Release date: June 15, 2020 Deadline: June 22, 2020: 23:00

Refer to recorded lectures on the secretary problem for this homework. Submission format: PDF is preferred. Photographs and scans are also acceptable, but it's your responsibility to ensure that they are clear and readable.

In the *real-time secretary* problem, numbers from an adversarially chosen set $\{x_1, \ldots, x_n\}$ appear at their respective arrival times $T_1, \ldots, T_n$ that are distributed independently and uniformly in $[0, 1]$. As usual, whenever a number appears, an algorithm is allowed to either pick it and discard the remaining input, or discard it and continue. Like in the secretary problem, our goal is to design an algorithm that maximizes the probability of "success"; we say that the algorithm succeeds if it picks $\max_i x_i$. Crucially, in this case, n is not known to the algorithm in advance (otherwise it could simply behave like the secretary algorithm, because the numbers appear in a uniformly random order).

1. **[6 points]** Consider the following algorithm for real-time secretary, which involves a parameter $\tau \in [0, 1]$.

   - Discard all numbers arriving before time τ. Let θ be the maximum of all numbers that arrive before time τ. ($\theta = -\infty$ if no number appears before time τ.)
   - Thereafter, accept the earliest arriving number which exceeds θ.

   Derive a lower bound on the success probability of this algorithm as a function of τ. Hence show that there exists a τ for which the success probability is at least $1/e$.

2. **[4 points]** Prove that no algorithm for the real-time secretary problem can have success probability $(1/e) + \varepsilon$ for any constant $\varepsilon > 0$.