CSL356: Analysis and Design of Algorithms

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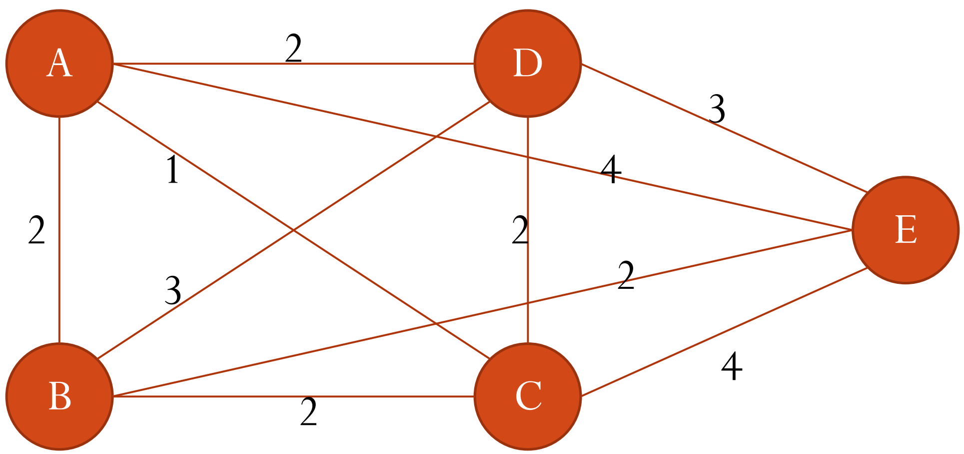
Dynamic Programming: Examples

- <u>Problem[travelling salesperson (TSP)]</u>: There are n cities and all the inter-city distances are given. Let $d(i,j)$ denote the distance between cities i and j. You are a salesperson and would like to visit each of the n cities starting and ending at your home-city. Give a tour that minimizes the total distance you have to travel.
- <u>Example</u>:



Dynamic Programming: Examples

- Suppose your home-city is city 1.
- How many different tours are possible?
  - $(n-1)!$
- Let S be a subset of cities containing city 1 and city j.
- Let $T(S,j)$ denote the shortest path between cities 1 and j such that all cities in S are visited once on this path.
- The first city in the path above is 1 and last city is j. Check all possibilities for second-to-last city.
- $T(S,j) = \min_{i \in S, i \neq j} (T(S - \{j\}, i) + d(i,j))$.

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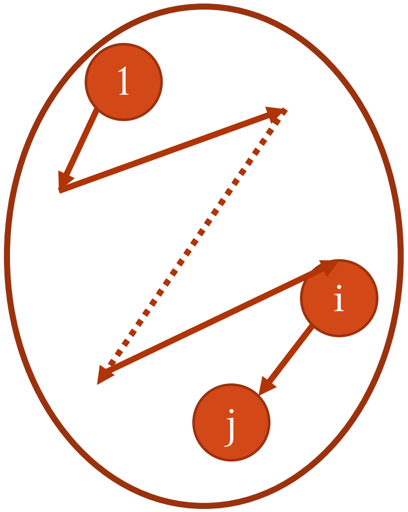

Dynamic Programming: Examples

TSP-Length

- $T(\{1\}, 1) = 0$
- for $s = 2$ to n
  - for all subsets S of $\{1, \dots, n\}$ of size s and containing 1
    - $T(S, 1) = \infty$
    - for all j in $S, j \neq 1$
      - $T(S, j) = \min\{T(S - \{j\}, i) + d(i, j), i \in S, i \neq j\}$
- return $min_j\ (T(\{1, \dots, n\}, j) + d(1, j))$

- Running time:
  - $O(n^2 \cdot 2^n)$.

Dynamic Programming: Examples

TSP-Length(S, i)

- if ($|S| = 1$) return(0)
- if ($|S| \neq 1$ and $i = 1$)return(∞)
- return$\left(\min_{j \in S, j \neq i} (TSP - Length(S - \{i\}, j))\right)$

- Running time:
  - $O((n-1)!)$.

Dynamic Programming: Examples

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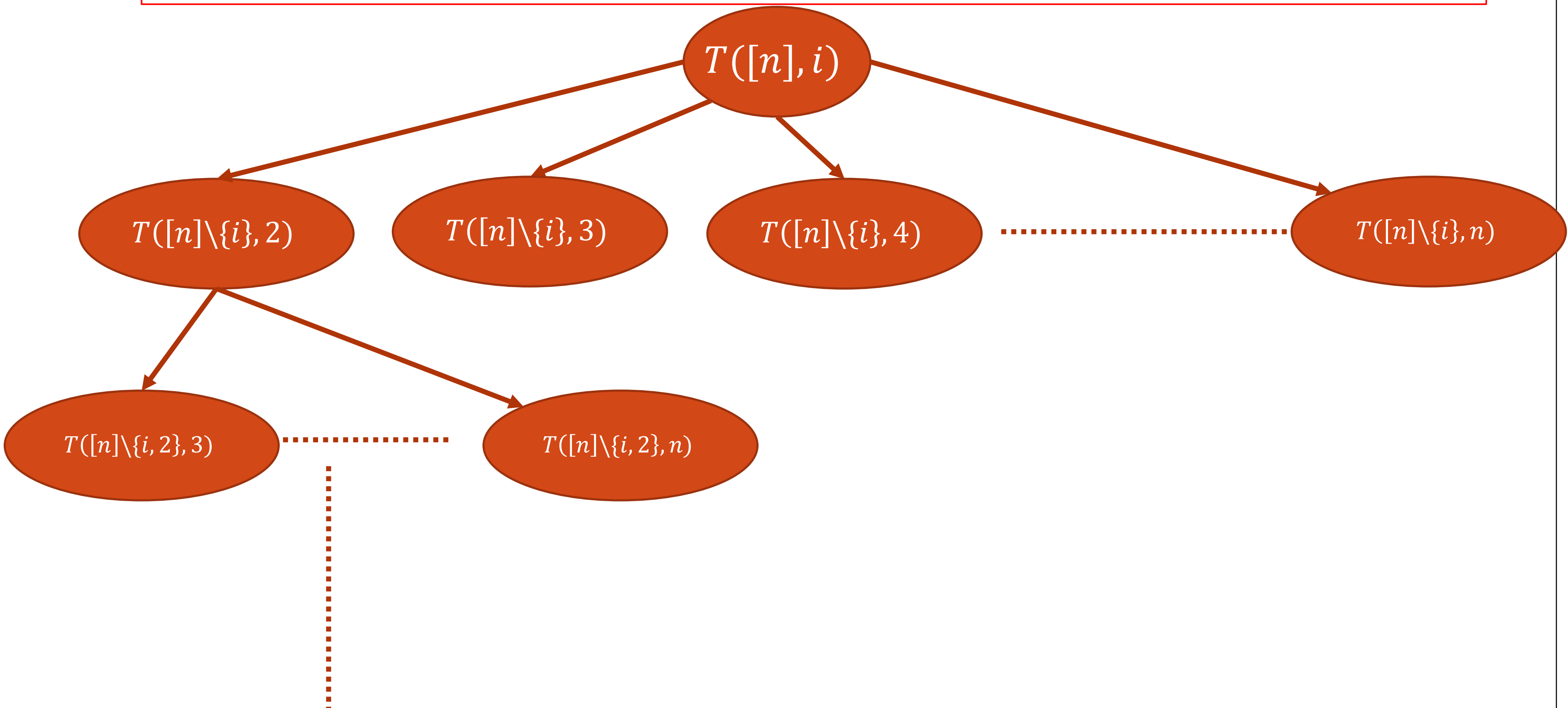
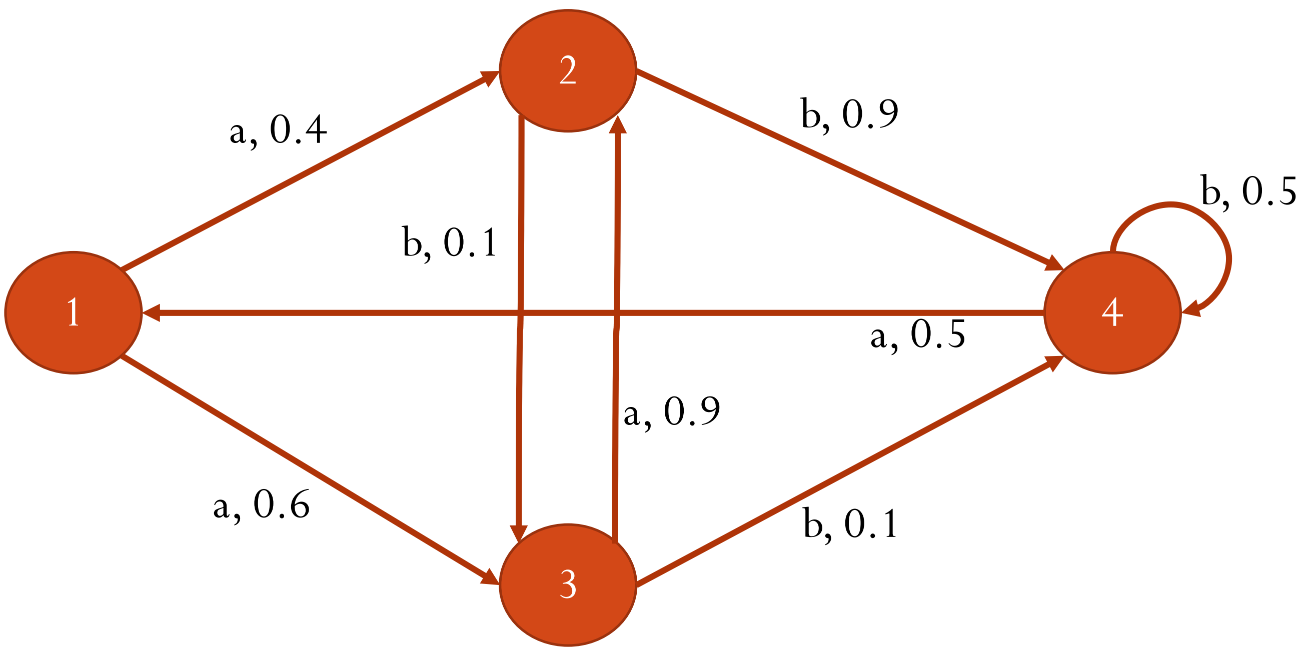


Dynamic Programming:
Some more examples

Viterbi's Algorithm

Dynamic Programming: Examples

- <u>Problem</u>: You are given a weighted, directed graph G. each directed edge is labeled with a symbol. Each edge (i, j) denotes the transition probability from i to j. So, sum of weights of out-going edges is 1. Given a string of n symbols, find the most probable path in G starting at a vertex v.



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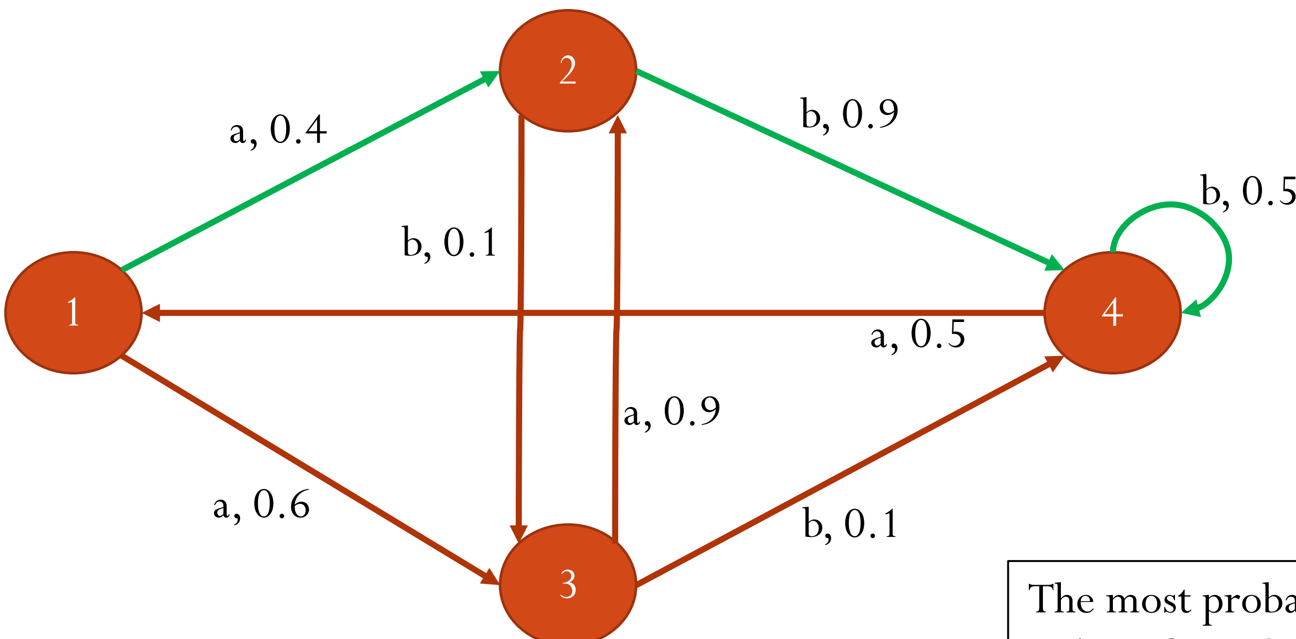

The most probable path generating "abbb" is $1 \to 2 \to 4 \to 4$

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- Let us consider the negation of log of edge weights instead of edge weights. Call this graph G'.
- Now we need to minimize the sum of edge weights in the path rather than maximize the product of edge weights.
- $M(i, w)$: The cost of the minimum weighted path with labels $s_i, \ldots, s_n$ starting at vertex w.

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- $M(i, w) = \min_{u \text{ s.t. label of } (w,u) \text{ is } s_i} \{M(i+1, u) + e(w, u)\}$
- Running Time:

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- Running Time:
  - $O(n \cdot |E|)$
- Space:
  - $O(|E|)$ in case we are only interested in the probability.

End