CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 8: Pigeonhole Principle

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1. If there are n people at a party, show that two of them know the same people of those present.
2. Given any $n+1$ distinct integers between 1 and $2n$, show that one of them is divisible by another. Is this the best possible i.e. is the conclusion still true for n integers between 1 and $2n$?
3. Prove that in any group of six people there are either three mutual friends (who all know each other) or three mutual strangers (who all don't know each other.)
4. Seventeen people correspond by mail with each other, each one with all the others. In each letter written only one of three topics is discussed. Prove that there are three people who write to each other about the same topic.
5. Suppose there are nine points in three dimensional Euclidean space, all with integral coordinates (i.e. lattice points). Show that there's a lattice point (i.e. a point with integral coordinates, but not necessarily one of the nine selected points) on the interior of one of the line segments joining two of these points.
6. Let a_j, b_j, c_j be integers for $1 \leq j \leq N$. Assume that for each j, at least one of a_j, b_j, c_j is odd. Show that there exist integers r, s, t, such that $ra_j + sb_j + tc_j$ is odd for at least $4N/7$ j, $1 \leq j \leq N$.