COL 352 Lecture 13

Pumping Lemma for Regular Language

For every regular language L,

there exists a sufficiently large n,

For all strings $s \in L$, $|s| \geq n$

there exist $u.v.w = s$

$|uv| \leq n$, $|v| \geq 1$

s.t. for all $i \geq 0$ $u.v^i.w \in L$

$L' = \{ 0^i 1^i \mid i \geq 0 \}$

Is L' regular?

Suppose L' is regular $\Rightarrow$ we can apply PL to L

Then there exists some n and let us choose $s = 0^n 1^n$ as the string

$$s = \underbrace{0\,0\,0\,0}_{u}\,\underbrace{0\,0\,0}_{v}\,0\,\underbrace{1\,1\,1\,1\,1\,1\,1\,1\,1}_{w}$$

$$\xleftrightarrow[n]{\qquad\qquad} \quad \xleftrightarrow[n]{\qquad\qquad}$$

By PL $\quad uv^iw \in L$ for all i and thereby produce strings $\in L'$ st $\#0s > \#1s$

So the assumption that L' is regular is fallacious

$L_{prime} = \{ 0^k \mid k \text{ is prime} \}$

$\{ 0^2, 0^3, 0^5, 0^7 \dots \}$

Suppose L_{prime} is regular (P.L constant n)

Choose a sufficiently long string, say $0^{\cancel{n}\,p}$

$p \geq n$

p is prime

$$\underbrace{0\;0\;0}_{u}\;\underbrace{0\;\dots\;0}_{v}\;\underbrace{0\;\dots\;0}_{w}$$

$|u| = k$ $\quad |v| = l$ $\quad |w| = m$ $\qquad k + l \leq n$

From P.L. $\quad uv^iw \in L_p$ for all $i \geq 0$

$\Rightarrow \quad 0^{k + il + m} \in L_p$

Choose $\quad i = k + m$

$k + (k+m)l + m \in P$

$(k+m)(1+l) \in P$

$L_c = \{ 0^k \mid k \text{ is } \boxed{\text{not prime}} / \text{composite} \}$

Is L_c regular?

<u>Claim</u> If L is regular then $\bar{L}$ $(\Sigma^* - L)$ is regular

Chose a DFA for L and invert the Final and non-Final states

It follows L_c is not regular

<u>Claim</u> : If L_1, L_2 are regular then $L_1 \cap L_2$ is regular.

DFA : M_1 M_2

Construct a DFA in the following manner

$M = M_1 \times M_2 : \{ Q_1 \times Q_2, \Sigma, (q_0^1, q_0^2)\ F_1 \times F_2,$

$\delta : ((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$

$p \in Q_1 \quad q \in Q_2$

$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$ (De Morgan's law proof)

Suppose for each symbol $a \in \Sigma$ we define a substitution function

$f(a) =$ regular expression