COL 352 Lecture 3 Jan 3

Finite alphabet (set of symbols) Σ

Strings : a sequence of symbols from Σ of finite length

$s \in \Sigma^*$ $\qquad$ ε : string of length 0

Language $L \subseteq \Sigma^*$

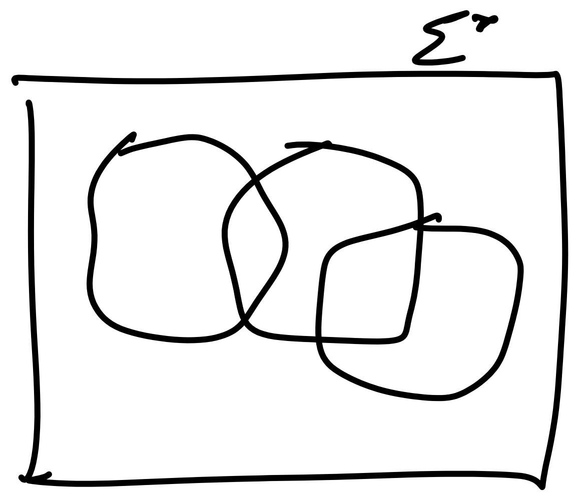
$$\chi_L(w) = \begin{cases} 1 & \text{if } w \in L \\ 0 & \text{otherwise} \end{cases}$$

characteristic function for membership problem in L

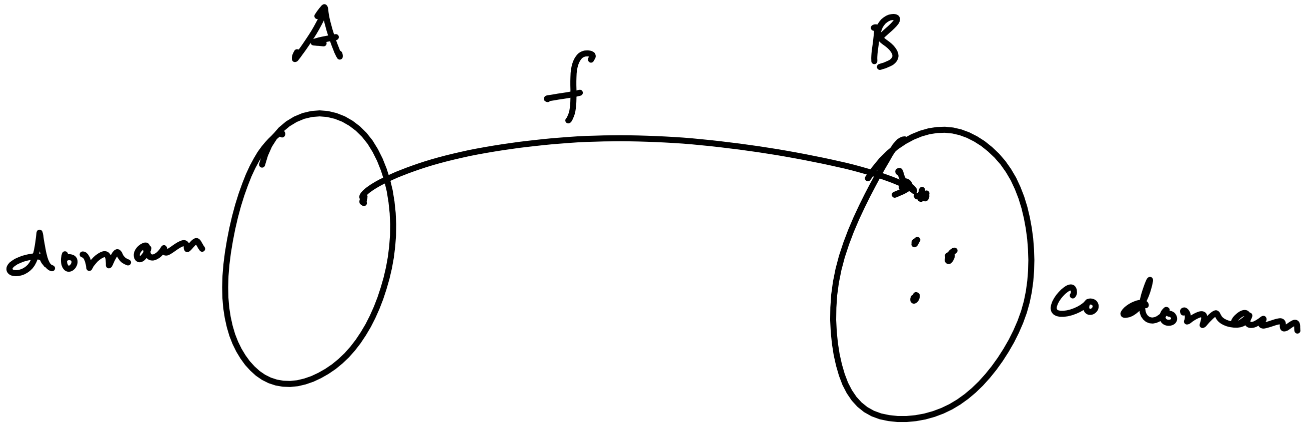
\# Languages $\quad 2^{\Sigma^*}$

\# programs $\quad \sim \quad \Sigma^*$

Classes of languages



Hierarchy of languages
using increasing resources for its recognition
"Chomsky" hierarchy



f is 1:1 (into)

f is 1:1 and onto : bijection

When sets are finite we can compare their cardinalities by the no. of elements in sets

How do we compare cardinalities of infinite sets

When we can define a bijection between two infinite sets, then we conclude that they have same cardinality

$\Rightarrow$ # even integers = $\mathbb{Z}$

\# odd integers = $\mathbb{Z}$

$f_E(i) \longleftrightarrow 2i$

$\Sigma^* \longrightarrow \mathbb{Z}$

strings of length 1, string of length 2,

.... – $\underbrace{\text{strings of length } i}_{\text{ordering}}$

0 0..

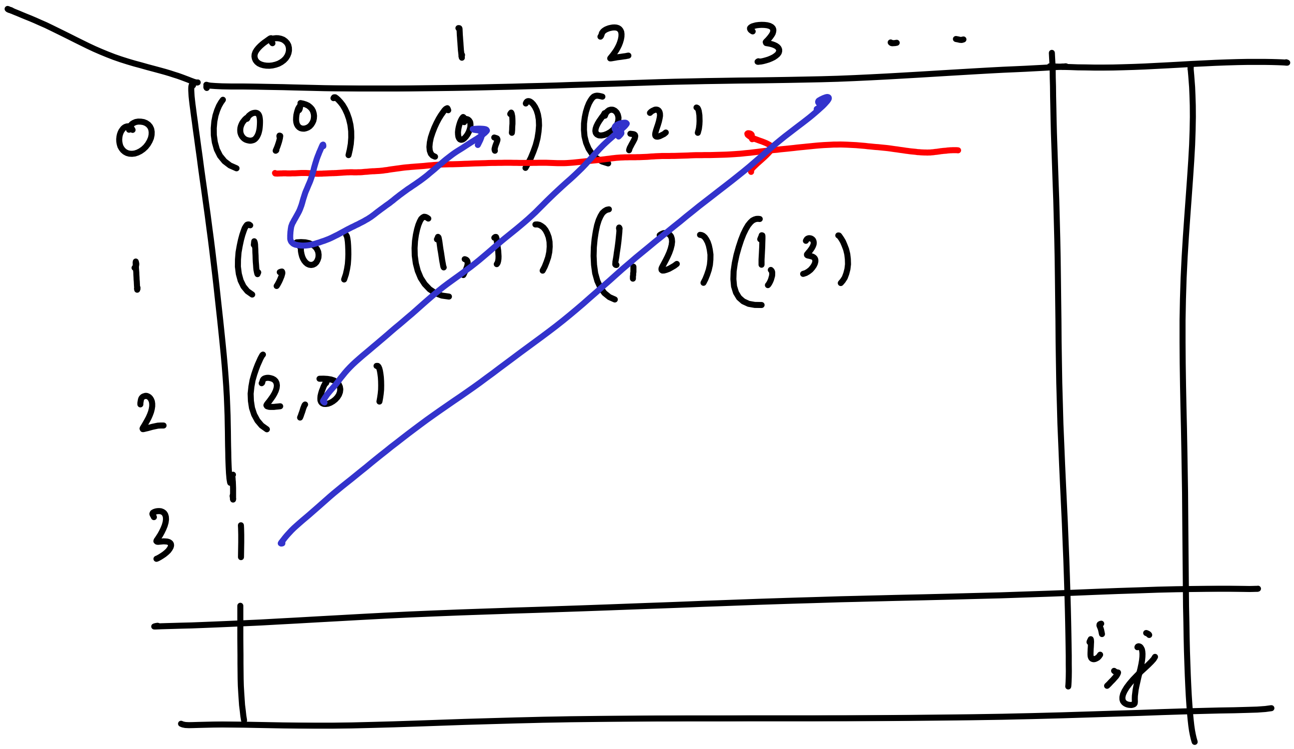
infinite

1.

$\mathbb{Z} \times \mathbb{Z} = \{ a, b \mid a, b \in \mathbb{Z} \}$

$(1, 3)$ $(5, 20) \in \mathbb{Z} \times \mathbb{Z}$

$\mathbb{Z} \longleftrightarrow \mathbb{Z} \times \mathbb{Z}$



$\mathbb{Z}^+$ $\qquad\qquad$ $\mathbb{Z}^- \cup \mathbb{Z}^+$

countable : all those infinite sets that have a bijection with $\mathbb{Z}$

"Countable union of countable sets is countable"

"Reals are not equinumerous with Integers" Cantor's diagonalization

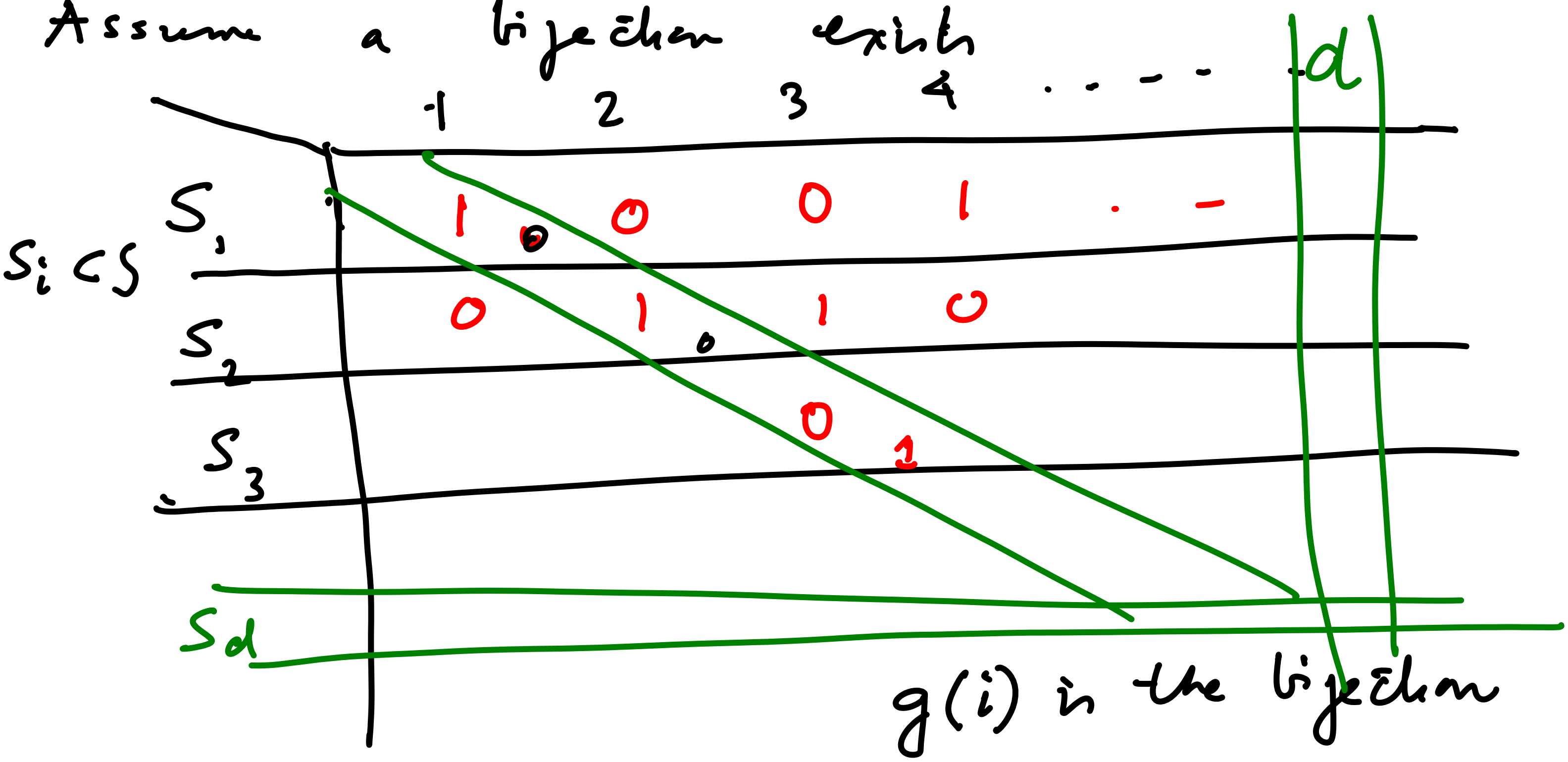
There cannot be a bijection between a set S and its powerset 2^S

__Case 1__ : S is finite — trivial

$i < 2^i$ for $i \geq 1$

Proof by contradiction for Infinite sets

Assume a bijection exists



$$S_d = \{ x \mid x \notin g^{-1}(x) \}$$

Question : $d \overset{?}{\in} S_d$

$d \in S_d$

$\Rightarrow$ according to defn $d \notin S_d$

$d \notin S_d$

$\Rightarrow$ according to defn $d \in S_d$

Diagonalization Proof

Math Induction	Complete Induction
$\forall i \; P(i)$ is true	
$P(0) \wedge \forall i \left(P(i) \Rightarrow P(i+1)\right)$	$P(0) \wedge \forall i \left[\left(\bigwedge_{j \leq i} P(j)\right) \Rightarrow P(i+1)\right]$