CSL705: Theory of Computation

Tutorial: Nondeterministic Push-down Automata

1. Construct NPDAs to recognise the following languages.
   (a) $L_1 = \{a^m b^n \ \mid \ m \neq n\}$
   (b) $L_2 = \{a^m b^n \ \mid \ m < n\}$
   (c) $L_3 = \{a^m b^n \ \mid \ m \geq n\}$
   (d) $L_4 = \{a^m b^n \ \mid \ n = 2m\}$
   (e) $L_5 = \{a^m b^n \ \mid \ n \neq 2m\}$
   (f) $L_6 = \{a^m b^n c^p \ \mid \ p = m + n\}$
2. Prove that for every NPDA $N = \langle Q, \Sigma, \Gamma, \Delta, q_0, \perp, F\rangle$ such that $\Sigma \cap \Gamma \neq \emptyset$ there exists an equivalent NPDA $N' = \langle Q, \Sigma, \Gamma', \Delta', q_0, \perp, F\rangle$ such that $\Sigma \cap \Gamma' = \emptyset$
3. Define a restricted NPDA, $N_r = \langle Q, \Sigma, \Gamma, \Delta_r, q_0, \perp, F\rangle$ such that with each transition, the stack changes by at most one symbol, i.e. $\Delta_r \subseteq_f (Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma) \times (Q \times (\Gamma.\Gamma \cup \Gamma \cup \{\epsilon\}))$ and for each $((q, a, A), (q', \alpha))$, $\alpha \in (\Gamma.\{A\} \cup \Gamma \cup \{\epsilon\})$ Prove that for every NPDA N there exists an equivalent restricted NPDA N_r (i.e. $\mathcal{L}(N) = \mathcal{L}(N_r)$).
4. Let L_1 and L_2 be context-free languages over a nonempty finite alphabet Σ accepted by NPDAs N_1 and N_2 respectively. Construct the NPDAs $N_{1\cup 2}$, $N_{1.2}$ and N_{1^*} which accept the union, concatenation and *-closure of the languages respectively.
5. Let $h : \Sigma_1 \longrightarrow \Sigma_2^*$ be a homomorphism and let $N_1 = \langle Q_1, \Sigma_1, \Gamma_1, \Delta_1, q_{1_0}, \perp, F_1\rangle$ be a NPDA. Define an NPDA $N_2 = \langle Q_2, \Sigma_2, \Gamma_2, \Delta_2, q_{2_0}, \perp, F_2\rangle$ such that $h(\mathcal{L}(N_1)) = \mathcal{L}(N_2)$