CS105L: Discrete Structures
I semester, 2005-06

Homeworks # 10 & 11

Due before 11 AM on **Tuesday, November 29, 2005**

Instructor: Amitabha Bagchi

November 20, 2005

This problem sheet is two homeworks combined. The total credit for this will be worth 10% of the grade.

1. A throws six dice and wins if he gets at least one occurence of the digit 5. B throws twelve dice and wins if he scores at least two occurences of the digit 4. Which of them has the greater probability of winning?

2. There are n urns of which the rth urn contains $r-1$ red balls and $n-r$ blue balls. You pick an urn at random and pick two balls at random from it. What is the probability that:
   (a) The second ball is blue.
   (b) The second ball is blue given that the first ball is blue.

3. A man has 5 coins. Two of them have heads on both sides, one has tails on both sides and two have heads on one side and tails on the other side.
   (a) He shuts his eyes, picks a coin at random and tosses it. What's the probability that the lower face is heads?
   (b) He opens his eyes and sees that the coin is showing heads. What's the probability that the lower face is heads?
   (c) He shuts his eyes again and tosses the coin again. What is the probability that the lower face is heads?
   (d) He opens his eyes again and sees that the coin is showing heads. What's the probability that the lower face is heads?
   (e) He discards this coin, picks another at random and tosses it. What is the probability that it shows heads?

4. Given n bins, what is the expected number of balls we have to throw to ensure that each bin gets one ball?
   Hint. Define a random variable X_i as the number of balls needed to be thrown, after $i-1$ bins already contain balls, before a ball lands in a new bin. You will need to use linearity of expectation.

5. Given a biased coin which turns up heads with probability $p < 1/2$ and tails with probability $q = 1 - p$, devise an experiment which will yield two unbiased outcomes (i.e. two outcomes, each occuring with probability 1/2.)

6. We select three numbers x, y, z independently and uniformly from the interval $(0, 1)$. What is the probability that $xy > z^2$?