CS105L: Discrete Structures
I semester, 2006-07

Homework # 6

Due before class on **Thursday, September 21st, 2006**

Instructor: Amitabha Bagchi

September 14, 2006

1. Find a recurrence relation for the number of ways to completely cover a $2 \times n$ chessboard with a 1×2 dominos. Solve the recurrence to determine this number of ways.

2. Find a recurrence relation for the number of *strictly* increasing sequences of positive integers that have 1 as their first term and n as their last term where n is a positive integer. That is, sequnces $a_1, a_2, \ldots, a_k$ where $a_1 = 1$ and $a_k = n$ and $a_j < a_{j+1}$ for $j = 1, 2, \ldots, k-1$. What are the initial conditions? How many such sequnces are there when n is a positive integer and $n \geq 2$?

3. Let $S(m, n)$ denote the number of onto functions from a set with m elements to a set with n elements. Show that $S(m, n)$ satisfies the recurrence:

$$S(m, n) = n^m - \sum_{k=1}^{n-1} \binom{n}{k} S(m, k)$$

whenever $m > n$ and $n > 1$ with the initial condition that $S(m, 1) = 1$.