COL 352 Lecture 19 Feb 22

1. $S \to ab$ Production rules

2. $S \to aSb$

3. $S \to \epsilon$

$S \xrightarrow{2} aSb \xrightarrow{2} aaSbb \xrightarrow{2} aaaSbbb$

$\xrightarrow{1} aaaabbbb$

$S \xrightarrow{*} w \mid w = a^i b^i, \; i \geq 0$

apply repeatedly one of the rules

Any sequence of substitution must begin with the special variable S

$G: \{ V = S \quad S = S$
$T = \{a, b, \epsilon\} \quad P = 1, 2, 3 \}$

Convention: Capital letters for variables
small case for terminals

$AB \longrightarrow ABB$

Context Free Grammar (CFG), the prodn rules have exactly one symbol on the LHS

Grammar $G = (V, T, P, S)$

set of variables that appear of LHS

Set of alphabet Terminals

Set of rules or productions

$\in V$ start symbol

L_{eq}: Equal number of a's and b's

$a^i b^i$ ababab, aababb,

Is L_{eq} regular?

Context Free Language (CFL) : all languages that can be generated using CFG

Is L_{eq} CFL?

$S, \{a, b, \varepsilon\}, S,$

$S \rightarrow \varepsilon \mid ba \mid ab \mid baS \mid abS$

$V = \{S, A, B\}$ $T = \{a, b\}$

$S \to aB \mid bA \mid \varepsilon$

$A \to a \mid bAA \mid aS$

$B \to b \mid aBB \mid bS$

aababb

$S \to aB \to aaBB \to aabB \to aabaBB \xrightarrow{*} aababb$

Claim 1. $S \xrightarrow{*} w$ iff w has equal # a's and b's

for $|w| \geq 1$ 2. $A \xrightarrow{*} w$ iff w has one more a than b

3. $B \xrightarrow{*} w$ iff w has one more b than a

Proof by induction on $|w|$

Base case $|w| = 1$ S: no strings of length 1, so true

A: $A \to a$ only strings of length 1

B: $B \to b$

Suppose true for all $|w| \leq k-1$

Consider any string $|w| = k$

S

$w \to aw_1$, $w \to bw_1$ $|w_1| = k-1$

$S \to aB \xrightarrow{*} aw_1$

since B generates all string $\leq$ length $k-1$ with one extra b

So $B \xrightarrow{*} w_1$

Conversely if $S \xrightarrow{*} w$ then $w = a w_1$ extra b

or $w = b w_2$ extra a

Let $S \to aB$ So $B \xrightarrow{*} w_1$

from I.H. w_1 has an extra b

Prove it for all the three assertions

A, B, S and their converse