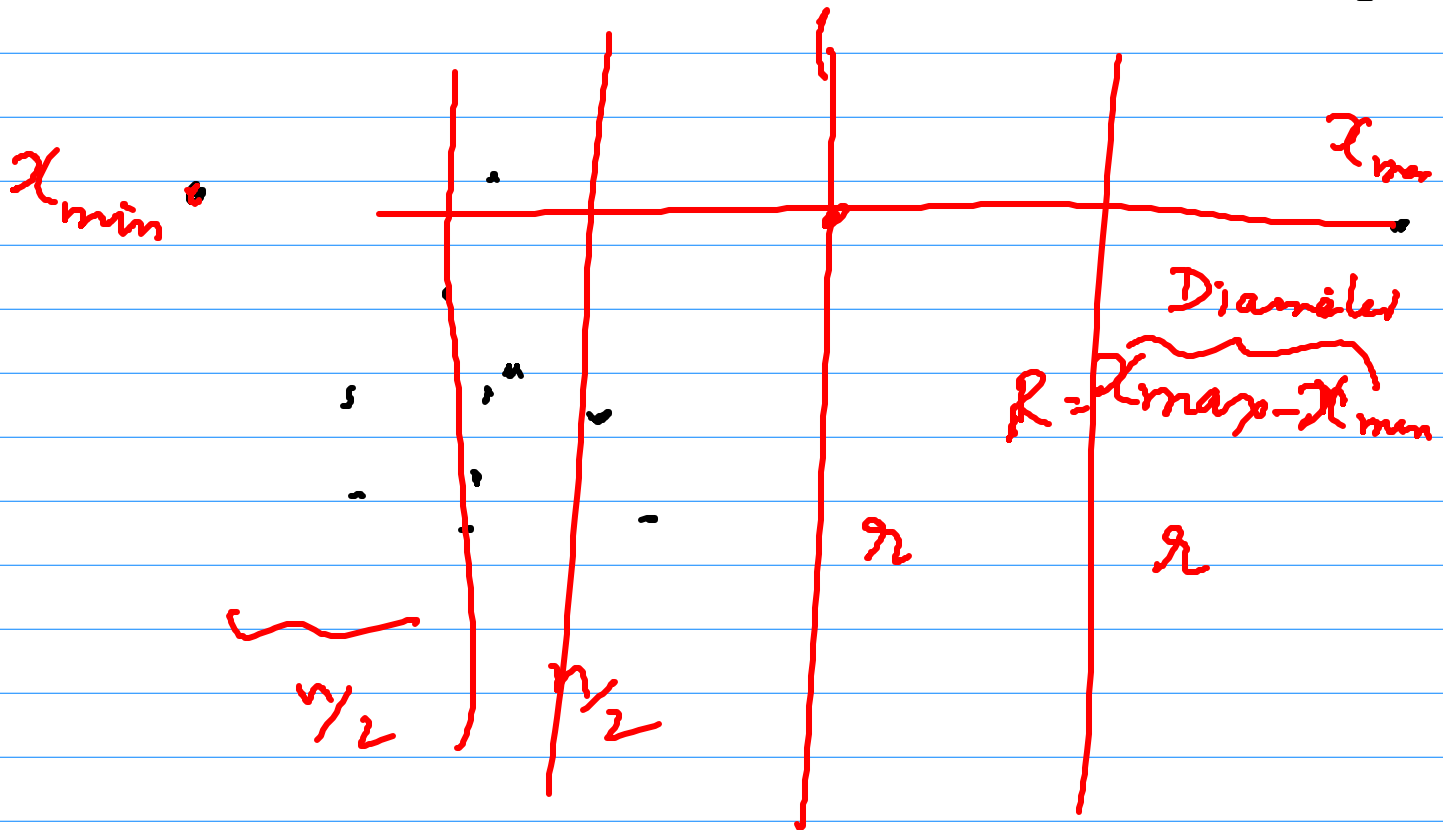


Computational Geometry : CSL 852

Lecture 29

Topic : Well-Separated Partitioning



$$\phi = \frac{R}{r} : \text{spread}$$

$$\log \phi \gg \log n$$

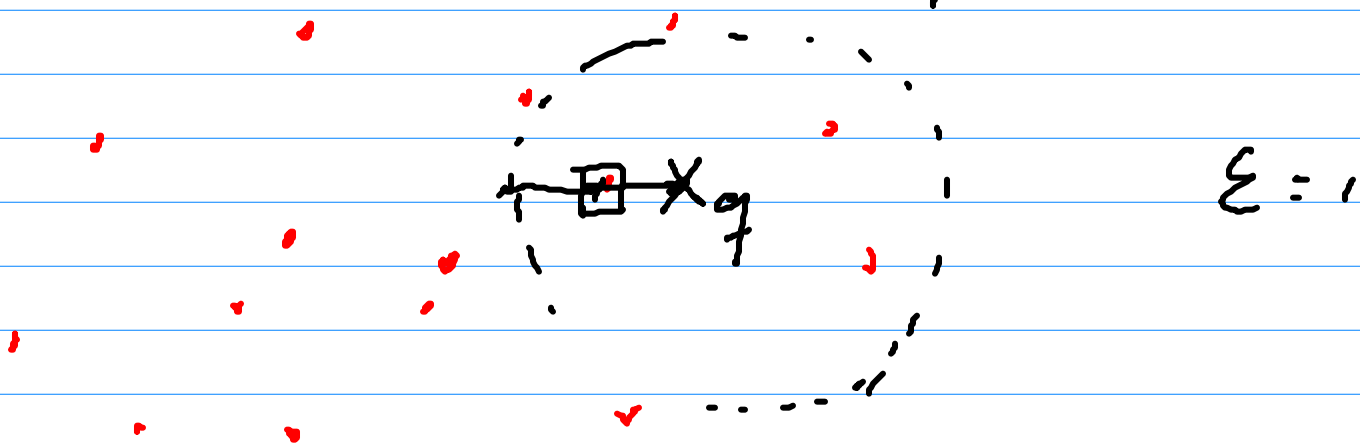
Build a data structure for nearest point query

ϵ . Approximate nearest neighbours

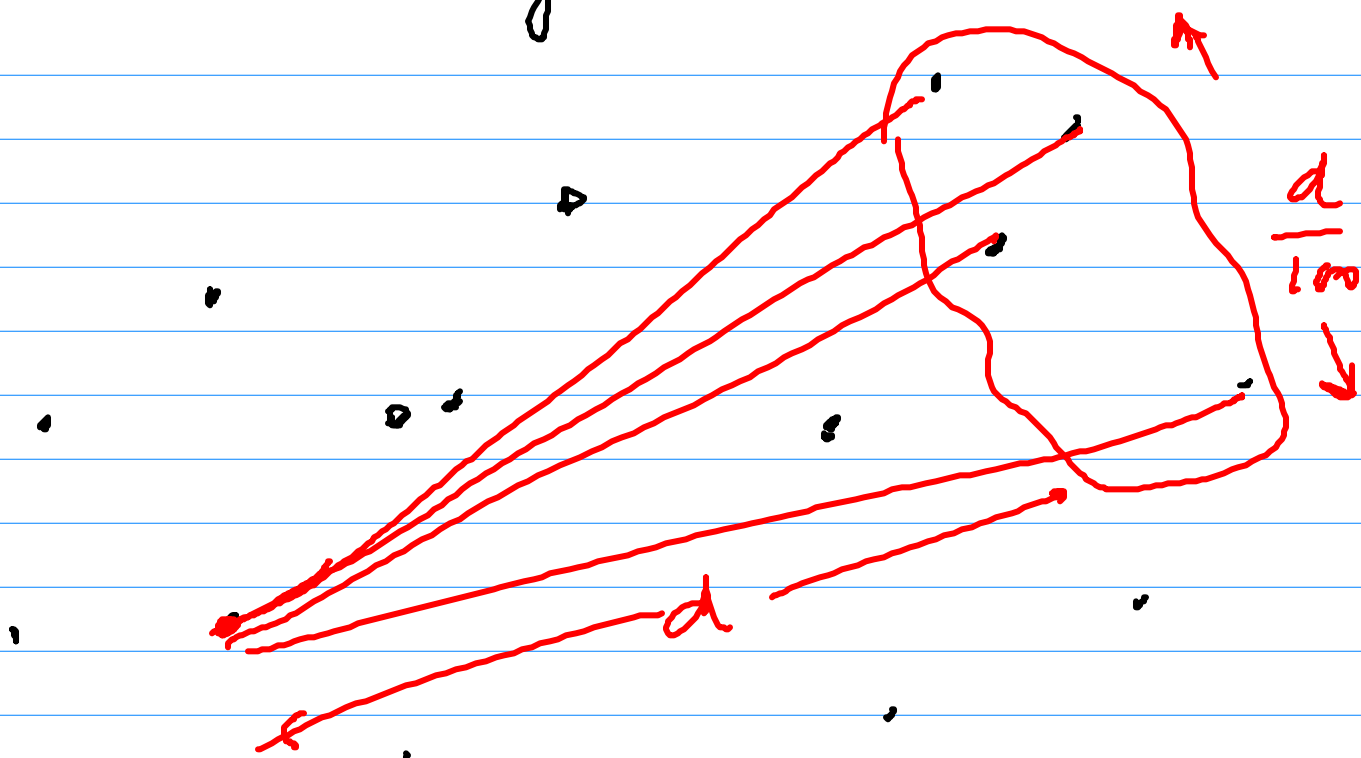
Given a set S of n points
an ϵ -approximate neighbour of some
arbitrary query point q is a
point $p \in S$, s.t.

$$\text{dist}(q, p) \leq (1 + \epsilon) \underbrace{\text{dist}(q, S)}_{\min_{t \in S} \text{dist}(q, t)}$$

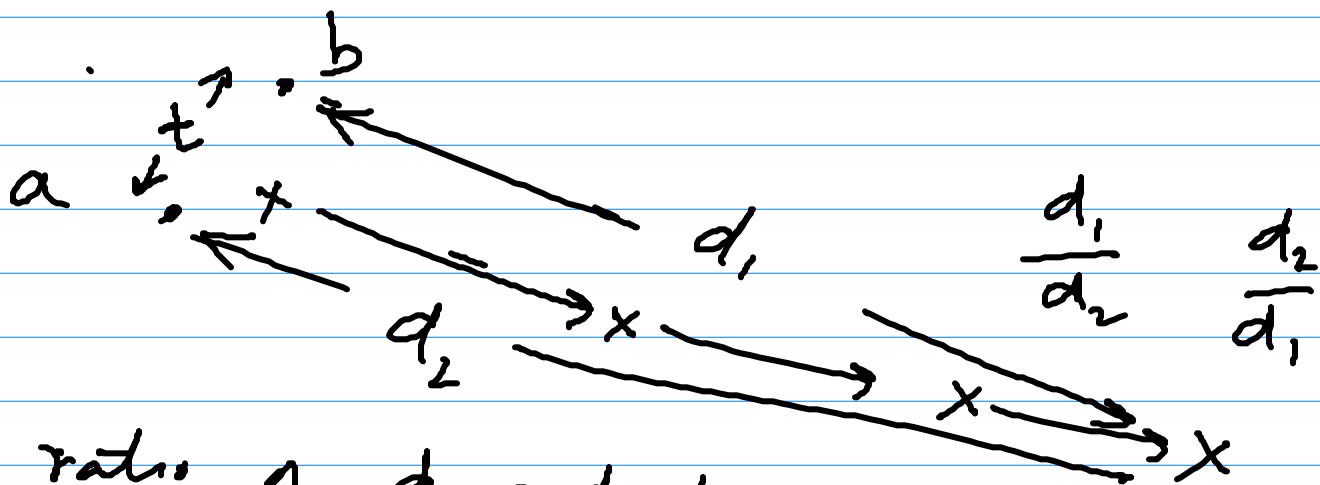
$$\text{dist}(P, Q) = \min_{p \in P, q \in Q} \text{dist}(p, q)$$



N-body simulation



Compute all pairwise forces $\binom{N}{2}$



ratio of d_1 and d_2 decreases as x moves further

By Δ inequality

$$d_1 \leq d_2 + t$$

t ↑
 t ↓

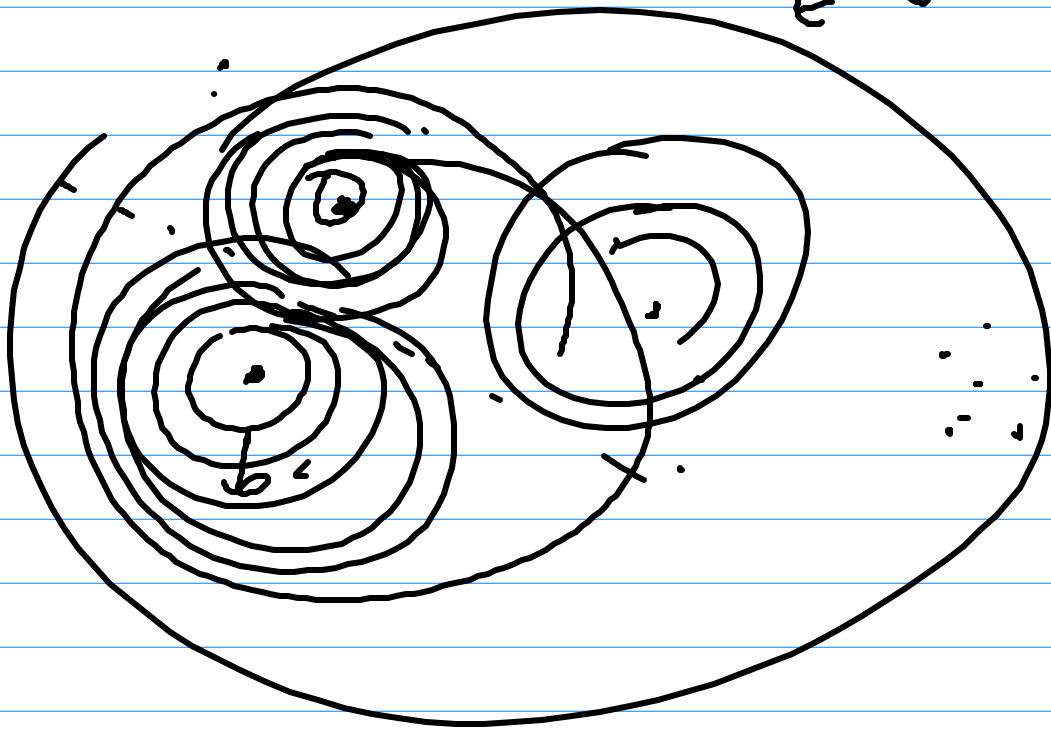
If we are interested in ϵ -approximate nearest neighbour, how far do we go before, a and b become indistinguishable.

$$d_2 \leq d_1 + t, \quad d_1 \geq \frac{t}{\epsilon}$$

$$\min\{d_1, d_2\} \geq \frac{t}{\epsilon}$$

$$\max\{d_1, d_2\} \leq (1 + \epsilon) \min\{d_1, d_2\}?$$

$$d_2 \leq \frac{t}{\epsilon} + t = \frac{t}{\epsilon} (1 + \epsilon)$$



t-spanner

Given a set P of n points in \mathbb{E}^d , we define a ^{wt.} graph $G = (P, E, w)$, such that for

any $p, q \in P$

$$\|p - q\| \leq d_G(p, q) \leq t \|p - q\|$$

$$t = 1 + \epsilon \quad \begin{array}{l} t > 1 \\ \uparrow \\ \text{stretch} \end{array}$$

Intuitively: $|E| = f\left(\frac{1}{\epsilon}\right)$ non-increasing
 $\left(\frac{1}{\epsilon}\right)^d \cdot n$

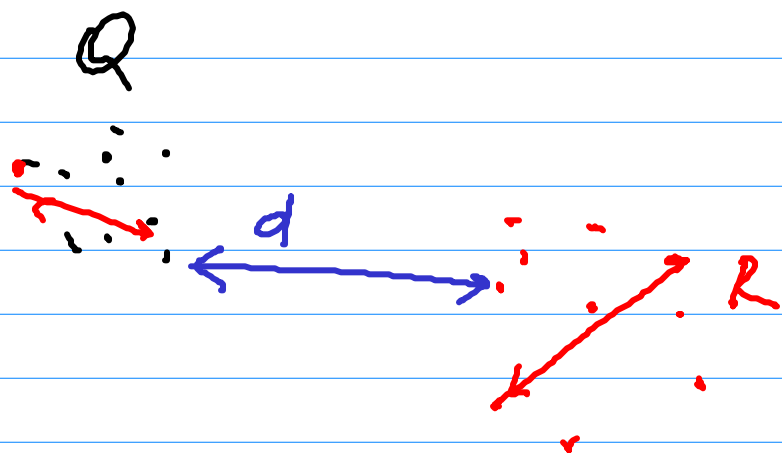
Delaunay Triangulation in 2D is a spanner with $t < 3$

Claim: The MST of the graph G (corresponding to t -spanner) is a t -MST of point set P .

A pair of point sets Q and R are $\frac{1}{\epsilon}$ -separated if

$$\max \{ \text{diam}(Q), \text{diam}(R) \}$$

$$< \epsilon \cdot d(Q, R)$$



Pair Decomposition: For a point set P , a pair decomposition of P is a set of pairs

$$W = \{ (A_1, B_1), (A_2, B_2), \dots, (A_s, B_s) \}$$

such that (i) $A_i, B_i \subset P$

$$(ii) A_i \cap B_i = \emptyset$$

$$(iii) \bigcup_i A_i \otimes B_i = P \otimes P$$

$$A \otimes B = \bigcup_{\substack{x \in A \\ y \in B}} (x, y)$$

ϵ -Well Separated
Pair
Decomposition
(ϵ -WSPD)