COL 352 Lecture 6 Jan 18

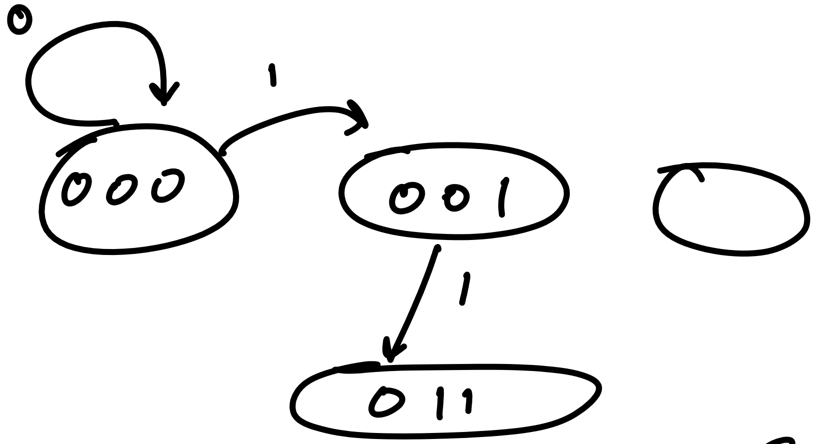
$L : \{ w \in \{0,1\}^* \mid$ the third last symbol is $1 \}$

(strings of length < 3 not in L)

$\boxed{1}01$ ✓ $\qquad 000\boxed{0}10$ ×

Design : Keep track of the window consisting of last three symbols

$\boxed{x_1\, x_2\, x_3}\, x_4\, x_5\, x_6\, x_7 \ldots \ldots x_n$



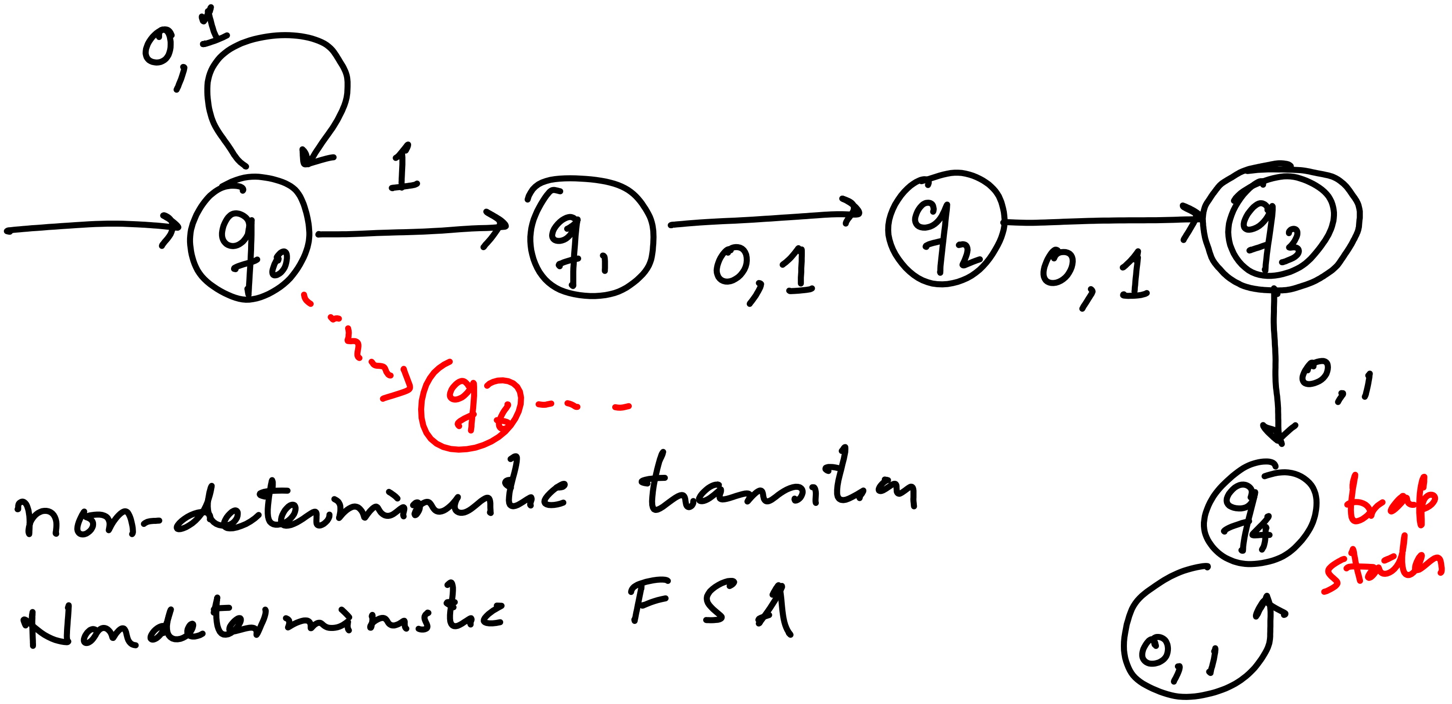
$\boxed{w_i\ w_{i+1}\ w_{i+2}}$ 1

0 0 0

Set of states

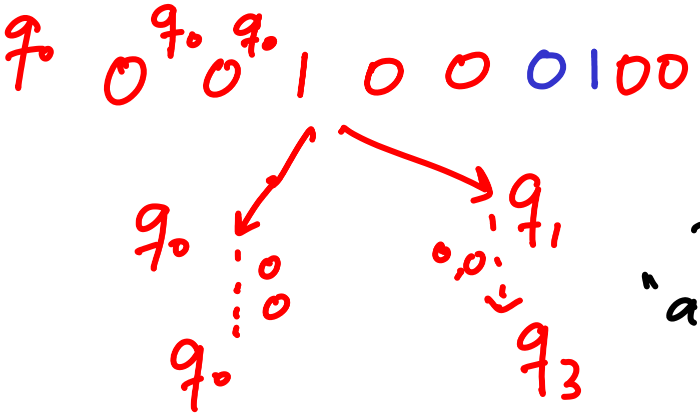
$w_1 w_2 w_3 \qquad w_i \in \{0,1\}$

$\delta(w_1 w_2 w_3, 0) = w_2 w 0$

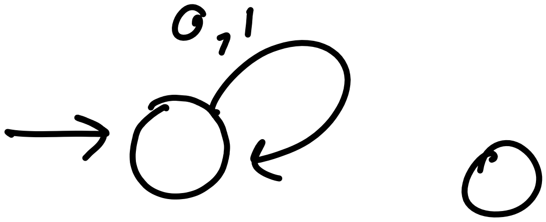


non-deterministic transition

Nondeterministic FSA



As long as one of the possible final states is "accepting" the string is accepted



$$w = \underbrace{x_1 \ x_2 \ x_3 \ \cdots}_{q_0} \boxed{x_{n-2}} x_{n-1} x_n$$

$$1 \quad q_1 \quad q_2 \quad q_3$$

① If the 3rd symbol from right is "1" then there is an <u>accepting path</u>

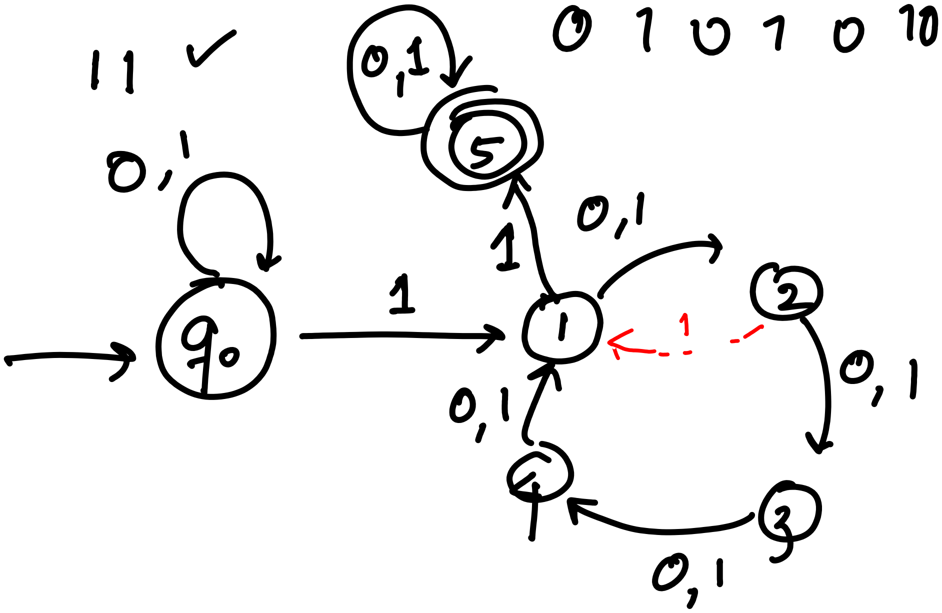
② No other strings should be accepted

$L' = \{ w \in \{0,1\}^*,$ some pair of 1's are separated by $4i$ symbols $i \geq 0 \}$

1 0 1 ×

1 1 ✓

0 1 0 1 0 101 0 0 0 101 0



NFA

(non-det finite automaton)

Converting NFA to DFA

NFA : $(Q, \Sigma, q_0, F, \delta)$

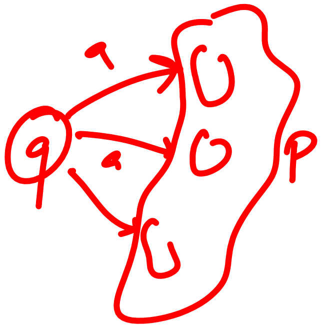
set of states (Q)

alphabet (Σ)

$$\delta : Q \times \Sigma \to Q^{\text{dfa}}$$

$$\to P \in 2^{Q}$$

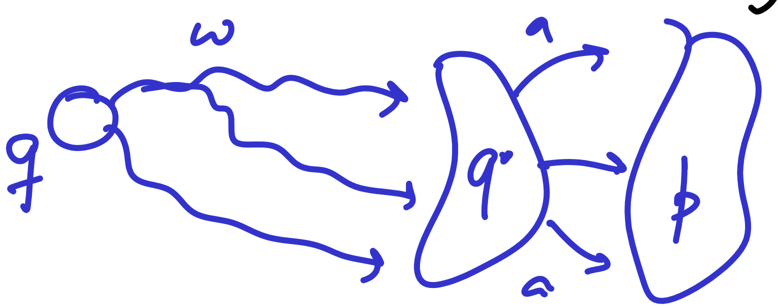
power set



$$\hat{\delta}(q, wa) = \{ p \in Q \mid p \in \delta(q', a) \text{ and } q' \in \hat{\delta}(q, w) \}$$

$w \in \Sigma^*$

$a \in \Sigma$



$$\hat{\delta}(q, \epsilon) = q$$

base case

$$\hat{\delta}(P, a) \overset{P \subseteq Q}{=} \{ p \mid p \in \underset{q \in P}{\delta(q,a)} \}$$

What strings are accepted by NFA N?

$$L(N) = \{ w \in \Sigma^* \mid \hat{\delta}(q_0, w) \cap F \neq \phi \}$$

↑

at least $\boxed{\text{one of}}$ the possible end states is an accepting state.

Note: Non-acceptance is when $\boxed{\text{all}}$ possible end states are non-accepting

The set of states for an equivalent DFA M : $L(N) = L(M)$

is the power set of N.

$$N : (Q, \Sigma, q_0, F, \delta)$$

↓

$$M = (2^Q, \Sigma, \underset{[q_0]}{\{q_0\}}, \underset{\substack{[P] : P \cap F \neq \phi \\ P \in 2^Q}}{F'}, \delta')$$

$$\delta' : 2^Q \times \Sigma \rightarrow 2^Q$$

$$\delta'([P], a) = \underset{[R]}{\{ q \mid q \text{ is a successor state of } \delta(p, a),\ p \in P \}}$$

To show $L(M) = L(N)$

(1) if $w \in L(M) \Rightarrow w \in L(N)$

(2) if $w \in L(N) \Rightarrow w \in L(M)$

To prove 2 we must show that for a string w, s.t.

$$\hat{\delta}(q_0, w) \cap F \neq \emptyset$$

$$\hat{\delta}(\{q_0\}, w) \cap F \neq \emptyset$$

then $\hat{\delta}'([q_0], w) \in F'$

We will prove that for all $P \in 2^Q$ and all $w \in \Sigma^*$

$$\hat{\delta}(P, w) = \hat{\delta}'([P], w)$$

" P' " $[P']$

$P' \in 2^Q$