CSL705/CS355N: Theory of Computation

Tutorial: DFAs and Regular Languages

1. Construct DFAs for each of the following languages over the alphabet $\mathbf{2} = \{0, 1\}$.
   (a) $L_0 = \{1x \mid x \in \mathbf{2}^*, 5|(1x)_2\}$.
   (b) $L_1 = \{x00 \mid x \in \mathbf{2}^*\}$.
   (c) $L_2 = \{x000y \mid x, y \in \mathbf{2}^*\}$.
   (d) L_3 is the set of all strings such that every substring of five consecutive symbols contains at least two 0s.
   (e) $L_4 = \{x0y \mid x, y \in \mathbf{2}^*, |y| = 4\}$.
2. Prove that any finite language over a finite alphabet is regular.
3. Let $L, M \subseteq \Sigma^*$ be languages on a nonempty finite alphabet Σ. For any $x, y \in \Sigma^*$, the shuffle of x and y is defined by induction as follows:

$$\begin{array}{lcl} \varepsilon \between y & = & \{y\} \\ x \between \varepsilon & = & \{x\} \\ ax' \between by' & = & a.(x' \between y) \cup b.(x \between y') \quad \text{if } x = ax' \text{ and } y = by' \end{array}$$

   It is extended to languages L and M as follows:

$$L \between M = \bigcup_{x \in L, y \in M} x \between y$$

   Prove that $L \between M$ is a regular language if L and M are regular.
4. For any language L over a finite alphabet Σ, let $\mathsf{Pref}(L)$ denote the set of all prefixes of L and let $\mathsf{Suff}(L)$ denote the set of all suffixes of L.
   (a) Prove that if L is regular then $\mathsf{Pref}(L)$ is also regular.
   (b) Is the converse statement true? If it is true prove it otherwise give an example of a language L such that $\mathsf{Pref}(L)$ is regular but L is not.
   (c) If L is regular, is $\mathsf{Suff}(L)$ necessarily regular? Prove or disprove your answer.
5. (a) Construct a DFA over the alphabet $\mathbf{2} = \{0, 1\}$ which accepts the language $L = \{x \in \mathbf{2}^* \mid \#_0(x) = \#_1(x), \forall y \preceq x[0 \leq |\#_0(y) - \#_1(y)| \leq 1]\}$.
   (b) Prove that your DFA accepts exactly the language L.