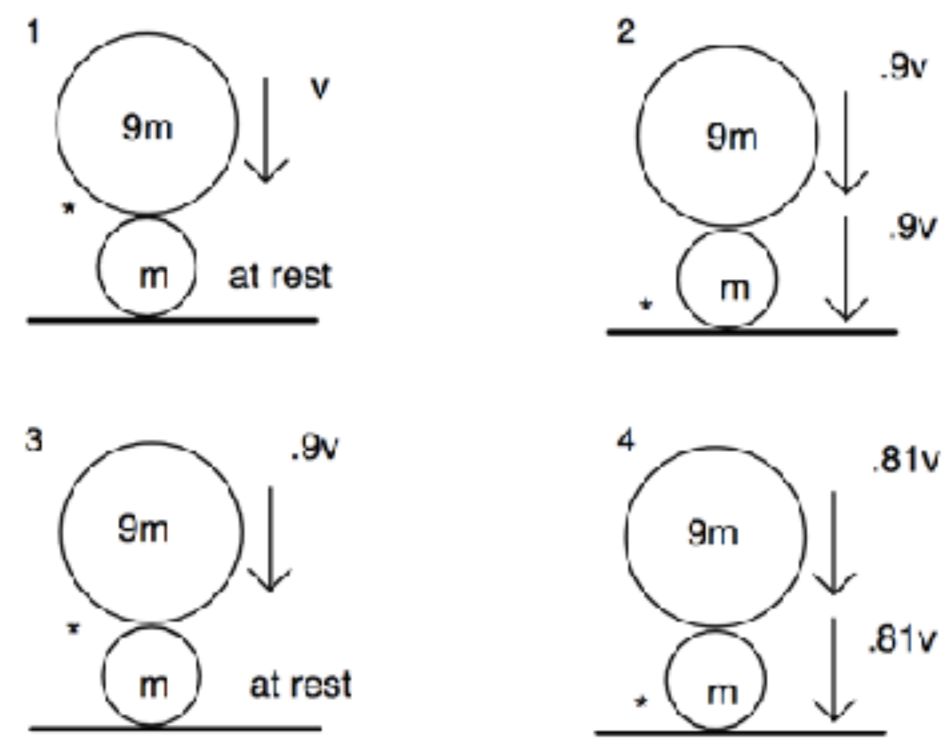
1. Unconstrained velocity update: $\tilde{\mathbf{u}}^1 = \mathbf{u}^0 + \mathbf{M}^{-1} (\mathbf{f}^0, \boldsymbol{\tau}^0)\,\Delta t$
2. Detect contact pairs
3. Contact resolution to get corrected velocity $\mathbf{u}^1$
    - Repeat steps 2 and 3 if necessary
4. Position update: $\mathbf{q}^1 = \text{normalize}(\mathbf{q}^0 + \mathbf{H}\,\mathbf{u}^1\,\Delta t)$

Multi-contact models

Propagation: contact forces act one at a time

Simultaneity: all contact forces act simultaneously



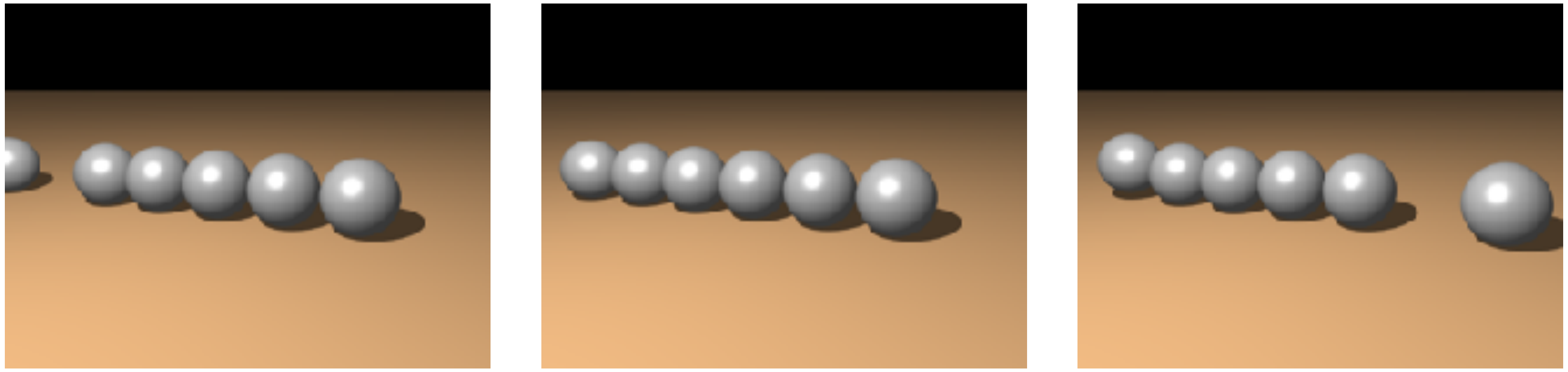


[Baraff 1989]

Corresponding algorithms: ***impulse-based*** and ***constraint-based***

Impulse-based methods

1. Pick an unresolved collision
2. Apply pairwise collision impulse
3. Repeat

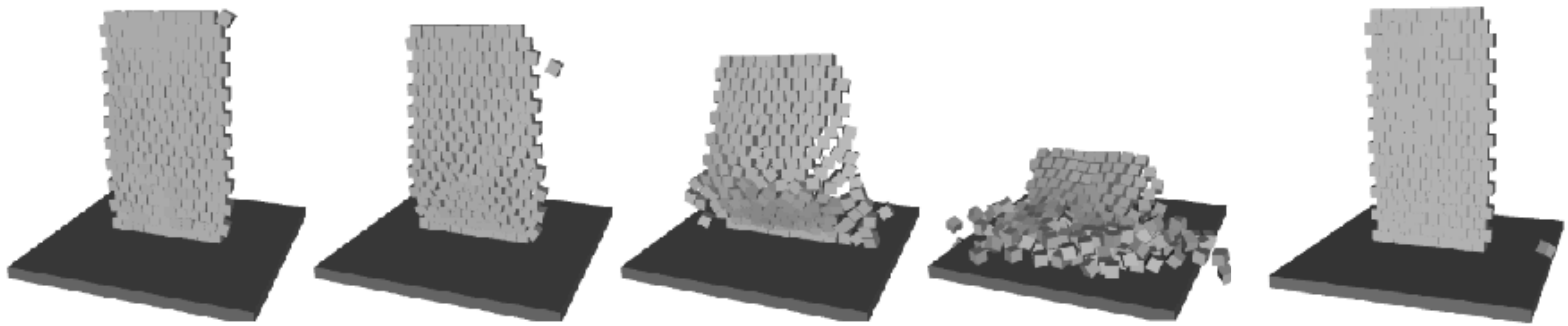


[Guendelman et al. 2003]

Guendelman et al. [2003] do this twice:

- Handle collisions before adding external forces
- Handle resting contact afterwards (set $\varepsilon = 0$)

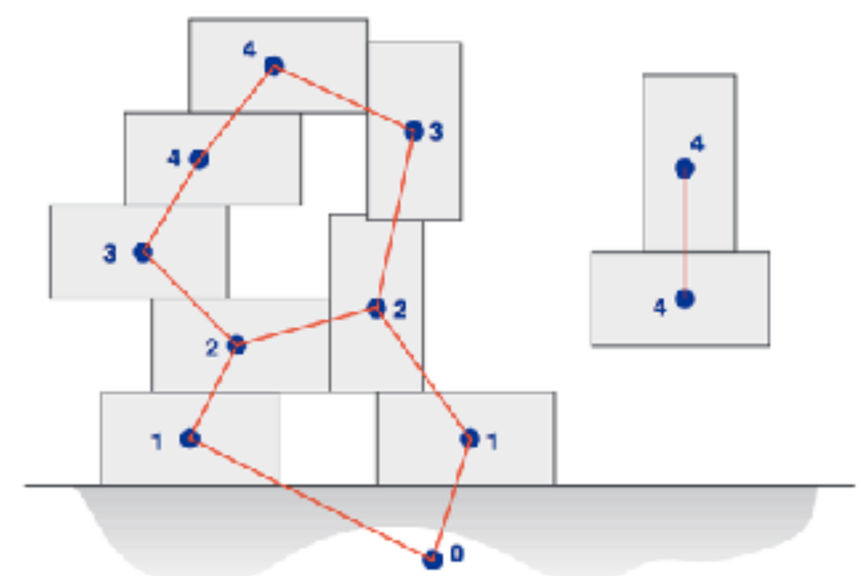
Shock propagation



without shock propagation

with shock prop.
[Erleben 2004]

1. Build DAG of layers from ground up
2. Resolve bottom layer with resting contact impulses, then freeze
3. Repeat with next layer up



[Erleben 2005]

Cloth collisions

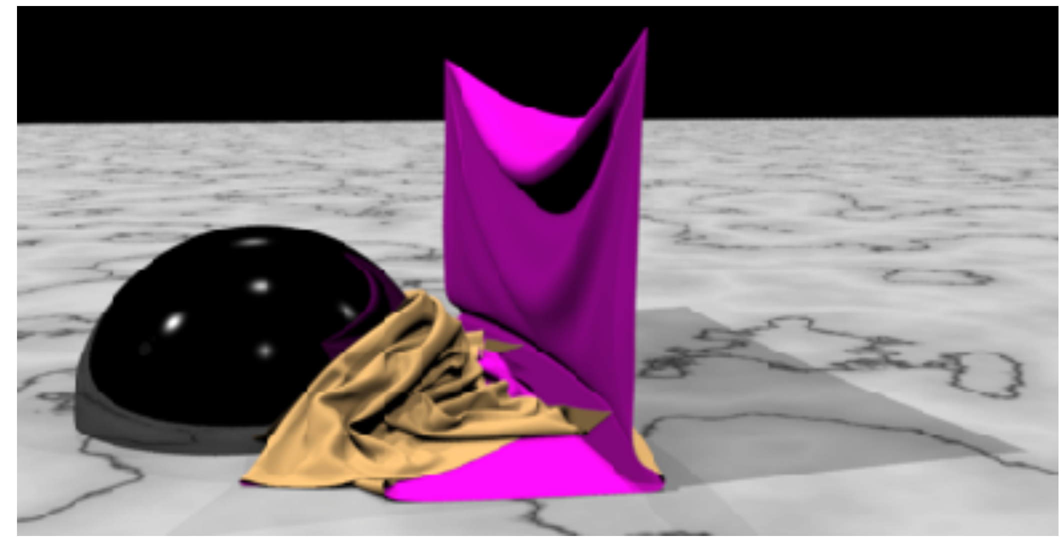
Collisions occur between face/vertex and edge/edge pairs

Typical strategy

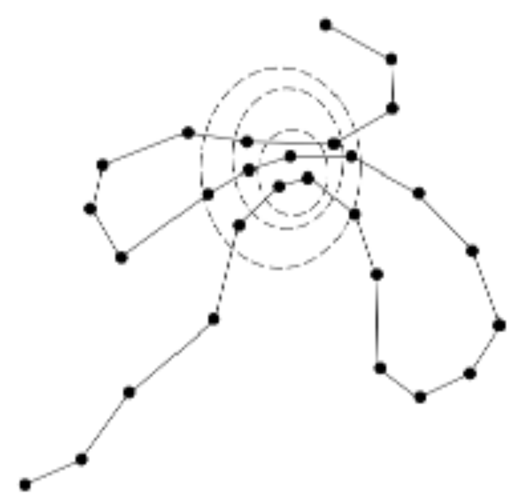
= ***repulsion springs***

+ ***geometric collision resolution*** (impulses with $\varepsilon = 0$)

+ ***impact zones*** (freeze sets of colliding vertices)



[Bridson et al. 2002]



[Provot 1997]

[Harmon et al. 2008]

Constraint-based methods

Designed primarily for contact ($v_{\text{rel,n}} = 0$)

Acceleration-level constraints:

$$a_{\text{rel,n}} \geq 0, \quad f_{\text{n}} \geq 0, \quad a_{\text{rel,n}} \cdot f_{\text{n}} = 0$$

or ***velocity-level***:

$$v_{\text{rel,n}} \geq 0, \quad j_{\text{n}} \geq 0, \quad v_{\text{rel,n}} \cdot j_{\text{n}} = 0$$

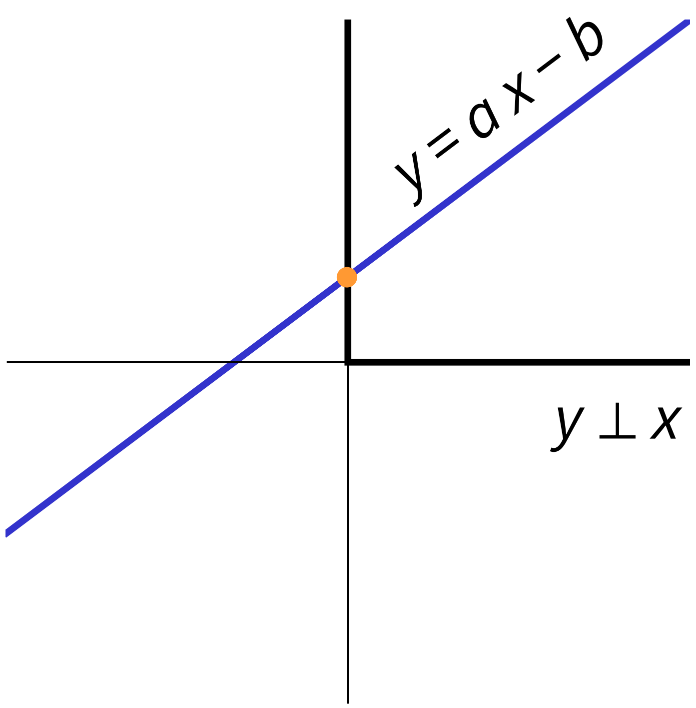
Shorthand:

$$0 \leq v_{\text{rel,n}} \perp j_{\text{n}} \geq 0$$

$$\mathbf{0} \leq \mathbf{A}\,\mathbf{j} + \mathbf{b} \perp \mathbf{j} \geq \mathbf{0}$$

The linear complementarity problem

$$\mathbf{0} \leq \mathbf{A}\,\mathbf{x} + \mathbf{b} \;\perp\; \mathbf{x} \geq \mathbf{0}$$



Theory: Does a solution exist? Is it unique?

- LCP has a unique solution if **A** is s.p.d.
- In this case, equivalent to constrained optimization:

$$\min \;\tfrac{1}{2}\,\mathbf{x}^{T}\mathbf{A}\,\mathbf{x} + \mathbf{b}\,\mathbf{x}$$
$$\text{s.t. } \mathbf{x} \geq \mathbf{0}$$

Practice: How to solve?

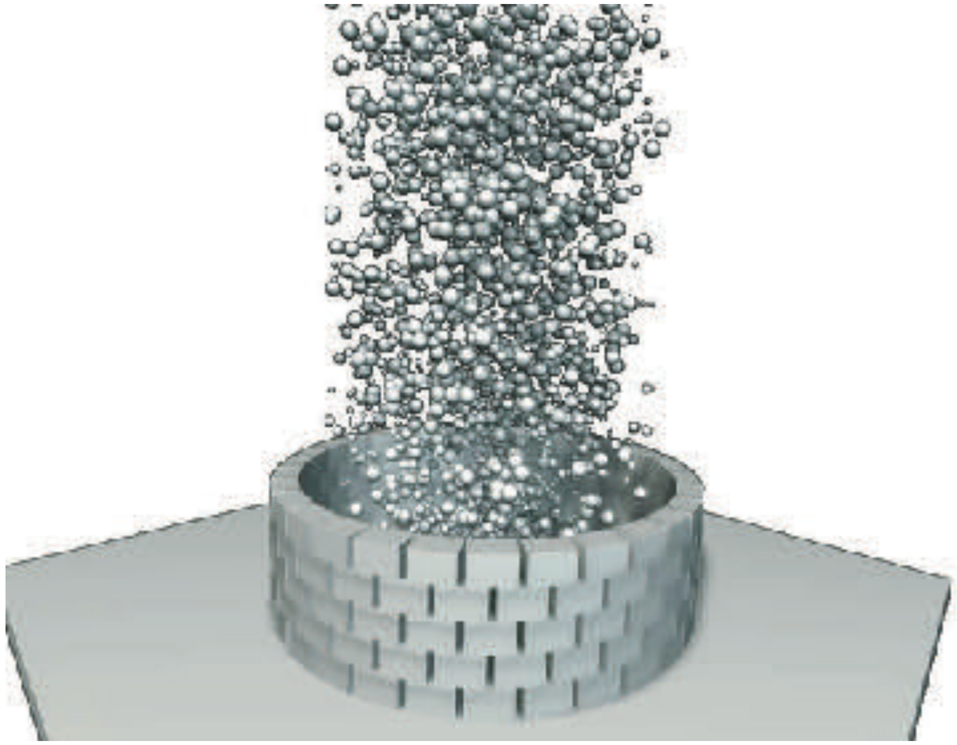
Dantzig's algorithm

Explained nicely by Baraff [1994]

1. Classify *i*th contact as "clamped" if $a_i = 0$, $f_i > 0$, "unclamped" if $a_i > 0$, $f_i = 0$
2. Pick unresolved contact $a_d < 0$
3. Find change in f_d, f_{clamped} so that $a_d = 0$, $a_{\text{clamped}} = 0$
4. Pick min step length which changes status of any contact
5. Repeat

Step 3 requires linear solve over all clamped contacts!

Projected Gauss-Seidel



[Erleben 2007]

If direct LCP solve is too expensive, can use a Gauss-Seidel approach:

1. Pick an unresolved contact
2. Compute appropriate change in contact response
3. Repeat

Nearly identical to a propagation method! What's the difference?

Still requires shock propagation

Summary and generalizations

Impulse-based methods: solve one contact at a time

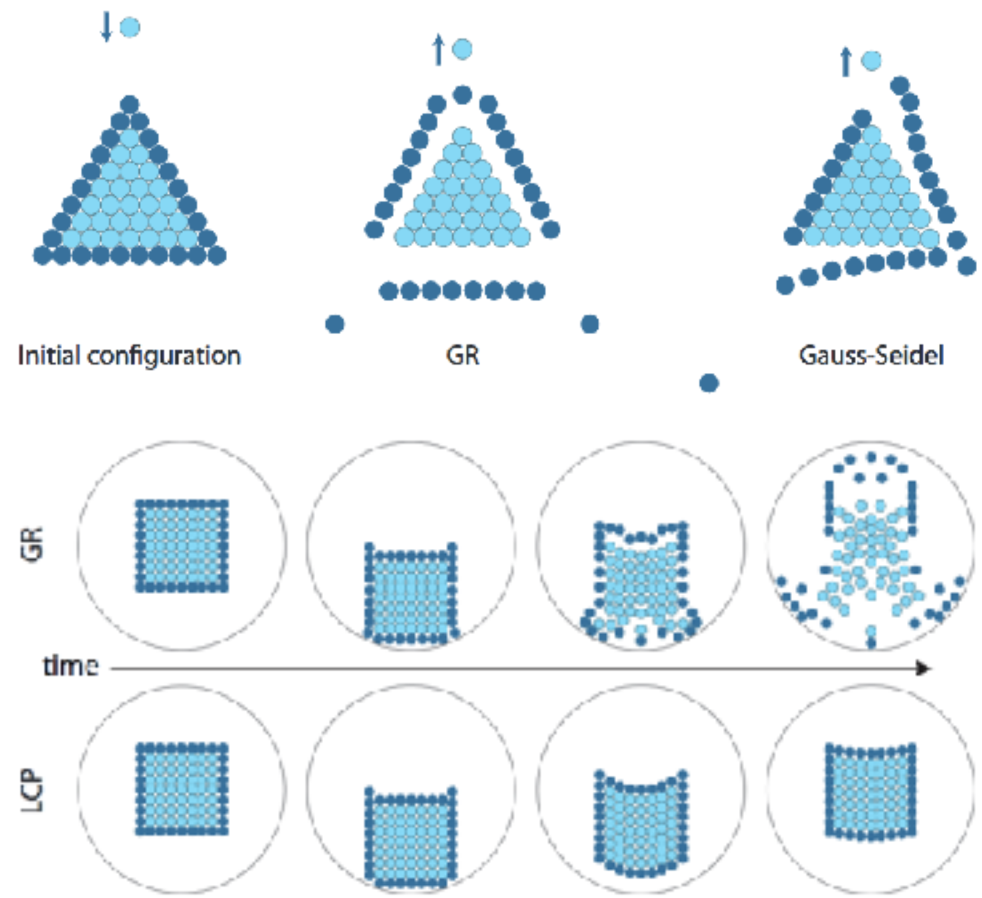
- Good for collision propagation

Constraint-based methods: solve all contacts together as LCP

- Good for symmetry, resting contact

Recent approach: ***generalized reflections*** [Smith et al. 2012]

- Solve one LCP at a time with only “violating” collisions
- Preserves symmetry and propagation



[Smith et al. 2012]