CS105L: Discrete Structures
I semester, 2005-06

Homework # 4

Due before class on **Thursday, September 15, 2005**

Instructor: Amitabha Bagchi

September 9, 2005

1. Prove that

$$\binom{r}{1}^2 + \binom{r}{2}^2 + \cdots + \binom{r}{i}^2 + \cdots + \binom{r}{r}^2 = \binom{2r}{r}$$

   And also that the generating funtion for $a_r = \binom{2r}{r}$ is

$$\hat{a}(z) = (1 - 4z)^{-1/2}$$

   Use recurrence equations to solve the following problems.

2. (**The Towers of Hanoi**) A set of r circular rings of tapering sizes are slipped onto a peg with the largest ring at the bottom and the smallest ring at the top. There are two other pegs available which are empty. All r rings are to be moved to the third peg. The second peg can be used temporarily. In how many moves can the rings be transferred to the third peg given that in the course of transferring the rings a ring of larger radius cannot be placed on top of a ring of smaller radius?

3. How many r-bit binary sequences contain no adjacent 0s?