CS105L: Discrete Structures
I semester, 2006-07

Homework # 12

Due before class on **Friday, November 24th, 2006**

Instructor: Amitabha Bagchi

November 16, 2006

1. A k-chromatic graph is called *critically k-chromatic*, or just critical if $\chi(G-v) < k$ for all $v \in V(G)$. Show that every k-chromatic graph has an induced subgraph that is critically k-chromatic and that any such subgraph has minimum degree at least $k-1$. (**Note.** This question deals with *vertex* colouring. The next two deal with *edge* colouring.)

2. Prove that $\chi'(G)$ (the edge-chromatic number of G) is equal to k for a k-regular bipartite graph without using the proof of the theorem which says $\chi'(G) = \Delta(G)$ for bipartite graphs i.e. come up with a simpler proof specifically for k-regular graphs. Please **do not** use induction.

3. Use the proof for k-regular bipartite graphs to prove that $\chi'(G) = \Delta(G)$ for all bipartite graphs.