

CS105L: Discrete Structures  
I semester, 2006-07

Homework # 8

Due before class on **Friday, October 6th, 2006**

Instructor: Amitabha Bagchi

September 28, 2006

1. Count the following:
  - (a) The number of 1-regular graphs on  $n$  vertices.
  - (b) The number of 2-regular graphs on  $n$  vertices.
  - (c) The number of paths of length  $k$  between two vertices  $u$  and  $v$  in a complete graph on  $n$  vertices.
  - (d) The number of bipartite graphs on  $n$  vertices.
2. For some natural number  $d$ , let's say the vertex set of a graph is labelled with the strings from  $\{0, 1\}^d$  i.e. each vertex has a unique label which is a  $d$ -bit string and every  $d$ -bit string corresponds to a vertex. Further we say that there's an edge between two vertices if their labels differ in exactly one position. This graph is known as the  *$d$ -dimensional cube*. Determine the average degree, number of edges, diameter, girth and circumference of this graph. Give proofs of all your claims.
3. Show that a graph is bipartite if and only if every induced cycle has even length.