CS105L: Discrete Structures
I semester, 2005-06

Tutorial Sheet 1: Propositional Logic

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All problems in this sheet are from Bogart, Drysdale and Stein's book. The numbers in bold in the parentheses indicate the location of the problem in that book.

1. (**3.1, Prob 1**) Give truth tables for the following expressions:
   (a) $(s \vee t) \wedge (\neg s \vee t) \wedge (s \vee \neg t)$
   (b) $(s \Rightarrow t) \wedge (t \Rightarrow u)$
   (c) $(s \vee t \vee u) \wedge (s \vee \neg t \vee u)$
2. (**3.1, Prob 8**) Use a truth table to show that $(s \vee t) \wedge (u \vee v)$ is equivalent to $(s \wedge u) \vee (s \wedge v) \vee (t \wedge u) \vee (t \wedge v)$.
3. (**3.1, Prob 9**) Use DeMorgan's Law, the distributive law and the previous problem to show that $\neg((s \vee t) \wedge (s \vee \neg t))$ is equivalent to $\neg s$.
4. (**3.1, Prob 13**) Suppose that for each line of a 2-variable truth table you are told the value in the final column, true or false. (For example, you might be told that the final column contains T, F, T and F in that order.) Explain how to create a logical statement using the symbols $s, t, \wedge, \vee$, and $\neg$ that also realizes the same truth table. Can you extend this procedure to an arbitrary number of variables?
5. (**3.2, Prob 1**) For what positive integers x is the statement $(x-2)^2 + 1 \leq 2$ true? For what integers is it true? For what real numbers is it true? If we expand the universe for which we are considering a statement about a variable, does this always increase the size of the statment's truth set?
6. (**3.2, Prob 6**) Using $s(x, y, z)$ to be the statement $x = yz$ and $t(x, y)$ to be the statement $x < y$, write a formal statement for the definition of the greatest common divisor of two numbers.
7. (**3.2, Prob 10**) Rewrite the following statement without any negations: It is not the case that there exists an integer n such that $n > 0$ and for all integers $m > n$, for every polynomial equation $p(x) = 0$ of degree m there are no real numbers for solutions.