CS105L: Discrete Structures
I semester, 2005-06

Homework # 1

Due before class on **Friday, August 12, 2005**

Instructor: Amitabha Bagchi

August 5, 2005

Write complete proofs for all problems. No credit will be given for answers lacking proper explanation.

1. A certain country is inhabited only by people who either always tell the truth or always tell lies and who will respond to questions only with a "yes" or a "no." A tourist comes to an unmarked fork in the road where one branch leads to the capital and the other does not. There is an inhabitant, Mr Z., standing at the fork. What *single* question should the tourist ask him to determine which branch to take?

2. Prove that $|A| < |2^A|$ for all sets $|A|$. Argue that this statement is equivalent to proving that there cannot exist a surjection from A to 2^A and then prove that no such surjection exists.

3. Prove that the strong principle of induction is not more powerful than the weak principle of induction. In other words, prove that the weak principle of induction implies the strong principle of induction.

4. Prove by induction:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

5. n couples arrived at a party and were greeted by the host and the hostess at the door. After several rounds of handshaking the host asked the guests as well as the hostess (his wife) to indicate the number of hands each one of them had shaken. He got $2n + 1$ different answers. Given that no one shook hands with his or her own spouse, how many hands had the hostess shaken? Prove your answer by induction.