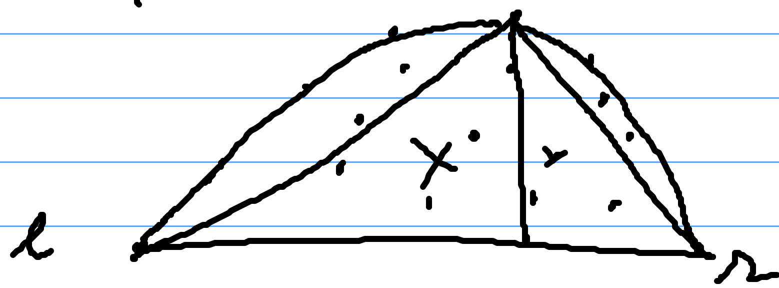
Computational Geometry Lecture 9

Topic Quickhull



$$T(n) = T(L) + T(R) + O(n)$$

$$L \gg R \Rightarrow T(n) \approx \Theta(n^2)$$

1. Pair up points arbitrary
   ($\frac{n}{2}$ pairs) $O(n)$

2. Among the $\frac{n}{2}$ pairs, find the pair, say (l^m, r^m) that has the "median slope" among all the $\frac{n}{2}$ pairs. $O(n)$
   random pair

3. Find the extreme point orthogonal in direction to (l^m, r^m), call p_m $O(n)$
   Draw a vertical line thru p_m and use it to subdivide the problem

Now prune the points on the basis of the following test

$O(n)$ In the left subproblem, consider a pair p_1, p_2 (p_2 is closer to the vertical line). If $p_m\, p_2\, p_1$ is a right turn, then we can discard p_2

(Likewise for the right subprob)

5. Call the algorithm recursively on the left and right subset of remaining points.

(if there are some output points)

No subproblem has size $\geq \frac{3}{4}n$

$$T(n, h) \leq T(n_\ell, h_\ell) + T(n_r, h - h_\ell - 1) + O(n)$$

\# input points

\# output points

$$T(n, 1) = c_1 \cdot n$$

$$n_\ell, n_r \leq \frac{3}{4}n$$

$$T(n,h) \le T(n_l,h_l) + T(n_r,h_r) + c\cdot n$$

<u>Claim</u> : Soln is of the form $T(n,h) = O(n \log_2 h)$ for $h>1$

$= k\, n \log_2 h$

Proof by induction

Plug in this soln and verify

$$T(n_l, h_l) = k\, n_l \log_2 h_l$$

$$T(n_r, h_r) = k\, n_r \log_2 (h - h_l)$$

R.H.S. $k\left(n_l \log h_r + n_r \log(h - h_r)\right) + cn$

Since $n_i \le \frac{3}{4} n$

$$\Rightarrow \quad kn\left(\alpha \log h_l + (1-\alpha)\log(h-h_l)\right) + cn$$

$$\frac{1}{2} < \alpha < \frac{3}{4}$$

To maximise the R.H.S. w.r.t h_l

it is obtained when $h_l = \alpha \cdot h$

$$\alpha \log(\alpha h) + (1-\alpha)\log((1-\alpha)\cdot h)$$

$$\le \log \alpha h = \log h - \log \frac{4}{3}$$

$$kn\left(\log h - \log 4/3\right) + cn \le kn \log h$$

$$\Rightarrow \quad k \ge \frac{c}{\log 4/3}$$

Mod. Quickhull runs in $O(n \log n)$ steps

If we chose a splitter at random, the running time of quicksort is expected $O(n \log n)$

Similar result follows for quickhull i.e. expected $O(n \log n)$.

→ Determin. version: Chan, Snoeyink, Yap 1995

→ Rand. version : Bhattacharya, Sen 1996