COL 352 Lecture 4 Jan 7

Attendence policy

Audit requirement $\geq 75\%$

'E' $\geq 75\%$

As per Institute policy

Σ : alphabet finite set of symbols

Σ^* : set of all strings over Σ including ε

$L \subseteq \Sigma^*$

Recognition problem of L / Membership

Given $w \in \Sigma^*$, does $w \in L$?

$\Sigma = \{0,1\}$ $\Sigma^* : \{0,1\}^*$

$L_1 = \{ w \in \{0,1\}^* \mid \#0\text{'s in } w \text{ is a multiple of } 3 \}$

$010 \notin L_1$, $\quad 0011001000 \in L_1$

L_1 : contains infinitely many strings

1st symbol $\qquad$ contains ε

1	0	1	1	0	0	1	1	0	1	0	1	1	1	$

we can count #0's and verify whether #0's is a multiple of 3.

① The entire string cannot be stored. We have limited / fixed amount of memory (independent of the length of the string)

② The string can scanned / read only once from left to right

③ No specific end of input

Can't count

Solution: do modulo counting (mod 3)

In general to do modulo counting, we need $\lceil \log_2 m \rceil$ many modulo m

m is not dependent on the length of input

Initially the mod 3 counter should be $0 \bmod 3$

Subsequently

Next symbol $\xrightarrow{0}$ $c+1 \bmod 3$

Next symbol $\xrightarrow{1}$ c

If the counter value is 0 at the end of the input - the answer is YES

otherwise NO

the string w is "accepted" (YES)

otherwise w is "not accepted"

"States" represent the relevant information about the string that is seen upto a certain point

- Initial state : when we start scanning
- States would get modified based on the next symbol
- #States is bounded (not dependent on input length)
- The final state at the end of the input should be correct.

In the previous example - there are 3 states $q_0 \quad q_1 \quad q_2$

q_i : i = # 0's mod 3

Computational model

Σ

Q : set of (finite) states

$q_0 \in Q$: initial state

$F \subset Q$: set of "accepting states"

$\delta : Q \times \Sigma \to Q$ state transition function

previous example

$Q \backslash \Sigma$	0	1
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_0	q_2

Q : (q_0, q_1, q_2)

F : q_0

Initial state q_0

Finite State Machine / Deterministic Finite Automaton (DFA) δ is extended to handle strings of length > 1

$$\delta^* : Q \times \Sigma^* \to Q$$

$$\delta^*(q, aw) = \delta^*(\delta(q,a), w)$$

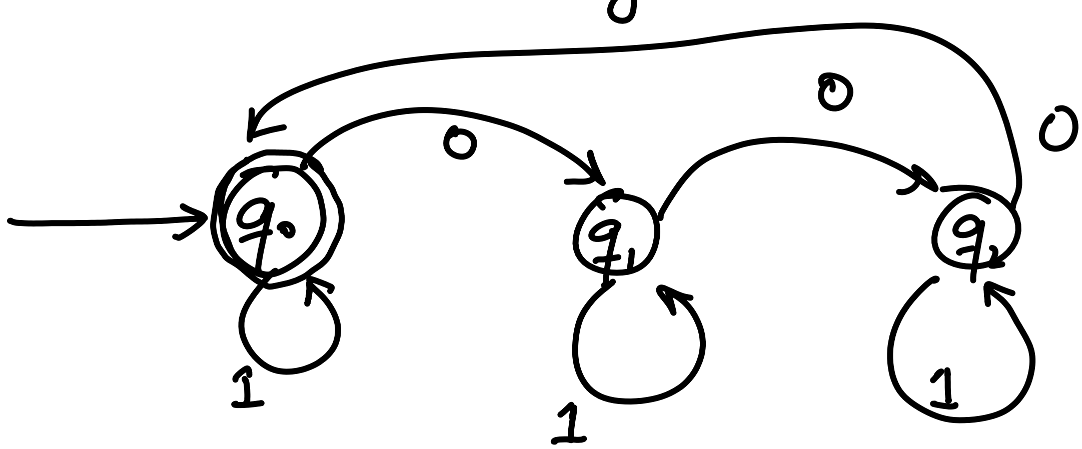
$$a \in \Sigma \quad w \in \Sigma^*$$

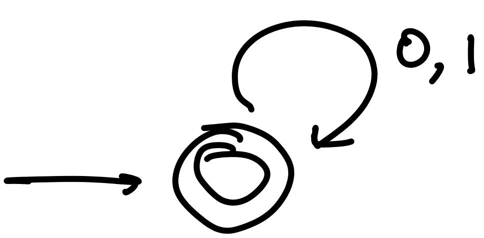
$$\delta^*(q, \varepsilon) = q$$

$$L_M = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \in F \}$$

for a DFA M

Given a machine M, L_M is the set of strings accepted by M





Σ^*

$\emptyset$