Mutual Exclusion
Tokenless and Token Based Algorithms

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Outline

1. Tokenless Algorithms
   - Ricart-Agarwala Algorithm
   - Maekawa’s Algorithm

2. Token Based Algorithms
   - Suzuki-Kasami Algorithm
   - Raymond’s Tree Algorithm
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1. **Tokenless Algorithms**
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   - Maekawa’s Algorithm

2. **Token Based Algorithms**
   - Suzuki-Kasami Algorithm
   - Raymond’s Tree Algorithm
Motivation

- Lamport’s algorithm required $3(N-1)$ messages.
- Insight:

**Question**
Do we really have to send a timestamped reply message?
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**Question**

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**Solution**

- Lamport’s algorithm sent an acknowledgement immediately.
- Let us hold on to the acknowledgement and piggy back it with a release message.
- We can reduce the number of messages per critical section to $2(N-1)$. 
Algorithm

Requesting the Lock

- $P_i$ sends a timestamped request message to all other nodes.
- When $P_j$ receives a request, it sends a reply if:
  - $P_j$ is neither holding the lock, not is it interested in acquiring it. OR
  - $P_i$’s request timestamp is smaller than $P_j$’s request timestamp, and $P_j$ is not holding the lock. (means that $P_i$ made an earlier request)
Acquiring and Releasing the Lock

Acquiring the Lock
A process acquires the lock when it has received $N - 1$ replies.

Releasing the Lock
A process replies to all pending requests after it releases the lock.
Proof

- Assume $P_i$ and $P_j$ both have a lock at the same time.
- Assume that $P_i$ has a lower request timestamp.
- This means that $P_i$ must have gotten $P_j$’s request after its request.
- According to the algorithm, $P_i$ cannot send a reply to $P_j$.
- Hence, $P_j$ does not have the lock.
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Can we do better?

The main reason for a linear number of messages is:
1. We send a message to all the sites.
2. We also expect replies from all of them.
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The main reason for a linear number of messages is:
1. We send a message to all the sites.
2. We also expect replies from all of them.

Can we send a message to a subset of sites?
- Let a set of processes associated with a process be called its request set \((R_i)\).
- For any two processes, \(P_i\) and \(P_j\), we have: \(R_i \cap R_j \neq 0\).
- The minimum value of \(|R_i|\) is \(\sqrt{N}\).
- It is possible to construct this using results from field theory.
Simpler construction $R_i = 2\sqrt{N}$
$P_i$ sends a timestamped request message to every node in $R_i$ including itself.

Upon receiving a request message a node $P_j \in R_i$ marks itself as locked, if it is not already locked. It returns a locked reply to $P_i$.
Acquiring a Lock

- $P_i$ sends a timestamped request message to every node in $R_i$ including itself.
- Upon receiving a request message a node $P_j \in R_i$ marks itself as locked, if it is not already locked. It returns a locked reply to $P_i$.
- If $P_j$ is already locked by a request from $P_k$.
  1. $P_j$ places the request in a wait queue.
  2. If the locking request or any other request in the queue precedes the current request, then send a failed message.
  3. Otherwise, send an inquire message to $P_k$. 
Acquiring a Lock - II

When $P_k$ receives an inquire message:
1. If $P_k$ has received a failed message, and knows that it cannot succeed, it sends a relinquish message.
2. Otherwise, it defers the reply.
When $P_k$ receives an *inquire* message:

1. If $P_k$ has received a *failed* message, and knows that it cannot succeed, it sends a *relinquish* message.
2. Otherwise, it defers the reply.

When $P_j$ receives the *relinquish* message:

1. It locks itself for the earliest message from $P_i'$ in its wait queue (might be $P_i$).
2. It sends a *locked* message to $P_i'$.
3. It adds the request from $P_k$ to its wait queue.
When $P_k$ receives an inquire message:
1. If $P_k$ has received a failed message, and knows that it cannot succeed, it sends a relinquish message.
2. Otherwise, it defers the reply.

When $P_j$ receives the relinquish message:
1. It locks itself for the earliest message from $P_i'$ in its wait queue (might be $P_i$).
2. It sends a locked message to $P_i'$.
3. It adds the request from $P_k$ to its wait queue.

A process acquires the lock when it has received locked messages from its entire request set.
Releasing the Lock

- A process sends *released* messages to all the processes in its request set.
- A process in the request set, locks itself for the earliest request in the wait queue. It sends it a *locked* message.
- If there is no such request, then it marks its status as *unlocked*.
Proof

**Mutual Exclusion**

Assume two processes $P_i$ and $P_j$ have the lock simultaneously.
- There must be a node $P_k$ that must have given locked messages to both the processes.
- This is **not possible**.

**Deadlock**

Not possible because we order the requests by their timestamp.

**Starvation**

Ultimately, a request will become the earliest message in the system.
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Main Idea

- A site can access the lock (critical section) if it has a token.
- Every process maintains a sequence number (request id)
  - A request is of the form \((i, m)\). This means that \(P_i\) wants its \(m^{th}\) access to the lock.
  - \(P_i\) keeps an array \(seq_i[1 \ldots N]\).
  - \(seq_i[j]\) is the largest sequence number received from \(j\).
  - When \(P_i\) receives \((j, m)\), it sets
    \[
    seq_i[j] = \max(seq_i[j], m)
    \]
- **Token**
  1. A queue (\(Q\)) of requesting sites.
  2. An array of sequence numbers \(C\).
  3. \(C[i]\) is the sequence number of the latest request that \(P_i\) executed.
Requesting the Lock

- $P_i$: $\text{seq}_i[i]++$, $\text{val} \leftarrow \text{seq}_i[i]$
- Sends $(i, \text{val})$ to all sites
- When $P_j$ receives $(i, \text{val})$
  - $\text{seq}_j[i] \leftarrow \text{max}(\text{seq}_j[i], \text{val})$
  - If the token is idle and with $P_j$ then it sends it to $P_i$ if $\text{seq}_j[i] \leftarrow C[i]+1$.
- $P_i$ enters the critical section when it has the token.
Releasing the Lock

$P_i$ releases the lock as follows:

- $C[i] \leftarrow \text{seq}_i[i]$
- $\forall j$, adds $P_j$ to $Q$ if $\text{seq}_i[j] = C[j] + 1$.
- Dequeues $P_k$ from $Q$, and sends the token to $P_k$.
Releasing the Lock

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- $\forall j$, adds $P_j$ to $Q$ if $\text{seq}_i[j] = C[j] + 1$.
- Dequeues $P_k$ from $Q$, and sends the token to $P_k$

Message Overhead: 0 or N
Proof

A requesting process gets the lock in finite time.

- The request will reach all the processes in finite time.
- By induction, one of these processes will have the token in finite time.
- Thus the current request will get added to $Q$.
- There can at most be $N - 1$ messages before it.
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Main Idea

- Nodes are arranged as a tree. Every node has a parent pointer.
- Each node has a FIFO queue of requests.
- There is one token with the root and the owner of the token can enter the critical section.
- Message Complexity: approximately $O(\log(N))$ for trees with high fan-out
As the token moves across nodes, the parent pointers change.

They always point towards the holder of the token.

It is thus possible to reach the token by following parent pointers.
Requesting a Token

- The node adds “self” in its request queue.
- Forwards the request to the parent.
- The parents adds the request to its request queue.
- If the parent does not hold the token and it has not sent any requests to get the token, it sends a request to its parent for the request.
- This process continues till we reach the root (holder of the token).
## Releasing a Token

<table>
<thead>
<tr>
<th>Releasing a Token</th>
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</thead>
<tbody>
<tr>
<td>Ultimately a request will reach the token holder.</td>
</tr>
<tr>
<td>The token holder will wait till it is done with the critical section.</td>
</tr>
<tr>
<td>It will forward the token to the node at the head of its request queue.</td>
</tr>
<tr>
<td>- It removes the entry.</td>
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<td>- It updates its parent pointer.</td>
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<tr>
<td>Any subsequent node will do the following:</td>
</tr>
<tr>
<td>- Dequeue the head of the queue.</td>
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<tr>
<td>- If “self” was at the head of its request queue, then it will enter the critical section.</td>
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<tr>
<td>- Otherwise, it forwards the token to the dequeued entry.</td>
</tr>
<tr>
<td>After forwarding the entry, a process needs to make a fresh request for the token, if it has outstanding entries in its request queue.</td>
</tr>
</tbody>
</table>
Correctness

**Mutual Exclusion**

1. No two nodes can have a token at the same time, and thus cannot be in the CS at one time.

**Deadlock**

1. Circular wait cannot occur because all the nodes wait on the node that holds the token.
2. Messages cannot get lost because all the time our parent pointers ensure that we have a rooted tree.

**Starvation**

1. Ultimately a starved process’s request will come to the front of all the request queues.
2. At this point it will have the highest priority, and the token will have to flow back to the starved process.
A $\sqrt{N}$ Algorithm for Mutual Exclusion in Decentralized Systems by Mamoru Maekawa, ACM Transactions on Computer Systems, 1985