Layered Clausal Resolution in the Multi-modal Logic of Beliefs and Goals

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Overview

- Motivation: Rational Agents
- The Logic of Beliefs and Goals
- Normal Form Transformation
- Resolution in the Logic

Rational Agents

Thus spake FIPA:

Assume a fixed set $Ag = \{1, 2, \ldots, n\}$ of agents.

- Each agent is autonomous and can work independently
- It possesses **mental attitudes** – **B**eliefs, **I**ntentions
- It also possesses **K**nowledge and **D**esires (not of interest in this talk).
- Each agent's beliefs, desires and intentions may change with time as it receives fresh input from the environment.
- Each agent tries to realize its Intentions by its own actions or with the help of other agents in the system.

skip to Revision of Mental State

Communication

Thus continued FIPA:

- Communication is an integral part of agent behaviour.
- Agents communicate with each other to
  - obtain fresh information
  - request other agents to perform some tasks, that they themselves may not be able to perform.
  - influence their beliefs, intentions and desires in various ways.

Agent communications are expressed through performatives in an agent communication language (ACL).

Examples of Agent-like behaviour

- Distributed games – Diplomacy, Age of Empires
- auction protocols – Dutch, English, Vickerey.
- playing the stock market

Revision of Mental State

- *Assumption:* Each agent starts off life with a certain finite **B**elief base and a finite **G**oal base which is internally consistent.
- In the *give-and-take* of life its mental state undergoes changes.
- To maintain its *consistency* it needs to constantly revise its **B**elief and **G**oal bases.
- It also needs to **reason** about its own **B**eliefs and **G**oals to determine whether there is any inconsistency.

goto Basic Assumptions

Revision Procedure

Function Revise(S, ϕ) #S is a belief or goal base and
  S= S $\cup$ $\{\phi\}$; #ϕ is a new formula
  return (Contraction(S));
End Revise.

Function Contraction(S)
  $S_0 = S$; i=0;
  while ($S_i \models$ **false**) do
    Find minimal $F_i \subseteq S_i$ s.t. $F_i \models$ **false**;
    $g_i = \gamma(F_i)$; #γ is a selection criterion
    $S_{i+1} = S_i - \{g_i\}$; # remove $g_i \in F_i$ from S_i
    $i = i + 1$;
  end while;
  return S_i;
End Contraction.

Basic Assumptions

- **B**elief is a modality satisfying the axioms **KD45**.
- **G**oal is a modality stisfying the axioms **KD**.
- **I**ntention is not a modality by itself (Why not?) but is instead a derived operator

$$I\phi \equiv G\phi \wedge B\neg\phi$$

skip to Logic of Beliefs & Goals

Goals vs. Intentions

- Many authors regard Intention as a modality satisfying **KD** but there is a “side-effect” problem (Cohen and Levesque). If ϕ is *desirable* property to achieve but has an *undesirable* consequence ψ, then does $I\phi$ imply $I\psi$ too?
- FIPA does not define a separate “Goal” attitude. But then it is silent about what an agent should do if it is committed to bring about ϕ even if it believes that ϕ already holds. This problem does not arise with intention as derived from beliefs and goals.

skip KD45 axioms

KD45 and KD

O is a modality.

- **K**: $\vdash O(\phi \Rightarrow \psi) \Rightarrow (O\phi \Rightarrow O\psi)$
- **D**: $\vdash O\phi \Rightarrow \neg O \neg \phi$
- **4**: $\vdash O\phi \Rightarrow OO\phi$
- **5**: $\vdash \neg O \neg \phi \Rightarrow O \neg O \neg \phi$.

Resolution: Propositional Logic

- $\mathcal{P}$ = Set of Atomic propositions, $p \in \mathcal{P}$.
- $\phi = p \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \neg\phi$
- Conjunctive Normal Form, $\phi_1 \wedge \cdots \wedge \phi_n$ where $\phi_i = l_1 \vee \cdots \vee l_{k_i}$.
- Any formula can be transformed into CNF.

Resolution: Propositional Logic

- From sentences
  $l_1 \vee l_2 \vee \cdots \vee l_n$ and $\bar{l_1} \vee l'_2 \vee \cdots \vee l'_m$
  infer
  $l_2 \vee \cdots \vee l_n \vee l'_2 \vee \cdots \vee l'_m$

- From p and $\neg p$ infer **false**.

The Logic of Beliefs & Goals

$Ag = \{1, \ldots, n\}$ is a set of agents.
Let $\mathcal{P} = \{p, q, r, ...\}$ of atomic propositions, and constants **true** and **false**

- any element of $\mathcal{P}$ is in WFF_{BG_n};
- **true** and **false** are in WFF_{BG_n};
- if ϕ and ψ are in WFF_{BG_n} then so are
  $\neg\phi, \phi \vee \psi, \phi \wedge \psi, B_i\phi, G_i\phi$ where $i \in Ag$.

Multi-Modal Logic BG_n

- Semantics of BG_n formulas are defined as usual on Kripke structures $M = (S, L, B_1, \ldots, B_n, G_1, \ldots, G_n)$
- B_i satisfies axioms $\mathbf{K}$, $\mathbf{D}$, $\mathbf{4}$, $\mathbf{5}$ and G_i satsifies axioms $\mathbf{K}$, $\mathbf{D}$
- convert formulas of BG_n to a Normal Form (NF_{BG_n}) using the notion of a view.
- View v is a sequence of B_i and G_j modal operators. $O_{1i_1} \ldots O_{ki_k}$ where $O_j \in \{B, G\}$ and $i_j \in Ag$.
- Example: $B_1B_2G_1$ is a view.

Normal Form NF_{BG_n}

- New symbol **start** such that $(M, s_0) \models \mathbf{start}$ for any initial state s_0.
- Formulas in NF_{BG_n} are of the general form

$$\bigwedge_h v_h : C_h$$

- v_h is a view and C_h is a $clause$.

Normal Form: Clauses

- Clauses are of the following form:

$\mathbf{start} \Rightarrow \bigvee_{b=1}^{r} l_b$	(an initial clause)	
$\mathbf{true} \Rightarrow \bigvee_{a=1}^{r} l_a$	(a literal clause)	
$\mathbf{true} \Rightarrow \bigvee_{a=1}^{r} m_{Bi_a}$	(a B_i clause)	$m_{B_i} = B_i l$ or $\neg B_i l$
$\mathbf{true} \Rightarrow \bigvee_{a=1}^{r} m_{Gi_a}$	(a G_i clause)	$m_{G_i} = G_i l$ or $\neg G_i l$

An Assumption

For a "rational" agent $i \in Ag$,

$$\begin{aligned} O_i O_i \phi &\equiv O_i \phi \\ O_i \neg O_i \phi &\equiv \neg O_i \phi \end{aligned}$$

or there cannot be consecutive nestings of the same modality.

Information Store

NF_{BG_n}: Transformation τ_0

the transformation is done in two steps:

$$\tau_0[F] \longrightarrow (\ \epsilon : \textbf{start} \Rightarrow f\ ) \wedge \tau_1[\ \epsilon :\ f \Rightarrow F\ ].$$

where f is a new propositional variable.

NF_{BG_n}: Transformation τ_1

x is a proposition, main operator on the right side $\wedge$ or $\neg$,

$\tau_1[v : x \Rightarrow (F \wedge H)] \longrightarrow \tau_1[v : x \Rightarrow F] \wedge \tau_1[v : x \Rightarrow H]$

$\tau_1[v : x \Rightarrow \neg(F \wedge H)] \longrightarrow \tau_1[v : x \Rightarrow (\neg F \vee \neg H)]$

$\tau_1[v : x \Rightarrow \neg(F \vee H)] \longrightarrow \tau_1[v : x \Rightarrow \neg F] \wedge \tau_1[v : x \Rightarrow \neg H]$

$\tau_1[v : x \Rightarrow \neg\neg F] \longrightarrow \tau_1[v : x \Rightarrow F]$.

Normal Form NF_{BG_n}: Transformation

Complex sub-formulas within the scope of $O_i \in \{B_i, G_i\}$, and F is not a literal.

$$\tau_1[v : x \Rightarrow O_i F] \longrightarrow \tau_1[v : x \Rightarrow O_i y] \wedge \tau_1[vO_i : y \Rightarrow F]$$

$$\tau_1[v : x \Rightarrow \neg O_i F] \longrightarrow \tau_1[v : x \Rightarrow \neg O_i \neg y] \wedge \tau_1[vO_i : y \Rightarrow \neg F]$$

where y is a new variable.

Normal Form NF_{BG_n}: Transformation

right hand side has $\vee$ as the main operator (D is a disjunction of formulas).

Case: D contains a disjunct of the form O'_j or $\neg O'_j$ where $O \neq O'$ or $i \neq j$.

$$\tau_1[v : x \Rightarrow D \vee O_i F] \longrightarrow \tau_1[v : x \Rightarrow D \vee y] \wedge \tau_1[v : y \Rightarrow O_i F],$$

$$\tau_1[v : x \Rightarrow D \vee \neg O_i F] \longrightarrow \tau_1[v : x \Rightarrow D \vee y] \wedge \tau_1[v : y \Rightarrow \neg O_i F],$$

where y is a new variable.

Normal Form NF_{BG_n}: Transformation

Case: F is not a literal and D contains only the modality O_i.

$$\tau_1[v : x \Rightarrow D \vee O_i F] \longrightarrow \tau_1[v : x \Rightarrow D \vee O_i y] \wedge \tau_1[vO_i : y \Rightarrow F],$$

$$\tau_1[v : x \Rightarrow D \vee \neg O_i \neg F] \longrightarrow \tau_1[v : x \Rightarrow D \vee \neg O_i \neg y] \wedge \tau_1[vO_i : y \Rightarrow F],$$

where y is a new variable.

Normal Form NF_{BG_n}

- Each modal clause may contain modal literals involving only one modal operator.
  Clause $\mathbf{true} \Rightarrow B_1 x \vee y \vee \neg B_1 z$ is allowed,
  but $\mathbf{true} \Rightarrow B_1 x \vee y \vee B_2 z$ and $\mathbf{true} \Rightarrow B_1 x \vee y \vee G_1 z$ are not allowed
- Finally (D is a disjunction of literals and modal literals only involving one modal operator.)

$$\tau_1[v : x \Rightarrow D] \longrightarrow v : true \Rightarrow \neg x \vee D$$

Example of transformation

$F = B_i(p \vee \neg B_j(q \vee \neg t))$ into normal form.

$$\tau_0[B_i(p \vee \neg B_j(q \vee \neg t))] = (\epsilon : start \Rightarrow f) \wedge \tau_1[\epsilon : f \Rightarrow B_i(p \vee \neg B_j(q \vee \neg t))]$$

$$\tau_1[\epsilon : f \Rightarrow B_i x_1] \wedge \tau_1[B_i : x_1 \Rightarrow p \vee \neg B_j(q \vee \neg t))]$$

$$\epsilon : true \Rightarrow \neg f \vee B_i x_1$$

$$\tau_1[B_i : x_1 \Rightarrow p \vee \neg B_j \neg x_2] \wedge \tau_1[B_i B_j : x_2 \Rightarrow \neg(q \vee \neg t)]$$

$$B_i : true \Rightarrow \neg x_1 \vee p \vee \neg B_j \neg x_2$$

$$\tau_1[B_i B_j : x_2 \Rightarrow \neg q \wedge t]$$

$$\tau_1[B_i B_j : x_2 \Rightarrow \neg q] \wedge \tau_1[B_i B_j : x_2 \Rightarrow t]$$

$$B_i B_j : true \Rightarrow \neg x_2 \vee \neg q$$

$$B_i B_j : true \Rightarrow \neg x_2 \vee t$$

Initial Resolution rules

$$[IRES1]\ \frac{\begin{array}{c}\epsilon : \mathbf{true} \Rightarrow (F \vee l) \\ \epsilon : \mathbf{start} \Rightarrow (H \vee \neg l)\end{array}}{\epsilon : \mathbf{start} \Rightarrow (F \vee H)}$$

$$[IRES2]\ \frac{\begin{array}{c}\epsilon : \mathbf{start} \Rightarrow (F \vee l) \\ \epsilon : \mathbf{start} \Rightarrow (H \vee \neg l)\end{array}}{\epsilon : \mathbf{start} \Rightarrow (F \vee H)}$$

Modal Resolution rules

$$[MRES1]\ \frac{\begin{array}{c} v : \mathbf{true} \Rightarrow D \vee m \\ v : \mathbf{true} \Rightarrow D' \vee \neg m \end{array}}{v : \mathbf{true} \Rightarrow D \vee D'}$$

$$[MRES2]\ \frac{\begin{array}{c} v : \mathbf{true} \Rightarrow D \vee O_i l \\ v : \mathbf{true} \Rightarrow D' \vee O_i \neg l \end{array}}{v : \mathbf{true} \Rightarrow D \vee D'}$$

$$[MRES3]\ \frac{\begin{array}{c} v : \mathbf{true} \Rightarrow D \vee \neg O_i l \\ vO_i : \mathbf{true} \Rightarrow D' \vee l \end{array}}{v : \mathbf{true} \Rightarrow D \vee mod_{O_i}(D')}$$

$$[MRES4]\ \frac{\begin{array}{c} v : \mathbf{true} \Rightarrow D \vee O_i l \\ vO_i : \mathbf{true} \Rightarrow D' \vee \neg l \end{array}}{v : \mathbf{true} \Rightarrow D \vee mod_{O_i}(D')}$$

where $mod_{O_i}(D')$ is defined below.

mod_{O_i}

$mod_{O_i}(F \vee G) = mod_{O_i}(F) \vee mod_{O_i}(G)$ for $O_i \in \{B_i, I_i\}$.

$mod_{O_i}(l) = \neg O_i \neg l$ for $O_i \in \{B_i, I_i\}$.

$mod_{B_i}(B_i l) = B_i l$

$mod_{B_i}(\neg B_i l) = \neg B_i l$

Note the difference in treatment between B_i and G_i. In case $O_i = G_i$, D' must be a disjunction of propositional literals in MRES3 and MRES4.

KD45 axioms

The intuition behind MRES3 for B_i

1. $v : \mathbf{true} \Rightarrow D \vee \neg B_i l \quad \in view(v)$
2. $vB_i : \mathbf{true} \Rightarrow D' \vee l = m_1 \vee ... \vee m_k \vee l \quad \in view(vB_i)$

Intuitively 2 is the same as

$$v : B_i(\neg D' \Rightarrow l)$$

i.e $v : B_i \neg D' \Rightarrow B_i l$. Further $B_i \neg D' \Leftrightarrow B_i \neg m_1 \ldots B_i \neg m_k$. By the transformation each m_i is a modal literal involving only B_i. So finally 2. is equivalent to

$$v : \mathbf{true} \Rightarrow \neg B_i \neg m_1 \vee \ldots \vee \neg B_i \neg m_k \vee B_i l$$

which may be resolved against 1 to yield

$$v : \mathbf{true} \Rightarrow D \vee \neg B_i \neg m_1 \vee \ldots \vee B_i \neg m_k$$

From the axioms of **KD45**, $\neg B_i \neg B_i \neg F \Leftrightarrow B_i \neg F$, which means the prefix "$\neg B_i \neg$" may be deleted from those that have such nestings.

Example of Resolution

Suppose agent i has the **belief base:** $B_i(\neg p \vee B_j q)$, $B_i B_j \neg q$. The question is, whether $B_i \neg p$ is implied by the belief base. We add $\neg B_i \neg p$ to the belief base and check if the resolution process results in the clause $\epsilon : \textbf{start} \Rightarrow \textbf{false}$.

Clauses :

$B_i(\neg p \vee B_j q)$	$B_i B_j \neg q$	$\neg B_i \neg p$
1. $\epsilon : \textbf{start} \Rightarrow f$	4. $\epsilon : \textbf{start} \Rightarrow f$	7. $\epsilon : \textbf{start} \Rightarrow f$
2. $\epsilon : \textbf{true} \Rightarrow \neg f \vee B_i x_1$	5. $\epsilon : \textbf{true} \Rightarrow \neg f \vee B_i y_1$	8. $\epsilon : \textbf{true} \Rightarrow \neg f \vee \neg B_i \neg p$
3. $B_i : \textbf{true} \Rightarrow \neg x_1 \vee \neg p \vee B_j q$	6. $B_i : \textbf{true} \Rightarrow \neg y_1 \vee B_j \neg q$	

Resolution :

3, 6 —MRES2→ 9. $B_i : \textbf{true} \Rightarrow \neg x_1 \vee \neg p \vee \neg y_1$; 8 —MRES3→ 10. $\epsilon : \textbf{true} \Rightarrow \neg f \vee \neg B_i x_1 \vee \neg B_i y_1$

2, 10 —MRES1→ 11. $\epsilon : \textbf{true} \Rightarrow \neg f \vee \neg B_i y_1$; 5 —MRES1→ 12. $\epsilon : \textbf{true} \Rightarrow \neg f$; 1 —IRES1→ $\epsilon : \textbf{start} \Rightarrow \textbf{false}$

Soundness & Completeness

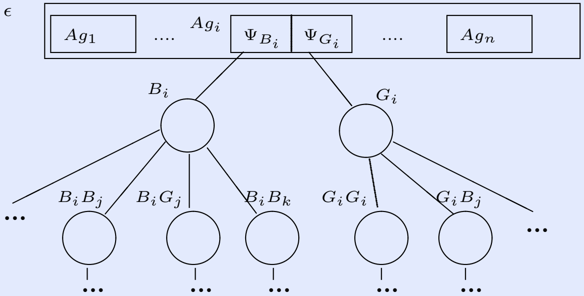
Soundness and Completeness of method is proved in three parts:

- Transformation preserves the satisfiability
- Set of Resolution rules are sound
- If a set of formulas is unsatisfiable, then there is a refutation using resolution rules.
- The rules are sound and complete modulo the assumption. The proof though standard, is extremely long and complicated.

Conclusions

- This is an extension of the method of Dixon, Fisher and Bolotov (AI vol 139 pp 47-89, 2002) for the unimodal case of a KD45 modality.
- The method satisfies a "locality" or "layering" property viz. that resolution is performed only within the same or adjacent levels of nesting. This allows for a simple representation of the Information store as in Benerecetti's work.
- Many tableau and resolution-based proof systems exist in the literature (too numerous to mention here). So this work is not really an advancement in proof techniques, but is perhaps more useful in
  - determining whether an agent's information store is inconsistent, and hence
  - in belief and goal revision.

Information Store



Thank you