COL870: Clustering Algorithms

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Streaming Clustering

Streaming Clustering

- Here are some of the results known for the streaming k-means/median.

Algorithm	Space requirement	Approximation
[GNMO00]	$O(n^c)$	$O(2^{1/c})$
[MCP03]	$O(k \cdot polylog(n))$	$O(1)$
[C09]	$O\left(\frac{dk}{\varepsilon}(\log n)^8\right)$	$(1+\varepsilon)$

- We already studied the first one. We shall skip the second result (project topic). The third result is through a concept known as coreset which we will not discuss (project topic).
- We will look at the notion of coresets and see how to use them to construct streaming algorithms.

- Before we study the concept of coresets, let us see whether it is possible to get a $(1+\varepsilon)$-approximation for the k-means problem.
- There are algorithms that run in time $O(nd \cdot 2^{\tilde{O}(k/\varepsilon)})$ and give a $(1+\varepsilon)$-approximation guarantee for the k-means problem.

Coreset

A corset is a subset of input such that we can get a good approximation to the original input by solving the optimisation problem directly on the coreset.

- (k, ε)-coreset for k-means/median: For a weighted point set $P \subset \mathbb{R}^d$, a weighted set $S \subset \mathbb{R}^d$ is a (k, ε)-coreset of P for k-means/median clustering, if for any set C of k points in $\mathbb{R}^d$, we have:

$$(1 - \varepsilon) \cdot \Phi_C(P) \leq \Phi_C(S) \leq (1 + \varepsilon) \cdot \Phi_C(P).$$

Streaming Clustering using Coresets

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$$(1 - \varepsilon) \cdot \Phi_C(P) \leq \Phi_C(S) \leq (1 + \varepsilon) \cdot \Phi_C(P).$$

- There is an algorithm that outputs a (k, ε) coreset of size $O(k^2 \varepsilon^{-2} (\log n)^2)$ in a general metric space and a (k, ε)-coreset of size $O(dk^2 \varepsilon^{-2} \log n \log (k/\varepsilon))$ in $\mathbb{R}^d$.

- Claim 1: If C_1 and C_2 are the (k, ε) for disjoint sets P_1 and P_2 respectively, then $C_1 \cup C_2$ is a (k, ε) coreset for $P_1 \cup P_2$.

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- How do we obtain a streaming algorithm for the k-means problem that uses only space that has logarithmic dependence on the stream size using the above claims?

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- Claim 3: There is an algorithm that outputs a (k, ε) coreset of size $O(k^2 \varepsilon^{-2} (\log n)^2)$ in a general metric space and a (k, ε)-coreset of size $O(dk^2 \varepsilon^{-2} \log n \log (k/\varepsilon)$.
- Claim 4: There are algorithms that run in time $O(nd \cdot 2^{\tilde{O}(k/\varepsilon)})$ and give a $(1 + \varepsilon)$-approximation guarantee for the k-means problem.

- Consider hypothetical buckets $P_0, P_1, ..., P_{\lceil \log n \rceil}$ such that $|P_i| = 2^i \cdot M$, where $M = (k/\varepsilon)^2$.
- As the data comes, we will try putting in the bucket P_0. In case this is full, we try to move the contents of P_0 to P_1 is possible and so on.
- We try to maintain (k, δ_j)-coreset for P_j at all times, where $1 + \delta_j = \prod_{l=0}^{j} (1 + \rho_l)$ and $\rho_j = \frac{\varepsilon}{c(j+1)^2}$.

Streaming Clustering using Coresets

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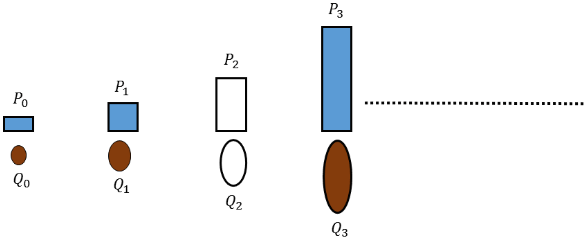

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- Claim 5: There is a streaming algorithm that outputs a $(1 + \varepsilon)$-approximate solution using space $O((dk/\varepsilon)^2 \cdot (\log n)^8)$ and amortised update time $O(dk \cdot polylog(ndk/\varepsilon))$.

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