COL 352 Lecture 2 Jan 6

We will focus our attention to decision problems

$\nearrow$ Yes

$\searrow$ No

Membership problems Given a set S (possibly infinite) and given an element x Does $x \in S$ $\nearrow$ Y $\searrow$ N

Suppose S is the set of primes P

then $10 \notin P$

$11 \in P$

If S is finite - the solution is trivial (enumerate and check exhaustively)

A language L is a set of strings over some finite alphabet

Σ : alphabet eg. $\{a, b, c, d \dots\}$

$\{0, 1\}$

Concatenation is the ordered joining of two strings.

$0 \cdot 1 \qquad 01$

often drop this symbol

Concatenation of alphabet

$$\Sigma_1 \cdot \Sigma_2 = \{ x \cdot y \mid x \in \Sigma_1, y \in \Sigma_2 \}$$

$\Sigma \cdot \Sigma$ will be shortened as Σ^2

$\Sigma^i, \ i \geq 1 \qquad \Sigma \cdot \Sigma^{i-1}$

$\Sigma^0 = \{ \varepsilon \}$ (empty string)

$$\epsilon \cdot x = x \cdot \varepsilon = x$$

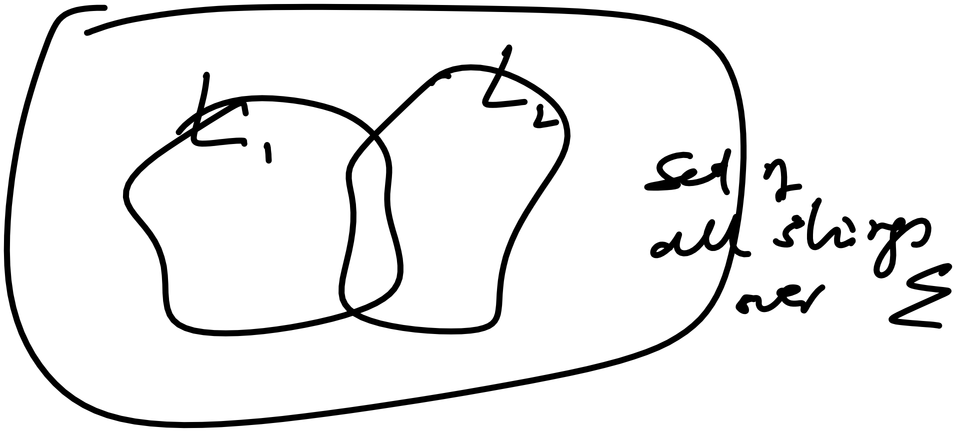
$$\Sigma^+ = \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \ldots \Sigma^i$$

i integer

$\Sigma = \{0, 1\} \qquad \Sigma^+$: set of all binary strings

$$\Sigma^* = \{\varepsilon\} \cup \Sigma^+$$

A language L over an alphabet (finite) Σ is a subset of Σ^*



Given a string $x \in \Sigma^*$, membership in L is $x \overset{?}{\in} L$

Characteristic function of L

$$\chi_L(y) = \begin{cases} 1 & \text{if } y \in L \\ 0 & \text{otherwise} \end{cases}$$

Suppose $f(x)$ is an arbitrary function $\mathbb{Z} \to \mathbb{Z}$

$$\chi_g(x, y) = \begin{cases} 1 & \text{if } y = f(x) \\ 0 & \text{otherwise} \end{cases}$$

<u>Claim</u> $\mathbb{Z} \times \mathbb{Z}$ can be mapped to $\mathbb{Z}$

How do compare infinite sets?
(cardinality)

Odd Int		Even Int
1	$\longrightarrow$	2
3	$\longrightarrow$	4
5		6
$\vdots$	
$2i+1$	$\longrightarrow$	$2i+2$

Odd int. $\qquad\qquad$ Integers

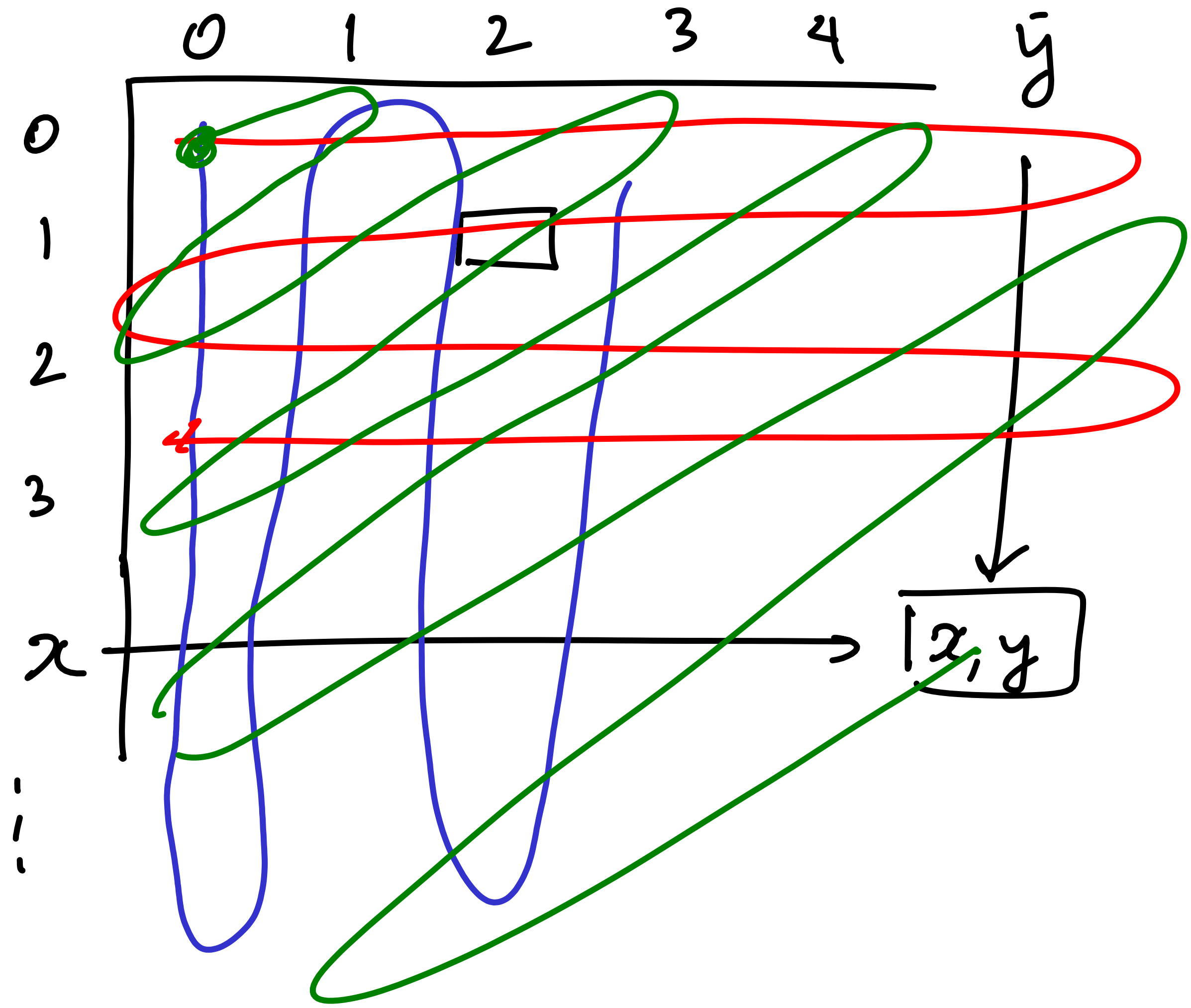
$$0 \quad \boxed{1} \quad 2 \quad \boxed{3} \quad 4 \quad \boxed{5}, \cdot \quad \boxed{7}$$

$$\begin{array}{cccc} \boxed{1} & \boxed{3} & \boxed{5} & \boxed{7} \\ \downarrow & | & | & | \\ 1 & 2 & 3 & 4 \end{array}$$

Pairs of Integers $\quad Z \times Z$

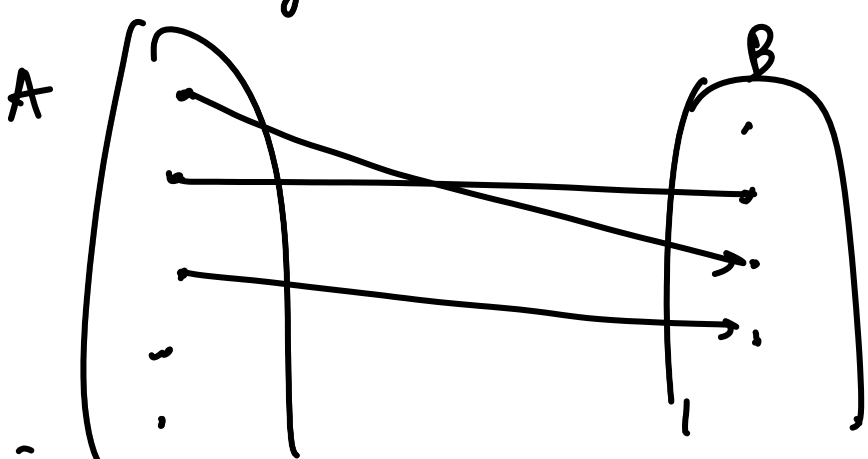
$(0,1) \quad (0,3) \quad (0,5) \quad (1,3) \quad (1,1)$

Is there a bijection between $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$



Given two sets A and B we say

$A \leq_{\#} B$ if there is a 1-1 mapping from A to B



<u>Claim</u>

If $A \subset B \Rightarrow A \leq_{\#} B$

Claim : The set of strings over any finite alphabet has a bijection with $\mathbb{Z}$.

(Exercise)

All strings are finite length

$|x|$: length of string and it is finite

$|1010|$: 4

A program written in any language (Java, ML, Python) is a finite length string over some appropriate alphabet.

The set of all possible programs $\approx \mathbb{Z}$

The possible # of subsets of $\mathbb{Z}$

$2^{\mathbb{Z}}$