COL 352 Intro Automata and Theory of Computation

Tutorial Sheet 0

1. (Practice problems for induction) Must state the induction assertion formally and what is the induction on.
(i) For all $n \geq 1$, $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$.
(ii) For all $n \geq 1$,
$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} \cdots \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1)$$
(iii) Prove that every integer divisible by 9 satisfies the property that, the sum of the digits is also divisible by 9.
(iv) The greatest-common-divisor (gcd) of two non-negative integers m, n is known to satisfy the identity $gcd(m, n) = gcd(m, n + m)$. Prove it.

2. Consider the following two definitions of the strings of balanced parentheses:

   A. A string w is balanced iff
   (i) w has an equal number of ”(” and ”)”
   (ii) Any prefix of w has at least as many ”(” as ”)”

   B. (i) ϵ is balanced.
   (ii) If w is balanced, so is (w).
   (iii) If w and x are balanced then so is $w \cdot x$
   (iv) Nothing else is balanced.

   Show that the above definitions A and B are equivalent.

3. Show that the Principle of Mathematical Induction and the Principle of Complete induction are equivalent.
Hint: Express them rigorously as sentences in first order logic.

4. Show two distinct bijective mappings between integers and rationals.

5. What is the fallacy in applying a diagonalization argument to the set of rationals ?

6. A relation $\leq_\#$ is defined as follows

   > If A, B are sets, then $A \leq_\# B$ iff there exists a 1-1 mapping $f : A \to B$ and an onto mapping $g : B \to A$.
   > If $f : A \to B$ is a bijection then, $A =_\# B$.

   What can you say about the pairs of sets
   (i) integers and rationals (ii) Reals in $[0, 1]$ and reals in $(10, 100)$ (open interval)
   The Bernstein-Schroeder theorem says that If $A \leq_\# B$ and $B \leq_\# A$ then $A =_\# B$.