CS105L: Discrete Structures
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Tutorial Sheet 8: Graph Theory continued

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1. An *edge colouring* of a graph is an assignment of colours (or just natural numbers) to the edges of a graph. A *colour class* in a edge coloured graph is a maximal set of edges which have been assigned the same colour. A graph is said to be properly edge coloured if no vertex has more than one edge incident on it from the same colour class. The *edge chromatic number* of a graph, $\chi'(G)$ is the minimum number of colours required to properly edge colour a graph. Prove the following theorem due to Vizing:

   Theorem 8.1 (**Vizing**) *For a graph G of maximum degree Δ:*

   $$\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1.$$

2. An *independent set* in a graph G is a set of vertices which induce a subgraph on G which has no edges. The *chromatic number* of G, denoted $\chi(G)$, is the minimum number of independent sets which cover the entire graph i.e. the minimum number of independent sets whose union is the entire vertex set, V, of G. If d is the maximum degree of the graph show that

   $$\chi(G) \leq d + 1.$$

3. A graph is said to be an (α, β, n)-*expander* if it has n vertices and α is the largest number such that every set of size at most βn has escape ratio at least α. Let us define G^k to be the graph obtained from G by putting edges between all pairs of vertices which have a path of length at most k between them in G. Can we say that G^2 is an (α_2, β_2, n)-expander? What are the values of α_2 and β_2? Generalize this to k.