CSL 356: Analysis and Design of Algorithms

Ragesh Jaiswal

CSE, IIT Delhi

Network Flow: Applications

Supply and Demand with capacity

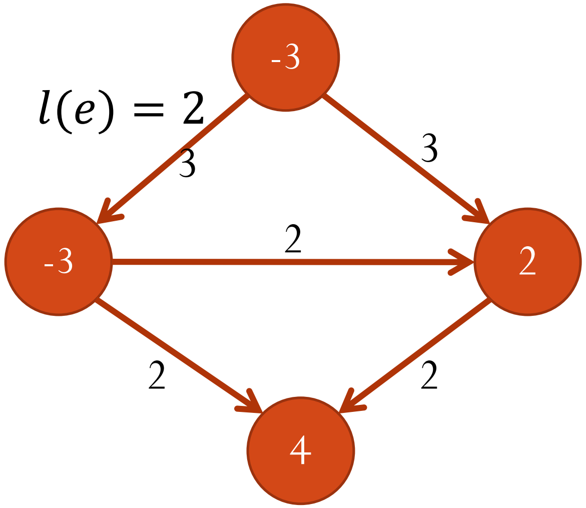
Network Flow: Applications

- Problem: Given a directed graph G with integer edge capacities $c(e)$ and a lower bound $l(e)$. For each node v, there is an associated demand value $t(v)$ denoting the demand at the node (for supply nodes this is $-s(v)$, for demand nodes $d(v)$, for other nodes 0). Find whether there exists a flow f such that for all nodes v:

$$f^{in}(v) - f^{out}(v) = t(v)$$

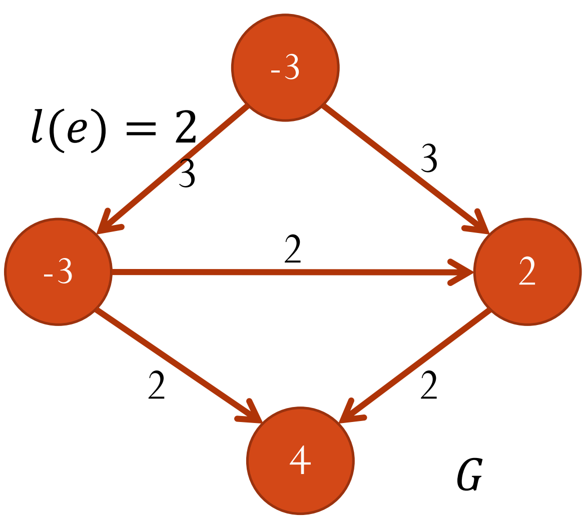
and the following capacity constraints are met. For each edge e:

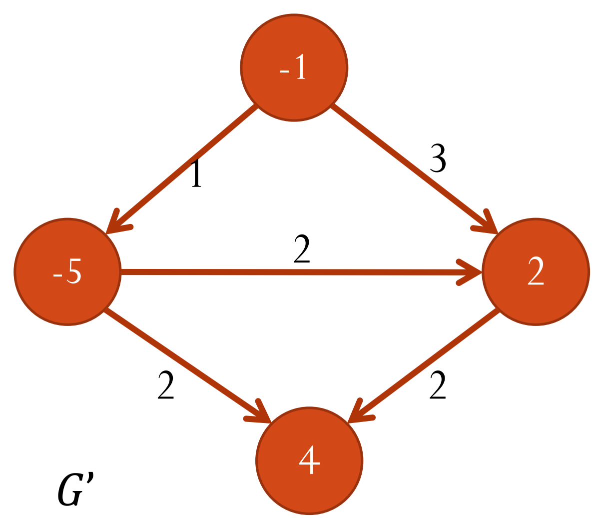
$$l(e) \leq f(e) \leq c(e)$$



Network Flow: Applications

- Consider a flow f such that for all edge e, $f(e) = l(e)$.
- For each vertex v, let $r(v) = f^{in}(v) - f^{out}(v)$.
- Construct a new graph G':
  - Each edge e has capacity $c(e) - l(e)$.
  - Each vertex v has demand $t(v) - r(v)$.
- Solve the feasible circulation problem without lower bounds on G'.

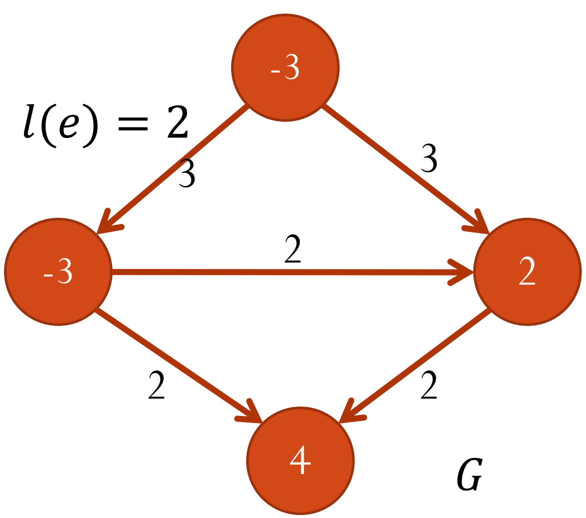


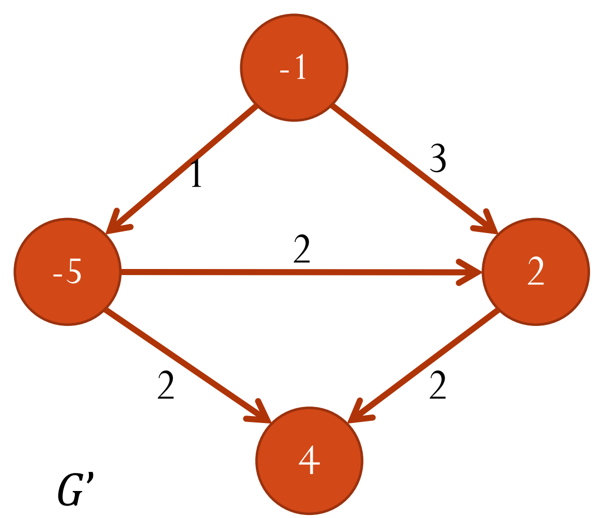


Network Flow: Applications

- <u>Claim 1</u>: There is a feasible circulation in G if and only is there is a feasible circulation in G'.
  - Proof:

    (➔) Let f be a feasible circulation in G. Consider $f'(e) = f(e) - l(e)$. Is f' a feasible circulation in G'?



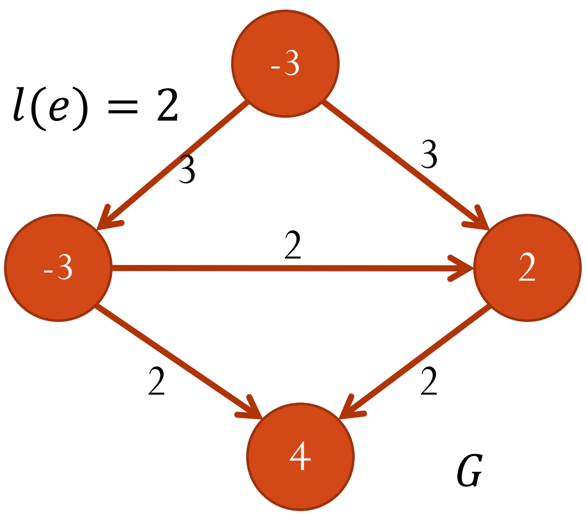


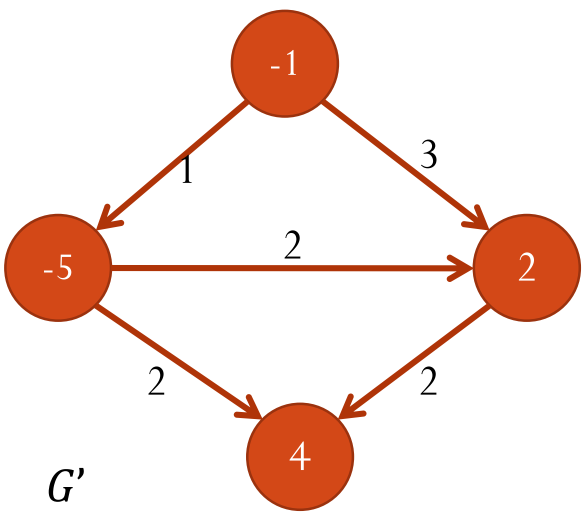
Network Flow: Applications

- <u>Claim 1</u>: There is a feasible circulation in G if and only is there is a feasible circulation in G'.
  - Proof:

    (➔) Let f be a feasible circulation in G. Consider $f'(e) = f(e) - l(e)$. Is f' a feasible circulation in G'?

    (🡐) Let f' be a feasible circulation in G'. Consider $f(e) = f'(e) + l(e)$. Is f a feasible circulation in G?



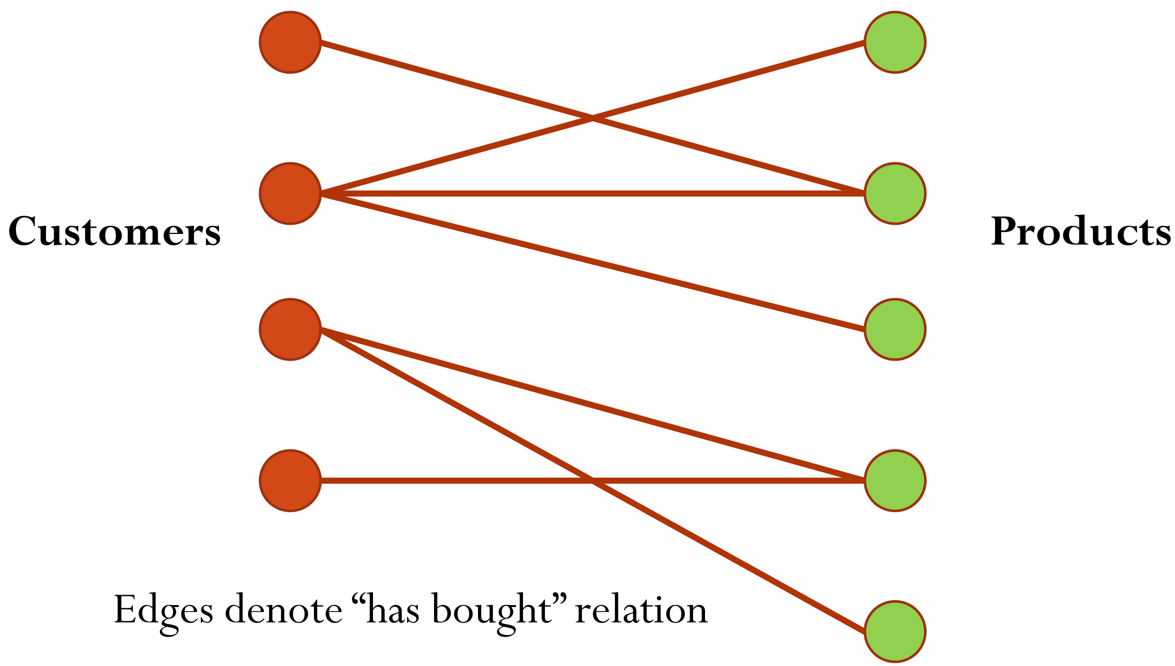


Network Flow: Applications

Survey Design

Network Flow: Applications

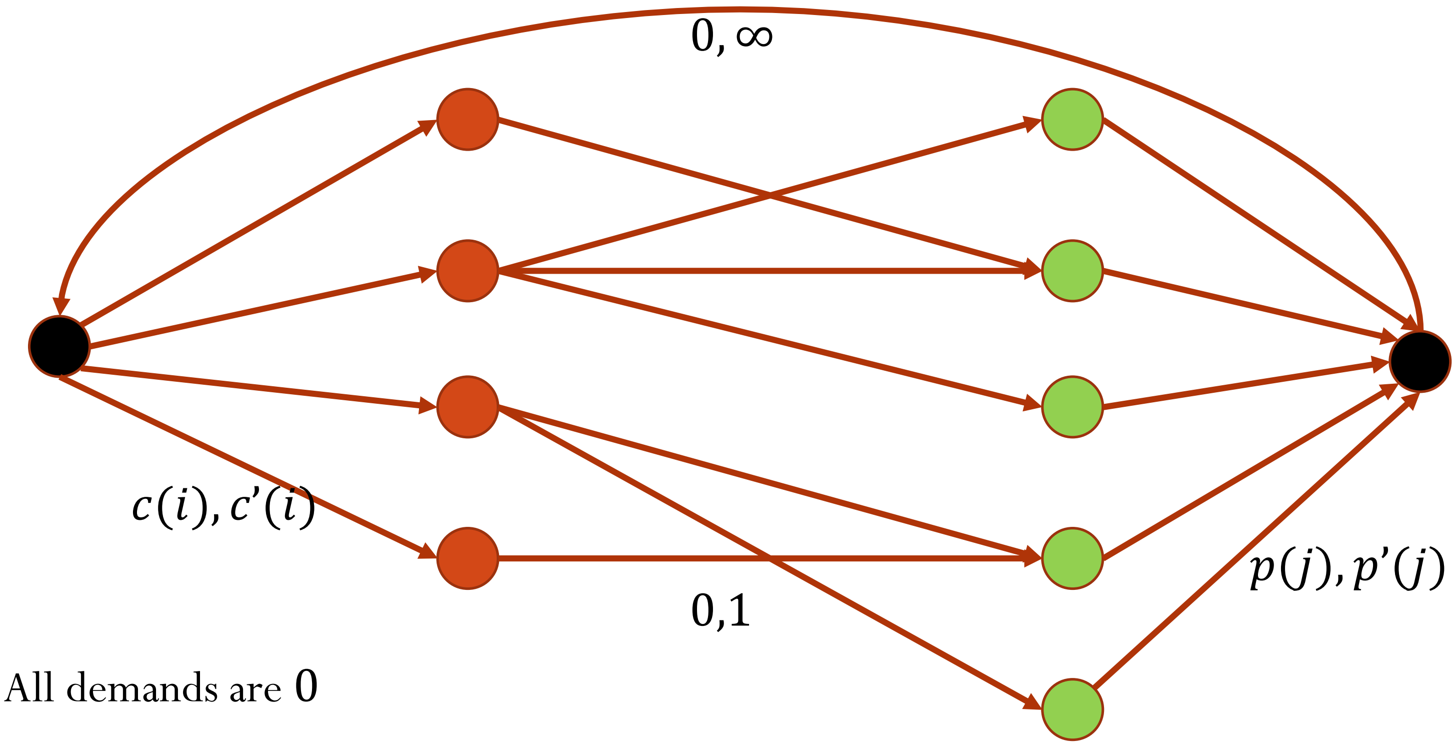
- Problem: There are n customers and m products. Each customer i is supposed to review between $c(i)$ and $c'(i)$ products that he has bought in the past and each product j should be reviewed by between $p(j)$ and $p'(j)$ customers. Find a way to do the survey.



Edges denote "has bought" relation

Network Flow: Applications

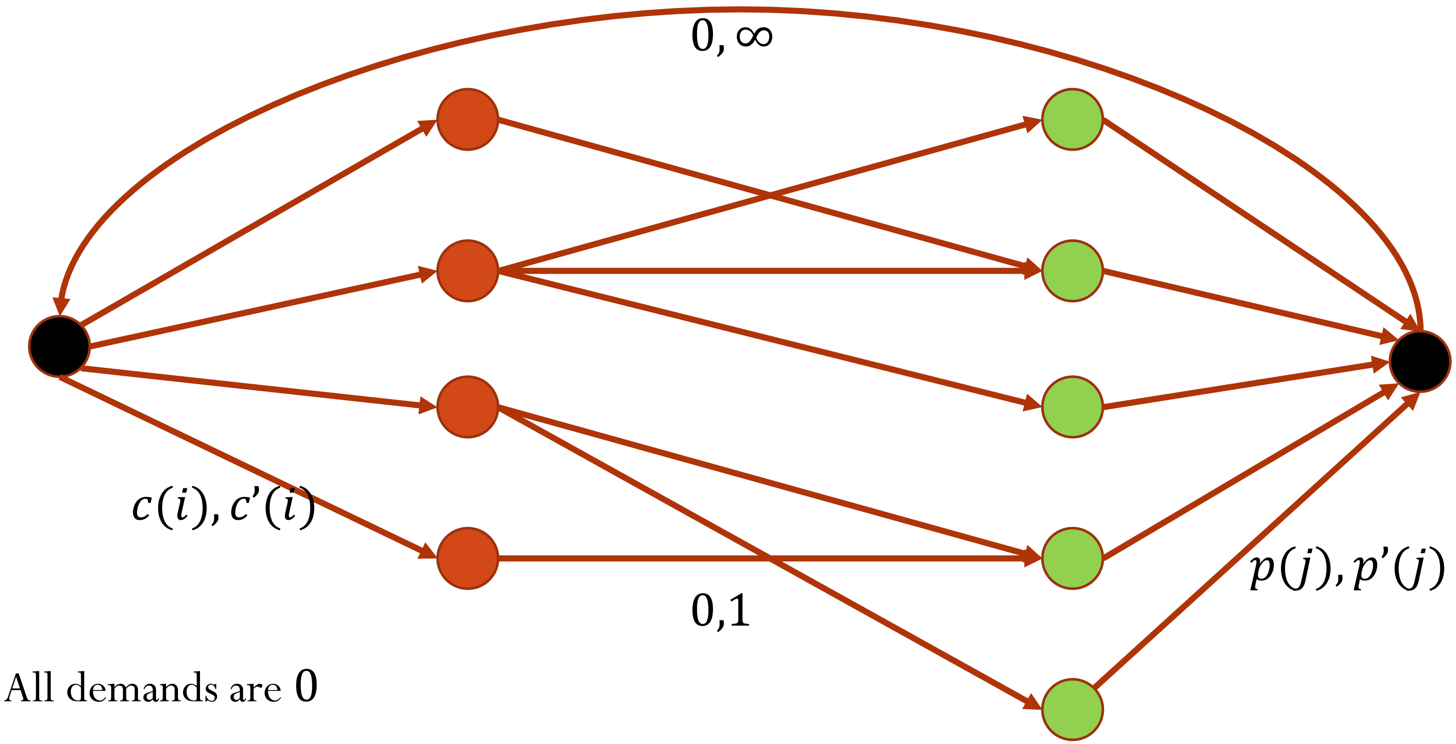
- Problem: There are n customers and m products. Each customer i is supposed to review between $c(i)$ and $c'(i)$ products that he has bought in the past and each product j should be reviewed by between $p(j)$ and $p'(j)$ customers. Find a way to do the survey.



All demands are 0

Network Flow: Applications

- <u>Claim</u>: The survey is feasible if and only if there is a feasible circulation in the network.



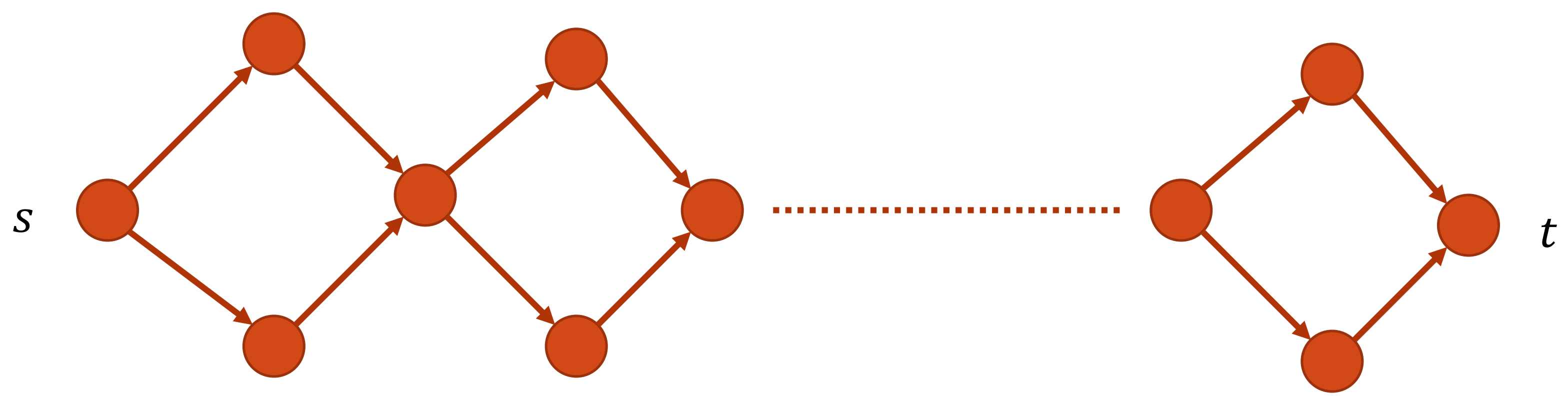
All demands are 0

Network Flow: Applications

Edge disjoint paths

Network Flow: Applications

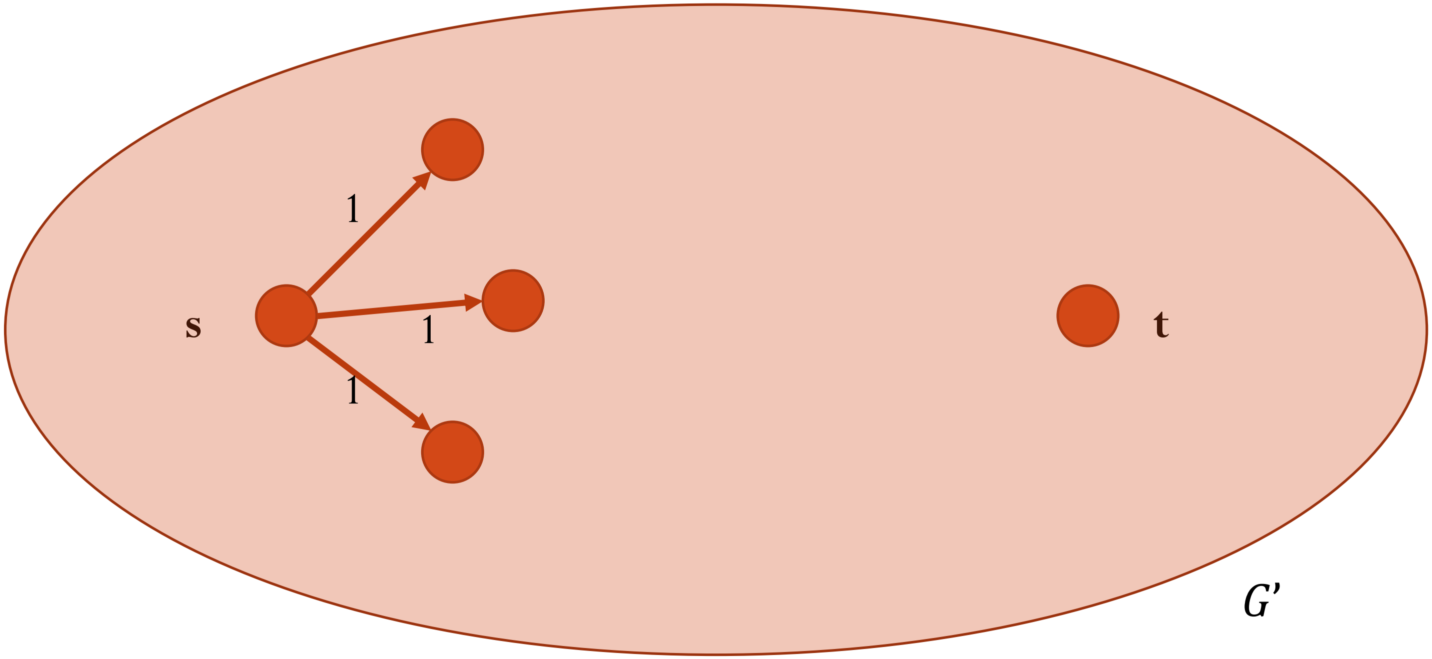
- Problem: Given an unweighted directed graph G find the maximum number of *edge disjoint paths* between s and t in G.
  - Edge disjoint paths: No two paths share an edge.



- How many different paths are present in the graph?
- How many edge disjoint paths present in the graph?

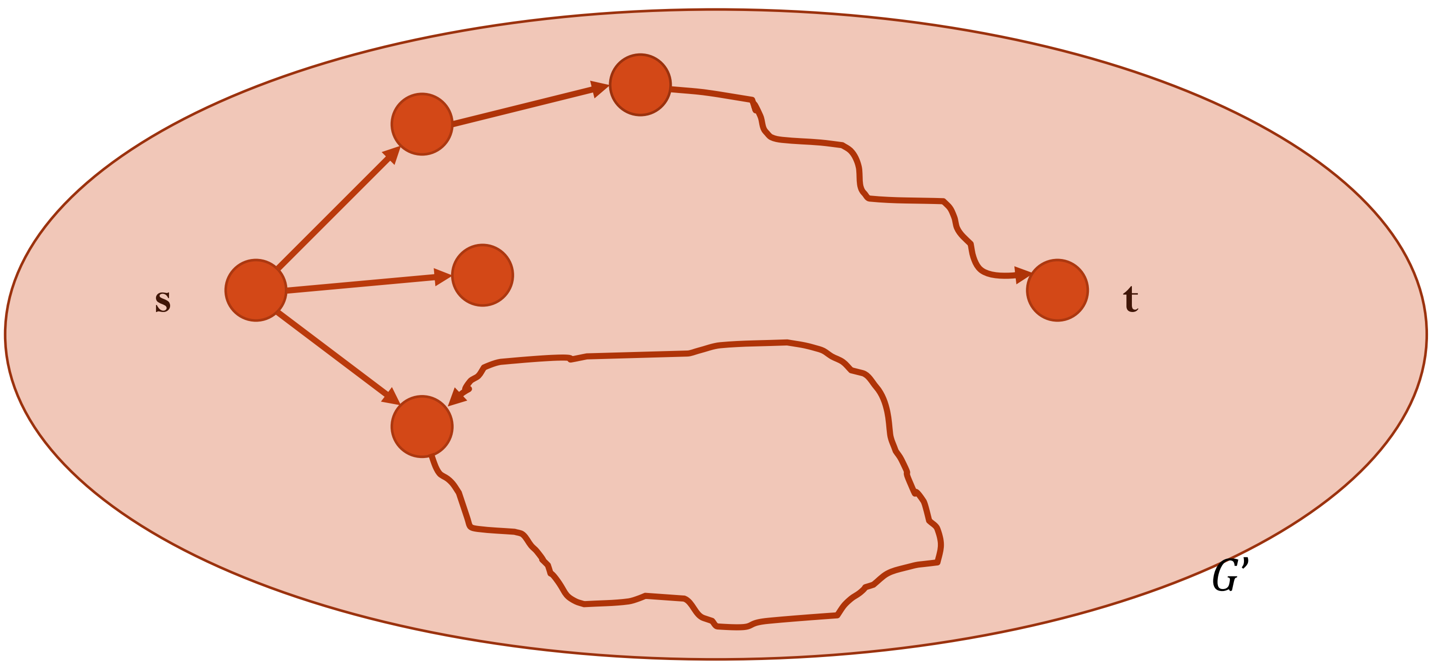
Network Flow: Applications

- Claim 1: If there are k edge disjoint paths in G then there is an $s - t$ flow in the graph with value at least k.



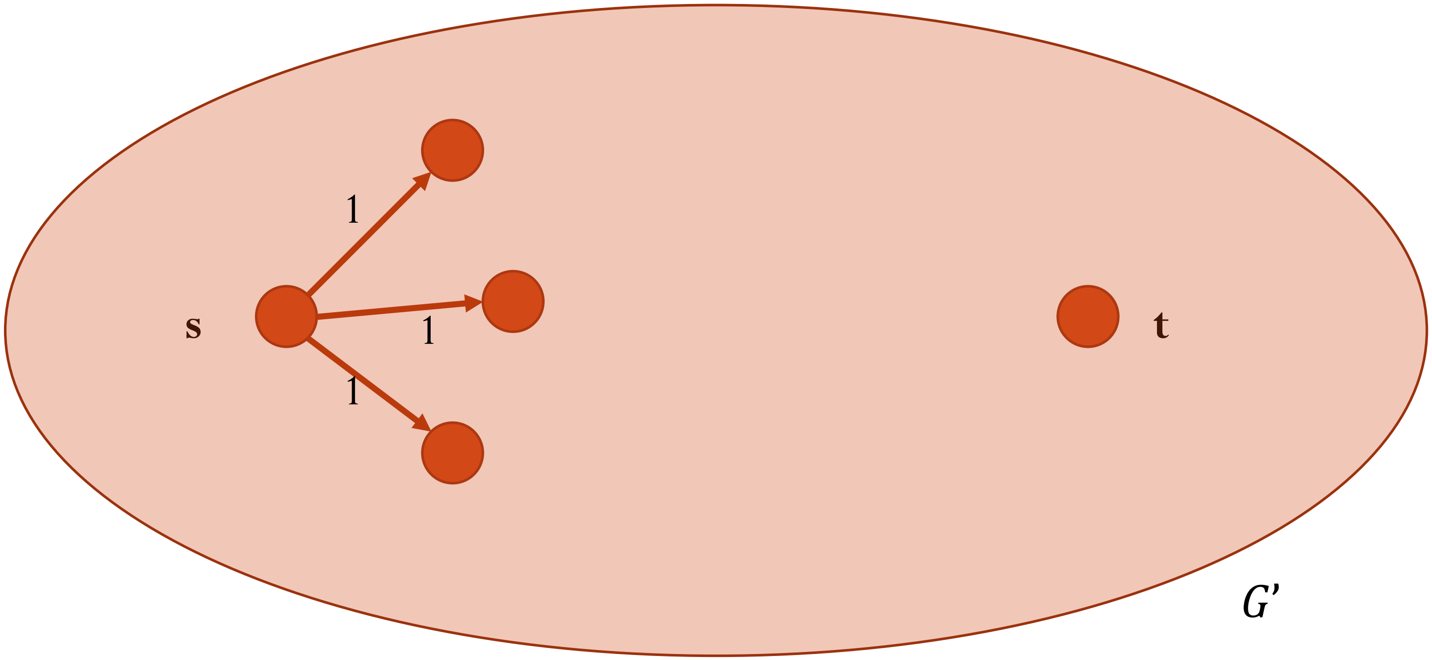
Network Flow: Applications

- Claim 2: If there is an $s - t$ flow in G' of value k, then there are at least k edge disjoint paths in G.
  - Proof idea: Induction on the number of edges with non-zero flow value.



Network Flow: Applications

- Theorem: There are k edge disjoint path in a graph G if and only if the maximum value of an $s-t$ flow in G' is at least k.



End