COL202: Discrete Mathematical Structures. I semester, 2020-21.
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Tutorial Sheet 1: Logic.
2 October 2020

Important: The question marked with a ♠ is to be submitted via gradescope by 11:59PM on the day that you have your tutorial.

Note: Please read *all* of Chapter 3 of [1] (even the parts not discussed in class) before attempting this sheet. All questions in this sheet are from that book, question numbers and page indicated in brackets refer to the pdf linked on the course page.

Problem 1 (Prob 1, pp 94)
Give truth tables for the following compound propositions

1. $(s \vee t) \wedge (\neg s \vee t) \wedge (s \vee \neg t)$
2. $(s \Rightarrow t) \wedge (t \Rightarrow u)$
3. $(s \vee t \vee u) \wedge (s \vee \neg t \vee u)$

Problem 2 (Prob 8, pp 94)
Use a truth table to show that $(s \vee t) \wedge (u \vee v)$ is equivalent to $(s \wedge u) \vee (s \wedge v) \vee (t \wedge u) \vee (t \wedge v)$.

Problem 3 (Prob 8, pp 107)
Write the following statement as a logical expression: The product of odd integers is odd. You may assume that $\texttt{odd} : \mathbb{Z} \rightarrow \{T, F\}$ is a predicate that maps odd integers to T and even integers to F.

Problem 4 (Prob 4, pp 106)
The definition of a prime number is that it is an integer greater than 1 whose only positive integer factors are itself and 1. Find two ways to write this definition so that all quantifiers are explicit. (It may be convenient to introduce a variable to stand for the number and perhaps a variable or some variables for its factors.)

Problem 5 (Theorem 3.2, pp 100)
Here is the statement of a theorem given in [1] written in slightly different terms.

Theorem 1
Suppose we have a domain D and two predicates $P, Q : D \rightarrow \{T, F\}$. Let $A = \{x \in D : Q(x) \text{ is } T\}$. Show that

1. $\forall x \in A : P(x)$ is logically equivalent to $\forall x \in D : Q(x) \Rightarrow P(x)$.
2. $\exists x \in A : P(x)$ is logically equivalent to $\exists x \in D : Q(x) \wedge P(x)$.

Write a proof for this. You may read the proof in the book and then write it in your own words.

Problem 6 (Prob 6, pp 106)
Using $s(x, y, z)$ to be the statement $x = yz$ and $t(x, y)$ to be the statement $x < y$, write a formal statement for the definition of the greatest common divisor of two numbers.

Problem 7 (Prob 10, pp 107)
Rewrite the following statement without any negations. It is not the case that there exists an integer n such that $n > 0$ and for all integers $m > n$, for every polynomial equation $p(x) = 0$ of degree m there are no real numbers for solutions.

Problem 8 ♠ (Prob 11, pp 107)
Consider the following slight modification of Theorem 3.2. For each part below, either prove that it is true or give a counterexample. Let U_1 be a universal set contained in another universal set U_2, i.e., $U_1 \subseteq U_2$. Suppose that $q(x)$ is a statement such that $U_1 = \{x \in U_2 \mid q(x) \text{ is true}\}$.

1. $\forall x \in U_1 : p(x)$ is equivalent to $\forall x \in U_2 : q(x) \wedge p(x)$.
2. $\exists x \in U_1 : p(x)$ is equivalent to $\exists x \in U_2 : q(x) \Rightarrow p(x)$.

Problem 9 (Prob 13, pp 107)
Each expression below represents a statement about the integers. Using $p(x)$ for "x is prime," $q(x, y)$ for "$x = y^2$," $r(x, y)$ for "$x \leq y$," $s(x, y, z)$ for "$z = xy$," and $t(x, y)$ for "$x = y$," determine which expressions represent true statements and which represent false statements.

1. $\forall x \in Z : \exists y \in Z : q(x, y) \vee p(x)$.
2. $\forall x \in Z : \forall y \in Z : s(x, x, y) \Leftrightarrow q(x, y)$.
3. $\forall y \in Z : \exists x \in Z : q(y, x)$.
4. $\exists z \in Z : \exists x \in Z : \exists y \in Z : p(x) \wedge p(y) \wedge \neg t(x, y)$.

References

[1] K. Bogart, S. Drysdale, C. Stein Discrete Math for Computer Science Students. 2005.