Parameterised Bisimulations: Some Applications

S. Arun-Kumar and Divyanshu Bagga

Department of Computer Science and Engineering
Indian Institute of Technology Delhi
Hauz Khas, New Delhi 110 016, India
Email: {sak@cse.iitd.ac.in, divyanshu@cse.iitd.ac.in}

Abstract. In [AK06] the first author had generalised the notion of bisimulation on labelled transition systems to that of a parameterised relation whose parameters were a pair of relations on the observables of a system. In this paper we present new results which show that notions of parameterised bisimilarity may be defined to capture congruences in process algebras. In particular, we show that observational congruence may be obtained as a parameterised bisimulation, thereby providing a co-inductive characterisation for the same. In another application, parameterisation is employed to prove that amortised bisimilarity is preserved under recursion in CCS by resorting to a generalisation of the so-called "upto"-technique. We then extend the framework to a name passing calculus and show that one can capture (hyper-)bisimulations in the fusion calculus [Vic98] as a parameterised (hyper-)bisimulation. However this involves giving a behaviourally equivalent alternative semantics for the fusion calculus, which is necessary for defining parameterised bisimulations in the fusion calculus and also allows for more natural definitions of bisimulations.

1 Introduction

In [AK06] the notion of bisimilarity was generalised to a bisimilarity relation induced by a pair of relations on the underlying set of observables. The notion was referred to as parameterised bisimilarity. Many of the well-known bisimilarity and pre-bisimilarity relations in the literature are special cases of this generalised notion. Further it was also shown that many of the nice properties that these bisimilarity relations exhibited were essentially inherited from the corresponding properties in the inducing relations. In particular, it was shown that a parameterised bisimilarity relation is a preorder (resp. equivalence) if and only if the inducing relations are themselves preorders (resp. equivalences). A generalised version of Park's induction principle also holds. Finally an efficient on-the-fly algorithm was described for computing parameterised bisimilarity for finite-state labelled transition systems.

In this paper we explore compatible parameterised bisimilarity relations (e.g. congruences and precongruences) in the context of process algebras. We present three very different applications using formulations of parameterised bisimilarity.

Inspired by the axiomatization of observational congruence by Bergstra and Klop [BK85] it was shown in [SAK09] that all parameterised bisimilarities which are preorders (resp. equivalences) are also precongruences (resp. congruences) on process graphs provided there are no “empty observables”. Loosely speaking, in the context of weak-bisimilarity, the silent action τ of CCS is an empty observable whereas in the context of strong bisimilarity it is not. The proofs in [SAK09] which axiomatized (pre-)congruences that did not involve empty observables, were as lengthy as those in [BK85]. But what eluded a solution therein was a coinductive characterization of observational congruence. The first application we present in this paper is the characterization of observational congruence as a parameterised bisimilarity. The characterization requires a careful analysis and definition of a certain kind of weak transition to capture observational congruence in the presence of empty observables.

In [KAK05] a cost-based notion called amortised bisimilarity was defined on a CCS-like language. The set of actions was augmented with a set of visible actions to which costs were associated. While it was possible to show fairly easily that amortised bisimilarity was preserved by most of the operators of CCS, the issue of whether it is preserved under recursion was left open. The second application we present is a proof that recursion does preserve amortised bisimilarity. However this proof requires casting amortised bisimilarity in the form of an equivalent parameterised bisimilarity and using a generalisation of the “upto”-technique used by Milner and Sangiorgi [SM92] to prove that the recursion operator preserves the equivalent parameterised bisimilarity.

We devote the last section of this paper to extending the theory of parameterised bisimulations to a name-passing calculus. We argue that the meaning of the actions and thus the transitions in name-passing calculi change according to the names being passed. This dynamic update in the meaning of actions needs to be incorporated in the definition of parameterised bisimulations for name-passing calculi. We will use the fusion calculus [Vic98] to develop a general theory of parameterised bisimulations. The notion of "fusion" as an equivalence relation on names comes in quite handy while defining parameterised versions of bisimilarity in a name-passing calculus. We will however, need to give an alternative but equivalent operational semantics for the fusion calculus which allows for a more natural definition of bisimulations and which, we argue is necessary in order to define parameterised bisimulations.

2 Parameterised Bisimulations

A *labelled transition system (LTS)* $\mathcal{L}$ is a triple $\langle \mathbf{P}, \mathcal{O}, \longrightarrow \rangle$, where $\mathbf{P}$ is a set of *process states* or *processes*, $\mathcal{O}$ is a set of *observables* and $\longrightarrow \subseteq \mathbf{P} \times \mathcal{O} \times \mathbf{P}$ is the *transition relation*. We use the notation $p \stackrel{a}{\longrightarrow} q$ to denote $(p, a, q) \in \longrightarrow$ and refer to q as a *(strong) a-successor* of p. The set of a-successors of p is denoted Succ^p_a. q is a *successor* of p if it is an a-successor for some observable a. A state q is *reachable* from p if either $p = q$ or q is reachable from some successor of p. An LTS of the form $\langle \mathbf{P}, \mathcal{O}, \longrightarrow \rangle$ may also be thought of as one of the form $\langle \mathbf{P}, \mathcal{O}^+, \longrightarrow \rangle$ such that for any $as \in \mathcal{O}^+$, $p \stackrel{as}{\longrightarrow} q$ iff for some p', $p \stackrel{a}{\longrightarrow} p'$ and $p' \stackrel{s}{\longrightarrow} q$. The notion of successor may be appropriately defined. Further by introducing the transition $p \stackrel{\epsilon}{\longrightarrow} p$ we may think of $\mathcal{L}$ also as an LTS of the form $\langle \mathbf{P}, \mathcal{O}^*, \longrightarrow \rangle$. A *rooted* labelled transition system is a 4-tuple $\langle \mathbf{P}, \mathcal{O}, \longrightarrow, p_0 \rangle$ where $\langle \mathbf{P}, \mathcal{O}, \longrightarrow \rangle$ is an LTS and $p_0 \in \mathbf{P}$ a distinguished *initial state*. In general we will consider the set of states of such an LTS as consisting only of those states that are reachable from the initial state. The term “process” will be used to refer to a process state in an LTS, as also to the sub-LTS rooted at that state and containing all the states and transitions reachable from that given state. Since an arbitrary disjoint union of LTSs is also an LTS, we shall often refer to $\mathbf{P}$ as the set of all processes. For each $p \in \mathbf{P}$, $\mathsf{Reach}(p)$ denotes the set of all reachable states of p.

Other notational conventions we use are the following.

- $\equiv$ denotes the identity relation on a set. It may be used in the context of observables, processes and also sets of processes.
- $\circ$ denotes relational composition i.e. for $R \subseteq A \times B$ and $S \subseteq B \times C$, $R \circ S = \{(a, c) \mid \exists b : aRbSc\}$.
- R^{-1} denotes the converse of the relation R.
- 2^U denotes the powerset of a set U.
- $|s|$ denotes the length of a sequence s.
- $\mathbf{0} \in \mathbf{P}$ is a process that is incapable of performing any observable action.
- Substitutions are applied in prefix form; two substitutions are composed using the relational composition operator $\circ$ so that $\{x/y\} \circ \{y/z\} = \{x/z\}$.

2.1 (ρ, σ)-Bisimulations

Definition 1. *Let $\mathbf{P}$ be the set of processes and let ρ and σ be binary relations on $\mathcal{O}$. A binary relation $R \subseteq \mathbf{P} \times \mathbf{P}$ is a (ρ, σ)*-**induced bisimulation** *or simply a (ρ, σ)*-**bisimulation** *if pRq implies the following conditions for all $a, b \in \mathcal{O}$.*

$$p \xrightarrow{a} p' \Rightarrow \exists b, q'[a\rho b \wedge q \xrightarrow{b} q' \wedge p'Rq']$$

$$q \xrightarrow{b} q' \Rightarrow \exists a, p'[a\sigma b \wedge p \xrightarrow{a} p' \wedge p'Rq']$$

The largest (ρ, σ)-bisimulation (under set containment) is called (ρ, σ)-**bisimilarity** *and denoted $\underline{\square}_{(\rho,\sigma)}$. A $(\equiv, \equiv)$-induced bisimulation will sometimes be called a* **natural bisimulation**[1]. *$\mathsf{B}_{(\rho,\sigma)}$ denotes the set of all (ρ, σ)-bisimulations.*

Proposition 1. (from [AK06]). *Let ρ and σ be binary relations on $\mathcal{O}$ and let R and S be binary relations on the set $\mathbf{P}$ of processes.*

1. *If R is a (ρ, σ)-bisimulation and pRq then so is $S = R \cap (\mathsf{Reach}(p) \times \mathsf{Reach}(q))$.*
2. *$p \,\underline{\square}_{(\rho,\sigma)}\, q$ iff pRq for some $R \in \mathsf{B}_{(\rho,\sigma)}$.* □

In Proposition 2 and Theorem 1 we quote important properties of (ρ, σ)-bisimulations. The reader is referred to [AK06] for some of the proofs.

Proposition 2. (Properties). *Let $R{:}p \,\underline{\square}_{(\rho,\sigma)}\, q$ denote that R is a (ρ, σ)-bisimulation containing the pair (p, q).*

1. **Point-wise Extension** *Let ρ^* and σ^* on $\mathcal{O}^*$ be respectively the point-wise extensions of the relations ρ and σ on $\mathcal{O}$. Then R is a (ρ, σ)-bisimulation iff it is a (ρ^*, σ^*)-bisimulation. Further,*
   - *$R : p \,\underline{\square}_{(\rho,\sigma)}\, q$ iff $R : p \,\underline{\square}_{(\rho^*,\sigma^*)}\, q$ and*

[1] A strong bisimulation on CCS processes with $\mathcal{O} = \mathit{Act}$ is an example of a *natural bisimulation*.

– $\sqsubseteq_{(\rho,\sigma)} \; = \; \sqsubseteq_{(\rho^*,\sigma^*)}$.

2. **Monotonicity.** *If* $\rho \subseteq \rho'$ *and* $\sigma \subseteq \sigma'$ *then every* (ρ,σ)*-bisimulation is also a* (ρ',σ')*-bisimulation and hence* $\sqsubseteq_{(\rho,\sigma)} \subseteq \sqsubseteq_{(\rho',\sigma')}$.
3. **Inversion.**
   – $R : p \sqsubseteq_{(\rho,\sigma)} q$ *implies* $R^{-1} : q \sqsubseteq_{(\sigma^{-1},\rho^{-1})} p$
   – $\sqsubseteq^{-1}_{(\rho,\sigma)} = \sqsubseteq_{(\sigma^{-1},\rho^{-1})}$.
4. **Composition.** $\sqsubseteq_{(\rho_1,\sigma_1)} \circ \sqsubseteq_{(\rho_2,\sigma_2)} \; \subseteq \; \sqsubseteq_{(\rho_1\circ\rho_2,\sigma_1\circ\sigma_2)}$ *since* $R_1 : p \sqsubseteq_{(\rho_1,\sigma_1)} q$ *and* $R_2 : q \sqsubseteq_{(\rho_2,\sigma_2)} r$ *implies* $R_1 \circ R_2 : p \sqsubseteq_{(\rho_1\circ\rho_2,\sigma_1\circ\sigma_2)} r$.
5. **Reflexivity.** *If* ρ *and* σ *are both reflexive then the identity relation* $\equiv$ *on* $\mathbf{P}$ *is a* (ρ,σ)*-bisimulation and consequently* $\sqsubseteq_{(\rho,\sigma)}$ *is reflexive.*
6. **Symmetry.** *If* ρ *and* σ *are both symmetric, the converse of each* (ρ,σ)*-bisimulation is a* (σ,ρ)*-bisimulation. In addition, if* $\rho = \sigma$ *then* $\sqsubseteq_{(\rho,\sigma)}$ *is a symmetric relation.*
7. **Transitivity.** *If* ρ *and* σ *are both transitive then the relational composition of* (ρ,σ)*-bisimulations is a* (ρ,σ)*-bisimulation, and* $\sqsubseteq_{(\rho,\sigma)}$ *is also transitive.*
8. **Preorder characterisation.** $\sqsubseteq_{(\rho,\sigma)}$ *is a preorder iff* ρ *and* σ *are both preorders.*
9. **Equivalence characterisation.** $\sqsubseteq_{(\rho,\sigma)}$ *is an equivalence iff* ρ *and* σ *are both preorders and* $\sigma = \rho^{-1}$.
10. *If* ρ *is a preorder then* $\sqsubseteq_{(\rho,\rho^{-1})}$ *is an equivalence.* □

Theorem 1. *Let* $2^{\mathbf{P}\times\mathbf{P}}$ *be the set of all binary relations on processes. Then*

1. $\langle \mathsf{B}_{(\rho,\sigma)}, \cup, \emptyset \rangle$ *is a commutative submonoid of* $\langle 2^{\mathbf{P}\times\mathbf{P}}, \cup, \emptyset \rangle$.
2. $\sqsubseteq_{(\rho,\sigma)}$ *is a preorder if* $\langle \mathsf{B}_{(\rho,\sigma)}, \circ, \equiv \rangle$ *is a submonoid of* $\langle 2^{\mathbf{P}\times\mathbf{P}}, \circ, \equiv \rangle$. □

3 On Observational Congruence

In this section we present a characterization of Milner's observational congruence relation $\approx^+$ for divergence-free finite-state CCS agents[2] as a parameterised bisimilarity. We achieve this by deriving an LTS which is observationally equivalent to the original LTS and show that observational congruence on CCS processes is a parameterised bisimilarity on the new LTS. In doing so we obtain a coinductive characterization of observational congruence.

Let $\mathcal{L} = \langle \mathbf{P}, Act, \rightarrow \rangle$ be the usual LTS defined by divergence-free finite-state CCS agents. A process q is a μ-*derivative* (or a *weak* μ*-successor*) of p if $p \stackrel{\mu}{\Longrightarrow} q$ i.e. for some $m, n \geq 0$, $p \stackrel{\tau^m \mu \tau^n}{\longrightarrow} q$ for any $\mu \in Act$. Similarly, q is a *derivative* of p if it is a μ-derivative for some $\mu \in Act$. For any $s \in Act^*$, let $\hat{s} \in A$ denote the sequence of visible actions obtained by removing all occurrences of τ in s. Given $s, t \in Act^*$, we define $s \mathrel{\widehat{=}} t$ if $\hat{s} = \hat{t}$. Recall from [Mil89] that

– Observational Equivalence (denoted $\approx$) is the largest symmetric relation on $\mathbf{P}$ such that for all $\mu \in Act$, if $p \stackrel{\mu}{\longrightarrow} p'$ then there exists q' such that $q \stackrel{\hat{\mu}}{\Longrightarrow} q'$ and $p' \approx q'$. Moroever $\approx \, = \, \sqsubseteq_{(\widehat{=},\widehat{=})}$ (from [AK06]).

[2] that is pure CCS agents without replication

– Observational Congruence (denoted as $\approx^+$) is the largest symmetric relation on $\mathbf{P}$ such that for all $\mu \in Act$, if $p \xrightarrow{\mu} p'$ then there exists q' such that $q \overset{\mu}{\Longrightarrow} q'$ and $p' \approx q'$.

It may be seen from the definition that $\approx^+$ is contained in $\approx$. While $\approx$ is easily shown to be the parameterised bisimilarity $\sqsubseteq_{(\hat{=},\hat{=})}$ on $\mathcal{L}$, a formulation of $\approx^+$ as parameterised bisimilarity is trickier. This difficulty comes from the fact that any parameterised bisimulation is defined coinductively while observational congruence is defined in literature everywhere in terms of observational equivalence (Note that in the above definition, observational congruence($\approx^+$) is defined in terms of observational equivalence($\approx$)). It is well known that $\tau.\tau.\mathbf{0} \approx^+ \tau.\mathbf{0}$ since they both have observationally equivalent τ-successors. However $\tau.\mathbf{0} \not\approx^+ \mathbf{0}$ because the preemptive power of the τ action may be used to distinguish them in choice contexts. We present a coinductive characterization of observational congruence as our first application.

Keeping the above requirement in mind, we define an LTS $\mathcal{L}_\dagger = \langle \mathbf{P}, Act.\tau^*, \longrightarrow_\dagger \rangle$ derived from $\mathcal{L}$ such that $Act.\tau^* = \{\mu\tau^n \mid \mu \in Act, n \geq 0\}$ and $p \xrightarrow{\mu\tau^n}_\dagger p'$ iff there exists $n \geq 0$ and processes $p_0, \cdots, p_n$ such that $p \xrightarrow{\mu} p_0 \approx\!\xrightarrow{\tau} p_1 \approx\!\xrightarrow{\tau} \cdots \approx\!\xrightarrow{\tau} p_n \equiv p'$ and there does not exists any p_{n+1} such that $p' \approx\!\xrightarrow{\tau} p_{n+1}$. (Note: We use $p \approx\!\xrightarrow{\tau} q$ to denote $p \xrightarrow{\tau} q$ and $p \approx q$ here). Since $Act^+ = (Act.\tau^*)^+$ we may identify $\mathcal{L}_\dagger$ with $\mathcal{L}_\dagger^+ = \langle \mathbf{P}, Act^+, \longrightarrow_\dagger^+ \rangle$ where $\longrightarrow_\dagger^+$ denotes one or more transitions via $\longrightarrow_\dagger$. We can prove that the LTS $\mathcal{L}_\dagger$ is observationally equivalent to the original LTS $\mathcal{L}$ for divergence-free[3] finite state processes through the following lemma. The proof of the lemma uses induction on the number of derivatives modulo observational equivalence that a process can have, where the finite state assumption helps. We refer the reader to Appendix A for the detailed proof.

Lemma 1. *If p is a finite state agent then*

1. *For all $\alpha \in Act^+$ if $p \xrightarrow{\alpha}_\dagger^+ p'$ then $p \xrightarrow{\alpha} p'$.*
2. *For all $\alpha \in Act^+$ if $p \xrightarrow{\alpha} p'$ then there exists α' with $\alpha' \hat{=} \alpha$ such that $p \xrightarrow{\alpha'}_\dagger^+ p''$ and $p' \approx p''$.* □

The main idea behind defining $\mathcal{L}_\dagger$ was to always ensure that no μ-derivative of any process p in $\mathcal{L}_\dagger$ has any τ-derivative which could be weakly bisimilar to itself. This helps us in ensuring that for any two observationally congruent processes, their derivatives in $\mathcal{L}_\dagger$ are observationally congruent to each other as well. For example, both $\tau.\tau.\mathbf{0}$ and $\tau.\mathbf{0}$ have $\mathbf{0}$ as their only derivative in $\mathcal{L}_\dagger$. Thus, we can now define a parameterised bisimulation on $\mathcal{L}_\dagger$ and show that it defines observational congruence. We refer the reader to Appendix A for the detailed proof of the following theorem.

Theorem 2. *For all divergence-free finite-state CCS agents p, q we have $p \sqsubseteq_{(\hat{=}_+,\hat{=}_+)} q$ in $\mathcal{L}_\dagger^+$ iff $p \approx^+ q$, where $\hat{=}_+$ is the restriction of $\hat{=}$ to Act^+.* □

[3] divergence-freeness gurantees that there are no infinite τ chains so that we do not lose any behaviour when defining $L_\dagger$

4 Amortised Bisimulations [KAK05]

In [KAK05] the notion of amortised bisimulations was introduced. The bisimilarity so defined uses priced actions to compare "functionally related" processes in terms of the costs incurred in the long run. The notion generalises and extends the "faster than" preorder defined by Vogler and Lüttgen in [LV06].

Amortised bisimulation is defined on the language of CCS, where in addition to the normal set of actions Act there is a set of priced actions CA as well. Priced actions cannot be restricted or relabelled and (since they do not have complements) cannot take part in synchronisation. They are assigned a cost by a function $c : CA \to \mathbb{N}$. This cost function is extended to $\mathcal{A} = CA \cup Act$ by assigning a zero cost to all actions in Act. For any $a \in \mathcal{A}$, c_a denotes the cost of a. The usual interleaving semantics of CCS is assumed.

Definition 2. *Let $\rho \subseteq \mathcal{A} \times \mathcal{A}$ such that ρ is the identity relation when restricted to Act. A family of relations $\mathscr{R} = \{R_i | \ i \in \mathbb{N}\}$ is called a* **strong amortised ρ-bisimulation***, if whenever $(p, q) \in R_i$ for some $i \in \mathbb{N}$,*

$$p \xrightarrow{a} p' \Rightarrow \exists b, q'[a\rho b \wedge q \xrightarrow{b} q' \wedge p' R_j q']$$

$$q \xrightarrow{b} q' \Rightarrow \exists a, p'[a\rho b \wedge p \xrightarrow{a} p' \wedge p' R_j q']$$

where $j = i + c_b - c_a$. Process p is said to be **amortised cheaper (more cost efficient)** *than q (denoted $p \prec_0^\rho q$ or simply $p \prec^\rho q$) if pR_0q for some strong amortised ρ-bisimulation $\mathscr{R}$. Further, p is said to be* **amortised cheaper** *than q* **up to credit** *i (denoted $p \prec_i^\rho q$) if $(p, q) \in R_i$. The index i gives the maximum credit which p requires to bisimulate q.*

One problem that has vexed the authors of [KAK05] is that of proving that amortised ρ-bisimilarity is preserved under recursion when ρ is known to be a preorder (it may not work otherwise), since standard techniques are not easily available for bisimulations defined as families of relations on processes.

We therefore characterise amortised bisimilarity as a parameterised one. We define $\mathbf{C}$ to be the set of states where each state is of the form $m : p$ where $m \in \mathbb{N}$. The intuition is that the state remembers the total cost incurred so far in reaching the current state. The following rule defines state transitions ($\longrightarrow_C$) in terms of the transitions of a process.

$$\boxed{p \xrightarrow{a} p' \Rightarrow m : p \xrightarrow{(a,n)}_C n : p', \text{ where } n = m + c_a}$$

The set of observables is $\mathcal{O} = \mathcal{A} \times \mathbb{N}$ and the LTS of interest is $\langle \mathbf{C}, \mathcal{O}, \longrightarrow_C \rangle$. The following theorem provides the required characterisation of amortised ρ-bisimilarity.

Theorem 3. *Let $\gamma_\rho = \{((a, m), (b, n)) \mid a\rho b, m \leq n\}$. Then $m : p \sqsubseteq_{(\gamma_\rho, \gamma_\rho)} m : q$ iff $p \prec^\rho q$, for all $m \in \mathbb{N}$.* □

The use of Theorem 3 in conjunction with the inheritance properties (Proposition 2) simplifies various proofs of properties of $\prec^\rho$ by rendering them in a more convenient form in terms of $\sqsubseteq_{(\gamma_\rho,\gamma_\rho)}$ on $\mathbf{C}$. Some notable examples are parts of Proposition 3, Lemma 4 and Proposition 5 in [KAK05]. For instance when ρ is a preorder, so is γ_ρ and hence $\sqsubseteq_{(\gamma_\rho,\gamma_\rho)}$ is a preorder too.

By Proposition 1.2, to show that a pair of processes is bisimilar it is necessary and sufficient to find a bisimulation containing the pair. However, it actually suffices to find a small relation (containing the pair) which by itself is not a bisimulation, but could be completed by relational composition with bisimilarity to yield a bisimulation. Such relations have been (awkwardly) called "upto"-relations [SM92]. But such a completion may not exist unless the underlying relations are preorders on the observables (see fact 4 and Theorem 5 below).

Definition 3. *Let ρ, σ be relations on observables. A relation $S \subseteq \mathbf{P} \times \mathbf{P}$ is said to be a* **potential** (ρ, σ)**-bisimulation** *if $\sqsubseteq_{(\rho,\sigma)} \circ S \circ \sqsubseteq_{(\rho,\sigma)}$ is a (ρ, σ)-bisimulation.*

Fact 4

1. *If ρ and σ are both transitive then every (ρ, σ)-bisimulation is also a potential (ρ, σ)-bisimulation.*
2. *If ρ and σ are both preorders and R is a potential (ρ, σ)-bisimulation, then so are $R \circ \sqsubseteq_{(\rho,\sigma)}$ and $\sqsubseteq_{(\rho,\sigma)} \circ R$.* □

A generalisation of a sufficiency condition which may be used in proving that recursion preserves bisimilarity is the following (see [Mil89] and [AKH92]).

Theorem 5. *Let ρ and σ both be preorders and R a relation such that $(p, q) \in R$ implies the following conditions for all $a, b \in \mathcal{O}$,*

– $p \xrightarrow{a} p' \Rightarrow \exists b \in \mathcal{O}, q'[a\rho b \wedge q \xrightarrow{b} q' \wedge (p', q') \in \sqsubseteq_{(\rho,\sigma)} \circ R \circ \sqsubseteq_{(\rho,\sigma)}]$,

– $q \xrightarrow{b} q' \Rightarrow \exists a \in \mathcal{O}, p'[a\sigma b \wedge p \xrightarrow{a} p' \wedge (p', q') \in \sqsubseteq_{(\rho,\sigma)} \circ R \circ \sqsubseteq_{(\rho,\sigma)}]$.

Then R is a potential (ρ, σ)-bisimulation. □

For CCS expressions e and f, $e \prec^\rho f$ and $m : e \sqsubseteq_{(\gamma_\rho,\gamma_\rho)} n : f$ if under any uniform substitution of processes for the free process variables the resulting processes (respectively states) are related likewise (see [Mil89] for a technically more accurate definition).

Theorem 3 may now be used in conjunction with Theorem 5 to render the problem as one of preserving the $\sqsubseteq_{(\gamma_\rho,\gamma_\rho)}$ relation under recursion on $\mathbf{C}$.

Theorem 6. *Let e and f be guarded CCS expressions and x a free process variable. If ρ is a preorder and $e \prec^\rho f$ then $\underline{\texttt{rec}}\, x[e] \prec^\rho \underline{\texttt{rec}}\, x[f]$.*

Proof outline: For simplicity we assume x may be the only process variable free in e and f. Let $E_m = m : e$, $F_n = n : f$, $p = \underline{\texttt{rec}}\, x[e]$, $q = \underline{\texttt{rec}}\, x[f]$, $P_0 = 0 : p$ and $Q_0 = 0 : q$. We have (by Theorem 3) $\underline{\texttt{rec}}\, x[e] \prec^\rho \underline{\texttt{rec}}\, x[f]$ iff $P_0 \sqsubseteq_{(\gamma_\rho,\gamma_\rho)} Q_0$. Further since ρ is a preorder, both γ_ρ and $\sqsubseteq_{(\gamma_\rho,\gamma_\rho)}$ are also preorders on $\mathcal{A}$ and $\mathbf{C}$ respectively.

Consider the following relation on $\mathbf{C}$.

$$S = \{(m : \{p/x\}g, n : \{q/x\}g) \mid FV(g) \subseteq \{x\}, m \leq n\}$$

$P_0 \equiv 0 : \{p/x\}x$ and $Q_0 \equiv 0 : \{q/x\}x$. So $(P_0, Q_0) \in S$. It then suffices to show that S is a potential $(\gamma_\rho, \gamma_\rho)$-bisimulation. It may be shown by transition induction ([Mil89], [AKH92]) that for all $(P, Q) \in S$ if $P \xrightarrow{\alpha}_C P'$, $\alpha \in \mathcal{O}$, then there exist Q' and β such that $\alpha\gamma_\rho\beta$, $Q \xrightarrow{\beta}_C Q'$ and $P'\ S \circ \sqsubseteq_{(\gamma_\rho,\gamma_\rho)} Q'$. Similarly if $Q \xrightarrow{\beta}_C Q'$, $\beta \in \mathcal{O}$, then we have for some P' and α with $\alpha\gamma_\rho\beta$, that $P \xrightarrow{\alpha}_C P'$ and $P' \sqsubseteq_{(\gamma_\rho,\gamma_\rho)} \circ S\ Q'$. By theorem 5, S is a potential $(\gamma_\rho, \gamma_\rho)$-bisimulation. □

5 Parameterised Bisimulations in Name-passing Calculi

Extending parameterised bisimulations to a value passing calculus requires more work in the theory of parameterised bisimulations. One cannot simply define a static relation on the labels of the transitions in a value passing calculus to define parameterised bisimulations over it. This is because the meanings of the labels/actions of the processes in a value passing calculus such as the π-calculus change dynamically based on the values passed over the input and output ports. For example, consider an agent $p \equiv \overline{u}(x)|(u(y).y.\mathbf{0})$ in the π-calculus[MPW92a,MPW92b], which is one of the most well known name-passing calculi. Suppose we wish to define a parameterised bisimulation using a relation ρ which relates the action y with z, i.e. $y\ \rho\ z$. Then by the semantics of the π-calculus, the value x is passed for y over the port u, which is given in the form of the transition $\overline{u}(x)|(u(y).y.\mathbf{0}) \xrightarrow{\tau} \{x/y\}y.\mathbf{0} \equiv x.\mathbf{0}$. Since the name y has now been identified with x according to the semantics of the π-calculus, the parameterised bisimulation must take this into account and should relate action x with z, since y was related to z. Formulating a theory of parameterised bisimulations which allows the dynamic update of the parameter relations on actions in accordance with the semantics of the value passing calculus is the challenging part which we will address in this section.

In this paper we will use the Fusion Calculus[Vic98] as our name-passing calculus of choice to describe the theory of parameterised bisimulations for value passing calculi. The most important reason in doing so is the explicit identification of names in the fusion calculus using equivalence relations called "fusions" instead of using substitutions to reflect the impact of communications. This explicit equivalence makes it possible for us to give a generalized theory of parameterised bisimulations which is not possible using substitution effects. To see this, consider a simple example of an action $t(u, a)$. Suppose we have a relation on names ρ such that $u\ \rho\ v$, $a\ \rho\ b$ and $t\ \rho\ t$. Then one can extend the relation ρ to relate the action $t(u, a)$ with $t(v, b)$. However, consider the case where some name w has been identified with both u and a, via some interactions between communication actions. Then the action $t(w, w)$ should be ρ-related to $t(v, b)$, however it is not possible to obtain this relationship by applying a substitutive effect on $t(w, w)$, which will replace w with a unique name, which may be either u or a. We need to consider the equivalence relation on names which identifies w with both u and a, in order to be able to relate $t(w, w)$ with $t(u, a)$ under ρ. Thus representing the identification of names as a result of communication by an equivalence relation on names is necessary to develop a general theory of parameterised bisimulations for name-passing calculi. The fusion calculus helps us in this regard as it represents the effects of communications by "fusion" actions which define equivalence relations on names.

Although the semantics of the fusion calculus does define equivalence relations on names via "fusion" actions, bisimulations in the fusion calculus are still defined in terms of the substitutive effects of fusions. As mentioned above, we would like the definitions of parameterised bisimulations to be independent of the substitutive effects. We therefore provide a modified operational semantics of the fusion calculus, which makes the effect of fusion actions explicit and allows for the definitions of bisimulations in a more natural manner without the use of any substitutive effects. We devote the next subsection to providing the details of the modified semantics for the fusion calculus with a brief comparision with the original semantics [Vic98].

5.1 An Alternative Operational Semantics for the Fusion Calculus

Assume an infinite set of names $\mathcal{N}$ ranged over by $u, v, \ldots, z$. Let $\tilde{x}$ denote a (possibly empty) finite sequence of names $x_1, \ldots, x_n$. Then the syntax of *fusions* (ranged over by φ), *free actions* (ranged over by α) and that of *agents* (*processes*) in the fusion calculus is given by the following BNF.

$$\begin{array}{ll} \varphi & ::= \{\tilde{x} = \tilde{y}\} \\ \alpha & ::= u\tilde{x} \mid \bar{u}\tilde{x} \mid \varphi \\ p, q, r & ::= \mathbf{0} \mid \alpha.q \mid q + r \mid q|r \mid (x)q \end{array}$$

where $u\tilde{x}$ and $\bar{u}\tilde{x}$ are polyadic versions of input and output respectively (collectively called *communication actions* with u and $\bar{u}$ called *subject*s and $\tilde{x}$ the *objects* of the communication) and φ is a *fusion action*. A fusion represented by $\{\tilde{x} = \tilde{y}\}$ (for sequences of names of equal length) is the smallest equivalence relation such that $x_i = y_i$ for $1 \leq i \leq n$. Thus φ ranges over total equivalence relations over $\mathcal{N}$ with only finitely many non-singular classes. For any name z and equivalence φ, $\varphi\backslash z$ is the equivalence $(\varphi \cap (\mathcal{N} - \{z\})^2) \cup \{(z, z)\}$ obtained from φ by deleting all occurrences of the name z. We denote the empty fusion (the identity relation) by $\mathbf{1}$. The set of names occurring in an action α is denoted as $n(\alpha)$. The name x is said to be *bound* in $(x)q$. We write $(\tilde{x})q$ for $(x_1) \cdots (x_n)q$. An *action* is either a fusion action or a communication action of the form $(\tilde{z})u\tilde{x}$ where $\tilde{z} = z_1, \ldots, z_n$, $n \geq 0$ and each $z_i \in \tilde{x}$. A communication action is *bound* if $n > 0$. There are no bound fusion actions.We will denote the set of all actions by Act (ranged over by α, β, γ) and the set of all agents $\mathbf{P}$. We denote sequences of actions by bold Greek letters $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$.

So far, the above description given for actions and processes in the fusion calculus is identical to that given in Victor's thesis[Vic98]. In a departure from Victor's treatment, we associate an *Environment* or “shared state” in the operational semantics of a process. This environment may be seen as the accumulation of the “side effects” of the fusion actions during transitions. Every action and hence a sequence of actions has a side effect, which is the creation of an equivalence on names. We define a function on action sequences which captures this side effect.

Definition 4.

$$E(\boldsymbol{\alpha}) = \begin{cases} \varphi \oplus E(\boldsymbol{\alpha'}) & \textit{if } \boldsymbol{\alpha} = \varphi.\boldsymbol{\alpha'} \\ E(\boldsymbol{\alpha'}) & \textit{if } \boldsymbol{\alpha} = \alpha.\boldsymbol{\alpha'} \\ \mathbf{1} & \textit{if } \boldsymbol{\alpha} = \epsilon \end{cases}$$

where α is a communication action and $\varphi \oplus \varphi' = (\varphi \cup \varphi')^*$ denotes the smallest equivalence relation containing both φ and φ'.

We formally define the environment or the shared state which is created during process execution in the following definition. We also define their substitutive effects which allow us to reduce an environment and process pair to a single process. Substitutive effects are important when we need to compare our bisimulations with the one's defined by Victor's semantics.

Definition 5. *Let Env, referred to as the set of* environments*, be the set of all equivalence relations on $\mathcal{N}$ defined by a finite set of pairs of non-identical names. Thus an environment $\psi \in Env$ is an equivalence relation on names and it is extended to actions as follows.*

- $(\tilde{y_1}).u.\tilde{x_1}\ \psi\ (\tilde{y_2}).v.\tilde{x_2}$ *iff* $|\tilde{y_1}| = |\tilde{y_2}|$, $|\tilde{x_1}| = |\tilde{x_2}|$, $\tilde{y_1}\ \psi\ \tilde{y_2}$, $\tilde{x_1}\ \psi\ \tilde{x_2}$ *and* $u\ \psi\ v$.
- $\{\tilde{y_1} = \tilde{x_1}\}\ \psi\ \{\tilde{y_2} = \tilde{x_2}\}$ *iff* $|\tilde{y_1}| = |\tilde{y_2}|$, $|\tilde{x_1}| = |\tilde{x_2}|$, $\tilde{y_1}\ \psi\ \tilde{y_2}$ *and* $\tilde{x_1}\ \psi\ \tilde{x_2}$.

Definition 6. *A* **substitutive effect** *of an environment ψ is a substitution θ such that $\forall x, y$ we have $x\psi y$ if and only if $\theta(x) = \theta(y)$ and $\forall x, y$ if $\theta(x) = y$ then $x\psi y$.*

$$\textsc{PrefL}\ \frac{-}{\alpha.p \xrightarrow{\alpha} p} \qquad \textsc{PrefR}\ \frac{-}{(\psi, \alpha.p) \xrightarrow{\alpha} (\psi \oplus \psi(\alpha), p)}$$

$$\textsc{SumL}\ \frac{p \xrightarrow{\alpha} p'}{p + q \xrightarrow{\alpha} p'} \qquad \textsc{SumR}\ \frac{(\psi, p) \xrightarrow{\alpha} (\psi', p')}{(\psi, p + q) \xrightarrow{\alpha} (\psi', p')}$$

$$\textsc{ParL}\ \frac{p \xrightarrow{\alpha} p'}{p|q \xrightarrow{\alpha} p'|q} \qquad \textsc{ParR}\ \frac{(\psi, p) \xrightarrow{\alpha} (\psi', p')}{(\psi, p|q) \xrightarrow{\alpha} (\psi', p'|q)}$$

$$\textsc{PassL}\ \frac{p \xrightarrow{\alpha} p'}{(z)p \xrightarrow{\alpha} (z)p'}, z \notin n(\alpha) \qquad \textsc{PassR}\ \frac{(\psi, p) \xrightarrow{\alpha} (\psi', p')}{(\psi, (z)p) \xrightarrow{\alpha} (\psi', (z)p')}, z \notin n(\alpha)$$

$$\textsc{ScopeL}\ \frac{p \xrightarrow{\varphi} p', z\varphi x, x \neq z}{(z)p \xrightarrow{\varphi \backslash z} p'\{x/z\}} \qquad \textsc{ScopeR}\ \frac{(\psi, p) \xrightarrow{\varphi} (\psi \oplus \varphi, p'), z\varphi x, x \neq z}{(\psi, (z)p) \xrightarrow{\varphi \backslash z} (\psi \oplus (\varphi \backslash z), p'\{x/z\})}$$

$$\textsc{OpenL}\ \frac{p \xrightarrow{(\tilde{y})a\tilde{x}} p', z \in \tilde{x} - \tilde{y}, a \notin \{z, \bar{z}\}}{(z)p \xrightarrow{(z\tilde{y})a\tilde{x}} p'} \qquad \textsc{OpenR}\ \frac{(\psi, p) \xrightarrow{(\tilde{y})a\tilde{x}} (\psi, p'), z \in \tilde{x} - \tilde{y}, a \notin \{z, \bar{z}\}}{(\psi, (z)p) \xrightarrow{(z\tilde{y})a\tilde{x}} (\psi, p')}$$

$$\textsc{ComL}\ \frac{p \xrightarrow{u\tilde{x}} p', q \xrightarrow{\bar{u}\tilde{y}} q', |\tilde{x}| = |\tilde{y}|}{p|q \xrightarrow{\{\tilde{x} = \tilde{y}\}} p'|q'} \qquad \textsc{ComR}\ \frac{(\psi, p) \xrightarrow{u\tilde{x}} (\psi, p'), (\psi, q) \xrightarrow{\bar{v}\tilde{y}} (\psi, q'), |\tilde{x}| = |\tilde{y}|, u\ \psi\ v}{(\psi, p|q) \xrightarrow{\{\tilde{x} = \tilde{y}\}} (\psi \oplus \{\tilde{x} = \tilde{y}\}, p'|q')}$$

Fig. 1. Original (L) and alternative (R) SOS rules (modulo structural congruence)

The left half of figure 1 shows the original operational semantics of the fusion calculus (modulo structural congruence)[Vic98]. The right half is the alternative operational semantics based on our notion that the state of process execution should be represented by an environment-process pair. An α denotes a free action in the rules. Every agent according to this semantics executes in an environment (possibly the identity denoted by $\mathbf{1}$) defined and regulated by scope. It should be noted that all the transitions allowed by the rules in the original semantics hold in the alternative semantics as well, except that the new semantics defines the transition on an environment-process pair. Thus, any transition that an agent p can perform under the rules of the original semantics also holds for (ψ, p) for any environment ψ. In fact, the COM rule in figure 1 allows more transitions in our semantics by specifying possible synchronizations between previously fused names. We illustrate this difference with the following example.

Example 1. Consider a process $r \equiv ua.a.\mathbf{0}|\bar{u}b.\bar{b}.\mathbf{0}$. Then by the semantics of figure 1 the following sequence of transitions can be derived starting with the identity environment $\mathbf{1}$.

$$(\mathbf{1}, ua.a.\mathbf{0}|\bar{u}b.\bar{b}.\mathbf{0}) \xrightarrow{\{a=b\}} (\{a = b\}, a.\mathbf{0}|\bar{b}.\mathbf{0}) \xrightarrow{\mathbf{1}} (\{a = b\}, \mathbf{0})$$

The second transition obtained by the interaction of a with $\bar{b}$ was made possible by the COMR rule as a and b have been fused, but it would not have been possible in the original semantics. However while defining behavioral relations the second transition is indeed taken into account by virtue of a substitutive effect of the first transition (which will either substitute a for b or vice-versa)[Vic98]. Thus even though the new semantics yields more transitions for the processes when compared with the original semantics, it still models the same intended behavior with the added advantage that we do not have to rely on any substitutions while defining bisimulations.

The following lemmas help in establishing the equivalence of behavior as given by original semantics using substitutive effect and the modified semantics using environments.

Lemma 2. ([Bag11]) *Let θ_1 be a substitutive effect of ψ_1 and s be a substitutive effect of $\theta_1(\psi_2)$. Then $s \circ \theta_1$ is a substitutive effect of $\psi_1 \oplus \psi_2 = \psi$.* □

Lemma 3. ([Bag11]) *For any $\psi \in Env$ and $x, z \in \mathcal{N}$ such that $x\psi z$, we have $(\psi, p) \xrightarrow{\alpha} (\psi \oplus E(\alpha), p')$ iff $(\psi\backslash x, \{z/x\}p) \xrightarrow{\{z/x\}\alpha} (\psi \oplus E(\alpha)\backslash x, \{z/x\}p')$* □

Since the main focus of this paper is on the results concerning parameterised bisimulations, we will limit our discussion to behaviours as described by the modified semantics. We refer the reader to [Bag11] for a formal proof of equivalence upto bisimilarity of the two semantics.

5.2 Parameterised Bisimulations in the Fusion Calculus

We first give the definition of bisimulations for the fusion calculus according to the original semantics[Vic98] before defining parameterised bisimulations for the modified semantics. In order to do so, we must first extend the transition relation for processes defined by Victor's semantics to Act^* by incorporating substitutive effects of the transitions.

Definition 7. *For all $p \in \mathbf{P}$, we have $p \xrightarrow{\epsilon} p$ and for all $\boldsymbol{\alpha} = \alpha.\boldsymbol{\alpha'}$ where $\boldsymbol{\alpha}, \boldsymbol{\alpha'} \in Act^*$, we have $p \xrightarrow{\boldsymbol{\alpha}} p'$ iff there exists some $p'' \in \mathbf{P}$ such that $p \xrightarrow{\alpha} p''$ and $(\theta p'') \xrightarrow{\theta\boldsymbol{\alpha'}} p'$ where θ is some substitutive effect of E(α).*

Definition 8. *A relation $\mathcal{R}$ is a strong bisimulation if $p\mathcal{R}q$ implies $\forall \boldsymbol{\alpha}, \boldsymbol{\beta} \in Act^*$,*

$$p \xrightarrow{\boldsymbol{\alpha}} p' \Rightarrow \exists q' : q \xrightarrow{\boldsymbol{\alpha}} q' \wedge (\theta_{\boldsymbol{\alpha}} p')\mathcal{R}(\theta_{\boldsymbol{\alpha}} q')$$
$$q \xrightarrow{\boldsymbol{\beta}} q' \Rightarrow \exists p' : p \xrightarrow{\boldsymbol{\beta}} p' \wedge (\theta_{\boldsymbol{\beta}} p')\mathcal{R}(\theta_{\boldsymbol{\beta}} q')$$

where $\theta_{\boldsymbol{\pi}}$ is a substitutive effect of $E(\boldsymbol{\pi})$ with $\boldsymbol{\pi} = \boldsymbol{\alpha}, \boldsymbol{\beta}$. $\dot{\sim}$ denotes **strong bisimilarity**.

As we argued before, the meanings of the actions in value passing calculi change in accordance with the names identified by transitions. In the fusion calculus, this identification of names is represented by the environment. This equivalence on names is taken into account using substitutive effects when defining bisimulations for the fusion calculus. With the modified semantics we can incorporate the effect of the environment more easily by arguing that given any state (ψ, p), any computation $\boldsymbol{\alpha}$ performed by process p should be considered exactly identical to a computation $\boldsymbol{\alpha'}$ if $\boldsymbol{\alpha}\psi\boldsymbol{\alpha'}$ holds, where ψ is extended point-wise to actions. Let ρ be a relation on actions which determines which computations should be considered ρ-related for the purpose of defining parameterised bisimulations. Then an action sequence $\boldsymbol{\beta}$ should be considered ρ-related to another action sequence $\boldsymbol{\alpha}$ if there exist action sequences $\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}'$ such that $\boldsymbol{\alpha}\psi\boldsymbol{\alpha}'$, $\boldsymbol{\beta}\psi\boldsymbol{\beta}'$ and $\boldsymbol{\alpha}'\rho\boldsymbol{\beta}'$. Equivalently, given an environment ψ, an action sequence $\boldsymbol{\beta}$ should be considered ρ-related to another action sequence $\boldsymbol{\alpha}$ iff $\boldsymbol{\alpha}\ \psi \circ \rho \circ \psi \quad bm\beta$ holds. With this formalisation of *"ρ-relatedness"* we modify our standard definition of (ρ, σ)-bisimulations to define a generalised (ρ, σ)-bisimulation for the fusion calculus.

Definition 9. *Let $\rho, \sigma \subseteq Act^2$. A relation $\mathcal{G} \subseteq (Env \times \mathbf{P})^2$ is a* **generalised** (ρ, σ)**-bisimulation** *if $(\psi, p)\ \mathcal{G}\ (\omega, q)$ implies for all $\boldsymbol{\alpha_1}, \boldsymbol{\beta_2} \in Act^*$,*

$$(\psi, p) \xrightarrow{\boldsymbol{\alpha_1}} (\psi', p') \Rightarrow \exists \boldsymbol{\alpha_2} \colon \boldsymbol{\alpha_1}\rho'\boldsymbol{\alpha_2}, (\omega', q') \colon (\omega, q) \xrightarrow{\boldsymbol{\alpha_2}} (\omega', q') \wedge (\psi', p')\ \mathcal{G}\ (\omega', q')$$
$$(\omega, q) \xrightarrow{\boldsymbol{\beta_2}} (\omega', q') \Rightarrow \exists \boldsymbol{\beta_1} \colon \boldsymbol{\beta_1}\sigma'\boldsymbol{\beta_2}, (\psi', p') \colon (\psi, p) \xrightarrow{\boldsymbol{\beta_1}} (\psi', p') \wedge (\psi', p')\ \mathcal{G}\ (\omega', q')$$

where $\rho' = \psi \circ \rho \circ \omega$ and $\sigma' = \psi \circ \sigma \circ \omega$.

While comparing processes, we may claim that they are related only if they display related behaviours under the same environment. This leads to the following definition of bisimulation.

Definition 10. *A generalised (ρ, σ)-bisimulation $\mathcal{G}$ is a* (ρ, σ)**-bisimulation** *if for all $\psi, \omega \in Env$ and $p, q \in \mathbf{P}$, $(\psi, p)\ \mathcal{G}\ (\omega, q)$ implies $\psi = \omega$. We refer the largest (ρ, σ)-bisimulation as* (ρ, σ)**-bisimilarity** *(denoted $\square_{(\rho,\sigma)}$).*

To be able to relate the parameterised bisimulations (Definition 10) with the bisimulations already defined for fusion calculus (Definition 8), we need to define a mapping from states to processes since our bisimulations are defined for states. This motivates the definition of the following translation relation.

Definition 11. *Let the relation $\mathbf{T} \subseteq States \times \mathbf{P}$ called the* **translation relation** *be defined by $(\psi, p)\mathbf{T}p'$ if and only if there exists θ a substitutive effect of ψ, such that $\theta p = p'$.*

Proposition 1 *Let* $\mathbf{T}$ *be the translation relation given in Definition 11. Then* $\mathbf{T}\circ\dot{\sim}\circ\mathbf{T}^{-1}$ *is a generalised* $(\equiv,\equiv)$*-bisimulation (Definition 10), where* $\dot{\sim}$ *is strong bisimilarity (Definition 8).*

Proof. Let $(\psi, p)\mathbf{T} \circ \dot{\sim} \circ \mathbf{T}^{-1}(\omega, q)$. Then by definition of the translation relation, for some substitutive effect θ_ψ of ψ and θ_ω of ω we must have $\theta_\psi p \mathrel{\dot{\sim}} \theta_\omega q$. Let $(\psi, p) \xrightarrow{\boldsymbol{\alpha}} (\psi', p')$ where $\psi' = \psi \oplus E(\boldsymbol{\alpha})$. Then by Lemma 3, $\theta_\psi p \xrightarrow{\theta_\psi(\boldsymbol{\alpha})} \theta_\psi p'$ which implies that there exists $q'' \in \mathbf{P}$ such that $\theta_\omega q \xrightarrow{\theta_\psi(\boldsymbol{\alpha})} q''$ and $\theta(\theta_\psi(p'))\dot{\sim}\theta(q'')$, where θ is a substitutive effect of $\theta_\psi(\boldsymbol{\alpha})$. Now by converse of Lemma 3 there must exist $q' \in \mathbf{P}$ and $\boldsymbol{\beta} \in Act^*$ such that $(\omega, q) \xrightarrow{\boldsymbol{\beta}} (\omega', q')$ where $\omega' = \omega \oplus E(\boldsymbol{\beta})$, $\theta_\omega(\boldsymbol{\beta}) = \theta_\psi(\boldsymbol{\alpha})$ and $\theta_\omega(q') = q''$. Now by Lemma 2, $\theta \circ \theta_\psi$ is a substitutive effect of ψ' and $\theta \circ \theta_\omega$ is a substitutive effect of ω' . Hence we have $(\omega, q) \xrightarrow{\boldsymbol{\beta}} (\omega', q')$ where $\boldsymbol{\alpha}\psi\circ \equiv \circ\omega\boldsymbol{\beta}$ and $(\psi', p')\mathbf{T} \circ \dot{\sim} \circ \mathbf{T}^{-1}(\omega', q')$. A similar proof may be given for a transition of q. □

Proposition 2 *Let* δ *be a one to one function mapping environments to substitutions such that* $\delta(\psi)$ *is a substitutive effect of* ψ*, for any* $\psi \in Env$*. Given* δ*, we define a sub-relation* $\mathbf{S}$ *of the translation relation* $\mathbf{T}$ *such that* $(\psi, p)\ \mathbf{S}\ q$ *iff* $(\delta(\psi))p = q$*. Then* $\mathbf{S}^{-1} \circ \Box_{(\equiv,\equiv)} \circ \mathbf{S} \subseteq \dot{\sim}$.

Proof. Let $p\mathbf{S}^{-1} \circ \Box_{(\equiv,\equiv)} \circ \mathbf{S}q$. Then for some ψ, p', q' we have $(\psi, p')\Box_{(\equiv,\equiv)}(\psi, q')$ where $(\delta(\psi))p' = p$ and $(\delta(\psi))q' = q$ (by definition of $\mathbf{S}$). Suppose $p \xrightarrow{\boldsymbol{\gamma}} p_d$. Then by Lemma 3, we have $(\psi, p') \xrightarrow{\boldsymbol{\alpha}} (\psi', p'')$ where $(\delta(\psi))p'' = p_d$, $\delta(\psi)(\boldsymbol{\alpha}) = \boldsymbol{\gamma}$ and $\psi' = \psi \oplus E(\boldsymbol{\alpha})$. Then by definition of $\Box_{(\equiv,\equiv)}$, there must exist $\boldsymbol{\beta}$ such that $(\psi, q') \xrightarrow{\boldsymbol{\beta}} (\psi', q'')$ where $\boldsymbol{\alpha}\psi\circ \equiv \circ\psi\boldsymbol{\beta}$ and $(\psi', p'')\Box_{(\equiv,\equiv)}(\psi', q'')$. Since $\boldsymbol{\alpha}\psi\circ \equiv \circ\psi\boldsymbol{\beta}$, we have $(\delta(\psi))(\boldsymbol{\alpha}) = \boldsymbol{\gamma} = (\delta(\psi))(\boldsymbol{\beta})$. Thus we have $q \xrightarrow{\boldsymbol{\gamma}} q_d$ where $q_d = (\delta(\psi))q''$. Now let s be any substitutive effect of $\boldsymbol{\gamma} = \delta(\psi)(\boldsymbol{\alpha})$, then by Lemma 2, $s \circ \delta(\psi)$ is a substitutive effect of $\psi' = \psi \oplus E(\boldsymbol{\alpha})$. Furthermore we can choose s and $\delta(\psi')$ to be such that $s \circ \delta(\psi) = \delta(\psi')$. We thus have $s(p_d)\ \mathbf{S}^{-1} \circ \Box_{(\equiv,\equiv)} \circ \mathbf{S}\ s(q_d)$ (as $(\delta(\psi))p'' = p_d$, $(\psi', p'')\Box_{(\equiv,\equiv)}(\psi', q'')$ and $(\delta(\psi))q'' = q_d$). A similar proof may be given for a transition of q. □

Corollary 1. $\Box_{(\equiv,\equiv)} = \mathbf{T} \circ \dot{\sim} \circ \mathbf{T}^{-1}$*, where* $\dot{\sim}$ *is strong bisimilarity (Definition 8).*

Proof. It follows from Proposition 2 that $\mathbf{S}^{-1} \circ \Box_{(\equiv,\equiv)} \circ \mathbf{S} \subseteq \dot{\sim}$ for some $\mathbf{S} \subseteq \mathbf{T}$. Since $\circ$ is monotonic in each argument with respect to the $\subseteq$ ordering, we get $\Box_{(\equiv,\equiv)} \subseteq \mathbf{S} \circ \dot{\sim} \circ \mathbf{S}^{-1} \subseteq \mathbf{T} \circ \dot{\sim} \circ \mathbf{T}^{-1}$. The reverse containment follows from Proposition 1. □

5.3 Parameterised Hyperbisimulations

A (ρ, σ)-bisimulation as defined above only compares two states under identical environments. However we are actually interested in comparing processes and not the states in which they operate. Intuitively speaking, two processes may be considered equivalent only if they are equivalent under all environments. Hence we need to extend (ρ, σ)-bisimulations to a bisimulation based ordering defined over processes.

Definition 12. *A relation $\mathscr{H} \subseteq \mathbf{P}^2$ is a (ρ, σ)*-**hyperbisimulation** *iff for all $p, q \in \mathbf{P}$, $p\mathscr{H}q$ implies for all $\psi \in Env$, there is a (ρ, σ)-bisimulation $\mathcal{G}$ such that $(\psi, p)\mathcal{G}(\psi, q)$. The largest (ρ, σ)-hyperbisimulation called (ρ, σ)*-**hyperbisimilarity** *is denoted $\underline{\square}_{(\rho,\sigma)}$.*

The concept of hyperbisimulations is unique to fusion calculus and it was originally defined by Victor in his work as the largest congruence contained within bisimulation. A very important property of interest for hyperbisimulations in the fusion calculus is the property of substitution closure which is necessary if one wishes to prove that hyperbisimilarity is a congruence.

Definition 13. *A relation ρ is* **conservative** *iff $\forall \boldsymbol{\alpha}, \boldsymbol{\beta}$ if $\boldsymbol{\alpha}\rho\boldsymbol{\beta}$ then $E(\boldsymbol{\alpha}) = E(\boldsymbol{\beta})$. It is* **substitution-closed** *iff $\forall x, y, \boldsymbol{\alpha}, \boldsymbol{\beta}$ if $\boldsymbol{\alpha}\rho\boldsymbol{\beta}$ then $(\{x/y\}\boldsymbol{\alpha})\rho(\{x/y\}\boldsymbol{\beta})$.*

The following result (see [Bag11] for a proof) shows that substitution-closure on processes can also be derived from certain properties of the relations on actions for parameterised hyperbisimulations.

Corollary 2. ([Bag11]) *If ρ and σ are both conservative and substitution closed relations on actions then for all $\psi \in Env$ and θ such that θ is a substitutive effect of ψ we have $(\psi, p)\square_{(\rho,\sigma)}(\psi, q)$ if and only if $(\mathbf{1}, \theta p)\square_{(\rho,\sigma)}(\mathbf{1}, \theta q)$. Furthermore if $p\underline{\square}_{(\rho,\sigma)}q$ then for all substitutions θ we have $(\theta p)\underline{\square}_{(\rho,\sigma)}(\theta q)$.* □

Our motivation in defining hyperbisimulations is the same as Victor's, i.e. hyperbisimulations should relate processes which have the same behaviour in all contexts. However we have defined hyperbisimulations as the natural lifting of bisimulations, which are relations defined over states, to relations defined over processes, whereas in [Vic98] hyperbisimulation is used to define the largest congruence contained in bisimulations defined on original semantics and is obtained by closing the relation under all substitutions.

Definition 14. *A strong bisimulation (Definition 8) $\mathcal{R}$ is a* **strong hyperbisimulation** *iff it is substitution-closed i.e. for all substitutions θ, if $p\mathcal{R}q$ then $(\theta p)\mathcal{R}(\theta q)$. We denote the largest strong hyperbisimulation, called* **strong hyperbisimilarity***, by $\sim$.*

The reason we choose to call the relation defined in Definition 12 as parameterised hyperbisimulation is because the relation defined by us turns out to be identical to the hyperbisimulations defined by Victor using the original semantics, as shown by the following result.

Corollary 3. $\underline{\square}_{(\equiv,\equiv)} = \sim$ *i.e. strong hyperbisimilarity (Definition 14), equals parameterised $(\equiv, \equiv)$-hyperbisimilarity (Definition 12).*

Proof. By Definition 14, $p \sim q$ iff for all substitutions θ we have $\theta p \dot{\sim} \theta q$. By Corollary 1 and noting that $(\mathbf{1}, p)\mathbf{T}p$ holds for all processes p, we have $\theta p \dot{\sim} \theta q$ if and only if $(\mathbf{1}, \theta p)\square_{(\equiv,\equiv)}(\mathbf{1}, \theta q)$. Let ψ be any environment such that θ is a substitutive effect of ψ, then by Corollary 2, we have $(\psi, p)\square_{(\equiv,\equiv)}(\psi, q)$. Since the result holds for all substitutions θ and hence for all environments ψ which have θ as its substitutive effect, by Definition 12, we have $p\underline{\square}_{(\equiv,\equiv)}q$. Each step of the proof is reversible, hence the converse also holds. □

6 Concluding Remarks

Some applications of parameterisation to the algebraic theory of bisimulations on process algebras were presented in this paper. While parameterisation has led to a more general notion of bisimulation, we have gone further in this paper by generalising this notion for name-passing calculi. In a manner similar to our earlier results, one can show that the properties of parameterised bisimulations for the value-passing calculus may be derived from the properties of the relations defined on actions, thereby providing a generalized framework for the study of bisimulations in value-passing calculi. In particular, one can show that the monotonicity, inversion, symmetry and reflexivity properties as shown in Proposition 2 also hold for these bisimulations, by simply noting that if ρ is symmetric or reflexive then so is $\psi \circ \rho \circ \psi$ and $(\psi \circ \rho \circ \psi)^{-1} = \psi \circ \rho^{-1} \circ \psi$. Also if one were to limit oneself to processes which can be represented in a non-value passing calculus like CCS, then the Definition 10 reduces to Definition 1. Therefore it strengthens our confidence that Definition 10 is the correct generalization of parameterised bisimulations to value passing calculus.

A more general proof can be given along the lines of corollary 3 to show that analogous notions such as weak (hyper-)bisimulations, efficiency preorder [AKH92] and elaborations [AKN95] over fusion calculus agents are special cases of $(\rho, \sigma)-$(hyper-)bisimulations by choosing ρ and σ appropriately. Further work may be done on investigating and extending the earlier results given for parameterised bisimulations, for example the axiomatization of parameterised bisimulations given in [SAK09], to the more generalised framework given for name passing calculus presented in this paper.

Acknowledgement. The research presented in this paper was partly sponsored by EADS Corp. We are thankful to the suggestions of anonymous reviewers who helped improve this paper. We are also thankful to Shibashis Guha for his careful review and suggestions.

References

[AKH92] Arun-Kumar, S., Hennessy, M.: An efficiency preorder for processes. Acta Informatica 29, 737–760 (1992)

[AKN95] Arun-Kumar, S., Natarajan, V.: Conformance: A precongruence close to bisimilarity. In: Structures in Concurrency Theory, pp. 55–68. Springer (1995)

[AK06] Arun-Kumar, S.: On bisimilarities induced by relations on actions. In: Software Engineering and Formal Methods, 2006. SEFM 2006. Fourth IEEE International Conference on, pp. 41–49. IEEE (2006)

[Bag11] Bagga, D.: Parametrised bisimulations for the fusion calculus. Master's thesis, Department of Computer Science and Engineering, IIT Delhi (2011). Available Online `http://www.cse.iitd.ac.in/~bagga/bag11.html`

[BK85] Bergstra, J. A., Klop, J. W.: Algebra of communicating processes with abstraction. Theoretical computer science 37, 77–121 (1985)

[KAK05] Kiehn, A., Arun-Kumar, S.: Amortised bisimulations. In: Formal Techniques for Networked and Distributed Systems-FORTE 2005, pp. 320–334. Springer (2005)

[LV06] Lüttgen, G., Vogler, W.: Bisimulation on speed: A unified approach. Theoretical Computer Science 360, 209–227 (2006)

[Mil89] Milner, R.: Communication and concurrency. Prentice-Hall, Inc. (1989)

[MPW92a] Milner, R., Parrow, J., Walker, D.: A calculus of mobile processes, i. Information and computation 100, 1–40 (1992)

[MPW92b] Milner, R., Parrow, J., Walker, D.: A calculus of mobile processes, II. Information and Computation 100, 41–77 (1992)

[PV98] Parrow, J., Victor, B.: The fusion calculus: Expressiveness and symmetry in mobile processes. In: Logic in Computer Science, 1998. Proceedings. Thirteenth Annual IEEE Symposium on, pp. 176–185. IEEE (1998)

[SM92] Sangiorgi, D., Milner, R.: The problem of "Weak Bisimulation up to". In: CONCUR'92, pp. 32–46. Springer (1992)

[SAK09] Singh, P., Arun-Kumar, S.: Axiomatization of a Class of Parametrised Bisimilarities. Perspectives in Concurrency Theory. Universities Press, India (2009)

[Vic98] Victor, B.: The fusion calculus: Expressiveness and symmetry in mobile processes. PhD thesis, Uppsala University (1998)

A Appendix: Observational Congruence Proof

We provide the proofs for the observational equivalence of the derived LTS $\mathcal{L}_\dagger$ with the original LTS $\mathcal{L}$ (see Lemma 1 in paper) and the observational congruence as parameterised bisimulation in $\mathcal{L}_\dagger$ (see Theorem 2 in paper) here. The formal proof of the results mentioned in the paper requires the proof of various other lemmas, which makes the proof quite lengthy but it ensures that we cover all the details, thus making them necessary.

Definition A1 *Let $\mathcal{L} = \langle \boldsymbol{P}, Act, \rightarrow \rangle$ be the usual LTS defined by divergence-free finite-state CCS agents. We define an LTS $\mathcal{L}_\dagger = \langle \boldsymbol{P}, Act.\tau^*, \longrightarrow_\dagger \rangle$ derived from $\mathcal{L}$ such that $Act.\tau^* = \{\mu\tau^n \mid \mu \in Act, n \geq 0\}$ and $p \xrightarrow{\mu\tau^n}_\dagger p'$ iff there exists $n \geq 0$ and processes $p_0, \cdots, p_n$ such that $p \xrightarrow{\mu} p_0 \approx\!\xrightarrow{\tau} p_1 \approx\!\xrightarrow{\tau} \cdots \approx\!\xrightarrow{\tau} p_n \equiv p'$ and there does not exists any p_{n+1} such that $p' \approx\!\xrightarrow{\tau} p_{n+1}$. (Note: We use $p \approx\!\xrightarrow{\tau} q$ to denote $p \xrightarrow{\tau} q$ and $p \approx q$ here).*

Since $Act^+ = (Act.\tau^*)^+$ we may identify $\mathcal{L}_\dagger$ with $\mathcal{L}_\dagger^+ = \langle \mathbf{P}, Act^+, \longrightarrow_\dagger^+ \rangle$ where $\longrightarrow_\dagger^+$ denotes one or more transitions via $\longrightarrow_\dagger$. We formally define the set of all weak μ-successors of p as the set Der_μ^p, i.e. $\mathsf{Der}_\mu^p = \{p' | p \xRightarrow{\mu} p', \mu \in Act\}$. We define the following preorder on these sets.

Definition A2 $\mathsf{Der}_\mu^p \subsetapprox \mathsf{Der}_\mu^q$ *if for every $p' \in \mathsf{Der}_\mu^p$ there exists $q' \in \mathsf{Der}_\mu^q$ such that $p' \approx q'$ and $\mathsf{Der}_\mu^p \underset{\not\approx}{\subset} \mathsf{Der}_\mu^q$ if $\mathsf{Der}_\mu^p \subsetapprox \mathsf{Der}_\mu^q$ and there exists $q' \in \mathsf{Der}_\mu^q$ such that $p' \not\approx q'$ for every $p' \in \mathsf{Der}_\mu^p$. $\mathsf{Der}_\mu^p \underset{\approx}{\subset\!\supset} \mathsf{Der}_\mu^q$ if $\mathsf{Der}_\mu^p \subsetapprox \mathsf{Der}_\mu^q$ and $\mathsf{Der}_\mu^q \subsetapprox \mathsf{Der}_\mu^p$.*

From Definition A2 it follows that $p \approx q$ iff $\mathsf{Der}_a^p \underset{\approx}{\subset\!\supset} \mathsf{Der}_a^q$ for every $a \in Act \setminus \{\tau\}$ and $p \approx^+ q$ iff $\mathsf{Der}_\mu^p \underset{\approx}{\subset\!\supset} \mathsf{Der}_\mu^q$ for each $\mu \in Act$. The following lemma follows from the preemptive power of the τ action.

Lemma A1 *For any processes $p, p' \in \boldsymbol{P}$, if $p \xRightarrow{\tau} p'$ then $\mathsf{Der}_\mu^{p'} \subsetapprox \mathsf{Der}_\mu^p$ for every $\mu \in Act$. Further, if $p \not\approx p'$ implies $\mathsf{Der}_\nu^{p'} \underset{\not\approx}{\subset} \mathsf{Der}_\nu^p$ for some $\nu \in Act$.*

Proof. Let $p'' \in \mathsf{Der}^{p'}_{\mu}$ for some $\mu \in Act$, then $p \stackrel{\tau}{\Longrightarrow} p' \stackrel{\mu}{\Longrightarrow} p''$ (by definition of $\mathsf{Der}^{p'}_{\mu}$), hence $p \stackrel{\mu}{\Longrightarrow} p''$. Thus we also have $p'' \in \mathsf{Der}^{p}_{\mu}$. Since p'' was arbitrary, we have $\mathsf{Der}^{p'}_{\mu} \subseteq_{\approx} \mathsf{Der}^{p}_{\mu}$ for all $\mu \in Act$. now if $p \not\approx p'$ implies $\exists \nu \in Act$ such that $p \stackrel{\nu}{\Longrightarrow} p''$ but $\not\exists p'''$ such that $p' \stackrel{\nu}{\Longrightarrow} p'''$ and $p'' \approx p'''$ (by definition of $\approx$). Clearly $p'' \in \mathsf{Der}^{p}_{\nu}$ hence $\mathsf{Der}^{p'}_{\nu} \subseteq_{\not\approx} \mathsf{Der}^{p}_{\nu}$ for some $\nu \in Act$. □

Lemma A2 *If p is a finite state agent then*

1. $p \stackrel{\tau}{\Longrightarrow} p'$ *implies* $p \xrightarrow{\tau^n}_{\dagger} p'' \approx p'$ *for some* $n > 0$, *and*
2. *for any* $a \in Act \setminus \{\tau\}$, $p \stackrel{a}{\Longrightarrow} p'$ *implies* $p \xrightarrow{\tau^m a \tau^n}{}^{+}_{\dagger} p'' \approx p'$ *for some* $m, n \geq 0$.

Proof. (part 1)

Let $p \stackrel{\tau}{\Longrightarrow} p'$ then we must have $p \xrightarrow{\tau^n}_{\dagger} q$ for some $n > 0$ such that either $q \approx p'$ or $q \xrightarrow{\tau^+}_{\dagger} q' \stackrel{\tau}{\Longrightarrow} q''$ such that $q'' \approx p'$ and $q' \not\approx p$. In case 1 the result holds. In case 2 we clearly have by Lemma A2, for all $\mu \in Act$ we have $\mathsf{Der}^{q'}_{\mu} \subseteq_{\approx} \mathsf{Der}^{p}_{\mu}$ and there exists $\nu \in Act$ such that $\mathsf{Der}^{q'}_{\nu} \subseteq_{\not\approx} \mathsf{Der}^{p}_{\nu}$. By using finite state agent assumption i.e. $\sum_{\mu \in Act} |\mathsf{Der}^{p}_{\mu}|$ is finite, applying above logic inductively with $\sum_{\mu \in Act} |\mathsf{Der}^{p}_{\mu}|$ as induction variable we have our result.

(part 2)

Let $p \stackrel{a}{\Longrightarrow} p'$ for some $a \in Act \setminus \{\tau\}$. Then we have the following two cases:

- case 1: $p \xrightarrow{\tau} p_1 \stackrel{a}{\Longrightarrow} p'$
  by part 1, in this case we have $p \xrightarrow{\tau^+}_{\dagger} q$ such that $q \approx p_1$. Hence $q \stackrel{a}{\Longrightarrow} q'$ such that $q' \approx p'$ (by definition of $q \approx p_1$). now if $q \xrightarrow{\tau} q'' \stackrel{a}{\Longrightarrow} q'$ then $q \xrightarrow{\tau^+}_{\dagger} r$ such that $r \approx q''$ (by part 1) but $r \not\approx p$ (by definition of $\mathcal{L}_{\dagger}$ and since $p \overset{\tau^+}{\rightarrow}{}^{2}_{\dagger} r$). Then by claim A2 we have $\sum_{\mu \in Act} |\mathsf{Der}^{r}_{\mu}| < \sum_{\mu \in Act} |\mathsf{Der}^{p}_{\mu}|$ and $r \stackrel{a}{\Longrightarrow} r'$ such that $r' \approx p'$.
  Thus under finite state assumption we can only have this case finitely many times and eventually we will get case 2.
- Case 2: $p \xrightarrow{a} p_1 \Rightarrow p'$
  Then by definition of $\mathcal{L}_{\dagger}$ we must have a q such that $p \overset{a.\tau^*}{\rightarrow}_{\dagger} q$ such that $q \approx p_1$. Since $q \approx p_1$, we have $q \Rightarrow q' \approx p'$. Now either $q \approx p'$ in which case we are done or else we have $q \stackrel{\tau}{\Rightarrow} q' \approx p'$ and $q \not\approx p'$. Then by part 1 we will have $q \xrightarrow{\tau^+}_{\dagger} r \approx q' \approx p'$. □

The following lemma shows that the LTS $\mathcal{L}_{\dagger}$ is observationally equivalent to the original LTS $\mathcal{L}$.

Lemma A3 *If p is a finite state agent then*

1. *For all* $\boldsymbol{\alpha} \in Act^+$ *if* $p \overset{\boldsymbol{\alpha}}{\rightarrow}{}^{+}_{\dagger} p'$ *then* $p \xrightarrow{\boldsymbol{\alpha}} p'$.

2. *For all* $\boldsymbol{\alpha} \in Act^+$ *if* $p \xrightarrow{\boldsymbol{\alpha}} p'$ *then there exists* $\boldsymbol{\alpha'}$ *with* $\boldsymbol{\alpha'} \hat{=} \boldsymbol{\alpha}$ *such that* $p \xrightarrow{\boldsymbol{\alpha'}}{}^{+}_{\dagger}\, p''$ *and* $p' \approx p''$.

Proof. The (1) result follows straightforward from the definition of $\mathcal{L}_\dagger$, since every transition in $\mathcal{L}_\dagger$ is defined only if the corresponding transition exists in $\mathcal{L}$. We prove (2) by induction on the length of $\boldsymbol{\alpha}$. Base case of induction, i.e. $\boldsymbol{\alpha} = a \in Act$, follows trivially from the definition of $\mathcal{L}_\dagger$. We assume by induction hypothesis that for all $\boldsymbol{\alpha}$ such that $|\boldsymbol{\alpha}| = k$ and $k \geq 1$, if $p \overset{\boldsymbol{\alpha}}{\rightarrow} p'$ then there exists $\boldsymbol{\alpha}' \in Act^+$ such that $\hat{\boldsymbol{\alpha}}' = \hat{\boldsymbol{\alpha}}$ and $p \overset{\boldsymbol{\alpha}'}{\rightarrow}{}^{+}_{\dagger} p''$ and $p' \approx p''$. Now Let $|\boldsymbol{\alpha}| = k+1$ then $\boldsymbol{\alpha} = \boldsymbol{\gamma}.a$ where $|\boldsymbol{\gamma}| = k, k \geq 1$. Now if $p \overset{\boldsymbol{\alpha}}{\rightarrow} p'$ then there exists a q such that $p \overset{\boldsymbol{\gamma}}{\rightarrow} q$ and $q \xrightarrow{a} p'$. By IH now there must exist $\boldsymbol{\gamma}' \in Act^+$ such that $\hat{\boldsymbol{\gamma}}' = \hat{\gamma}$ and $p \overset{\boldsymbol{\gamma}'}{\rightarrow}{}^{+}_{\dagger} q'$ and $q \approx q'$. Since $q \approx q'$ and $q \xrightarrow{a} p'$ implies $q' \overset{a}{\Longrightarrow} p'''$ such that $p' \approx p'''$. Then by Lemma A2, there must exist p'' such that $q' \overset{\tau^n.a.\tau^m}{\rightarrow}{}_{\dagger} p''$ for some $n, m \geq 0$ and $p'' \approx p''' \approx p'$. Thus we have $p \overset{\boldsymbol{\alpha}'}{\rightarrow}{}^{+}_{\dagger} p''$ and $p' \approx p''$ where $\boldsymbol{\alpha}' = \boldsymbol{\gamma}'.\tau^n.a.\tau^m$. □

Theorem A1 *For all divergence-free finite-state CCS agents* p, q *we have* $p \,\square_{(\hat{=}_+, \hat{=}_+)}\, q$ *in* $\mathcal{L}^+_\dagger$ *iff* $p \approx^+ q$*, where* $\hat{=}_+$ *is the restriction of* $\hat{=}$ *to* Act^+

Claim A1 *Observational Congruence,* $\approx^+$ *is a* $(\hat{=}_+, \hat{=}_+)-$*bisimulation in* ${\mathcal{L}_\dagger}^+$

Proof. Let $p \approx^+ q$. Then for some $\boldsymbol{\alpha} \in Act^+$ such that $p \overset{\boldsymbol{\alpha}}{\rightarrow}{}^{+}_{\dagger} p'$ we must have $p \overset{\boldsymbol{\alpha}}{\rightarrow} p'$ (by Lemma A3). Since $p \approx^+ q$, there must exist a q' and $\boldsymbol{\beta} \in Act^+$ such that $\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\beta}}$ and $q \overset{\boldsymbol{\beta}}{\rightarrow} q'$ and $p' \approx q'$. Now by Lemma A3 there must exist $\boldsymbol{\beta}' \in Act^+$ such that $\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}'$ and $q \overset{\boldsymbol{\beta}'}{\rightarrow}{}^{+}_{\dagger} q''$ and $q' \approx q''$. i.e. $q \overset{\boldsymbol{\beta}'}{\rightarrow} q''$ and $p' \approx q''$. Now by our definition for ${\mathcal{L}_\dagger}^+$ there does not exists any τ child of p' and q'' which are bisimilar to them, hence p' or q'' cant do a ϵ transition to match a τ transition for the other process and still reach bisimilar states. Hence $p' \approx^+ q''$. Hence we have $q \overset{\boldsymbol{\beta}'}{\rightarrow}{}^{+}_{\dagger} q''$ and $p' \approx^+ q''$ and $\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\beta}}'$. Since $\boldsymbol{\alpha}$ was arbitrary, this holds for all $\boldsymbol{\alpha}$. We can show the result for all transitions of q in the similar way. Hence we have $\approx^+$ as a $(\hat{=}_+, \hat{=}_+)-$bisimulation by definition.

Claim A2 $(\hat{=}_+, \hat{=}_+)-$*bisimulation in* ${\mathcal{L}_\dagger}^+$ *is a observational congruence upto weak bisimulation.*

Proof. Let $p \square_{(\hat{=}_+, \hat{=}_+)} q$ in ${\mathcal{L}_\dagger}^+$. Then for some $\boldsymbol{\alpha} \in Act^+$ such that $p \overset{\boldsymbol{\alpha}}{\rightarrow} p'$ there must exist $\boldsymbol{\alpha}' \in Act^+$ such that $\hat{\boldsymbol{\alpha}}' = \hat{\boldsymbol{\alpha}}$ and $p \overset{\boldsymbol{\alpha}'}{\rightarrow}{}^{+}_{\dagger} p''$ and $p' \approx p''$ (by Lemma A3)

Hence there exists $\boldsymbol{\beta} \in Act^+$ such that $\hat{\boldsymbol{\alpha}}' = \hat{\boldsymbol{\beta}}$ and $q \overset{\boldsymbol{\beta}}{\rightarrow}{}^{+}_{\dagger} q'$ and $p'' \square_{(\hat{=}_+, \hat{=}_+)} q'$. Therefore, $q \overset{\boldsymbol{\beta}}{\rightarrow} q'$ and $\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\beta}}$ and $p' \approx p'' \square_{(\hat{=}_+, \hat{=}_+)} q'$ where $\boldsymbol{\beta} \in Act^+$.Since $\boldsymbol{\alpha}$ was arbitrary, this holds for all $\boldsymbol{\alpha}$. We can show the result for all transitions of q in the similar way. Hence proved. □