## CSL705: Theory of Computation

Tutorial: Turing Machines and Turing Computability

- 1. Prove that for every NPDA, there exists a Turing machine which accepts the same language.
- 2. Design a deterministic turing machine to multiply two non-negative integers.
- 3. Any relation R between various sets is said to be Turing-computable if there exists a Turing machine which implements the *characteristic function* of R. If  $R \subseteq A_1 \times A_2 \times \cdots \times A_n$  then the *characteristic* function of R is the function  $\chi_R$  such that for any  $\vec{a} \in A_1 \times A_2 \times \cdots \times A_n$ ,  $\chi_R(\vec{a}) = 1$  whenever  $\vec{a} \in R$  and  $\chi_R(\vec{a}) = 0$  otherwise.
- 4. Design a deterministic turing machine which implements the "greater than" relation on the naturals.
- 5. Prove that the addition and comparison of rational numbers are Turing computable
- 6. Let T be a (deterministic) Turing machine which accepts a language  $L \subseteq \Sigma^*$ . Design a Turing machine T' which decides membership in L.
- 7. Let  $T_f$  be a Turing machine which implements a partial function  $f: D \to E$  and let  $T_g$  be a Turing machine which implements another function  $g: C \to D$ . Prove that there exists a Turing machine  $T_{f \circ g}$  which implements  $f \circ g$ .
- 8. Prove that every Turing-computable function may be transformed into a language. Further show that there exists a Turing machine that accepts the language.