CSL356: Analysis and Design of Algorithms

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Divide and Conquer: Recurrence Relations

Master Theorem

Divide and Conquer: Recurrence Relation

- Master Theorem: Let

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + c \cdot n^k \quad \text{and} \quad T(1) = c$$

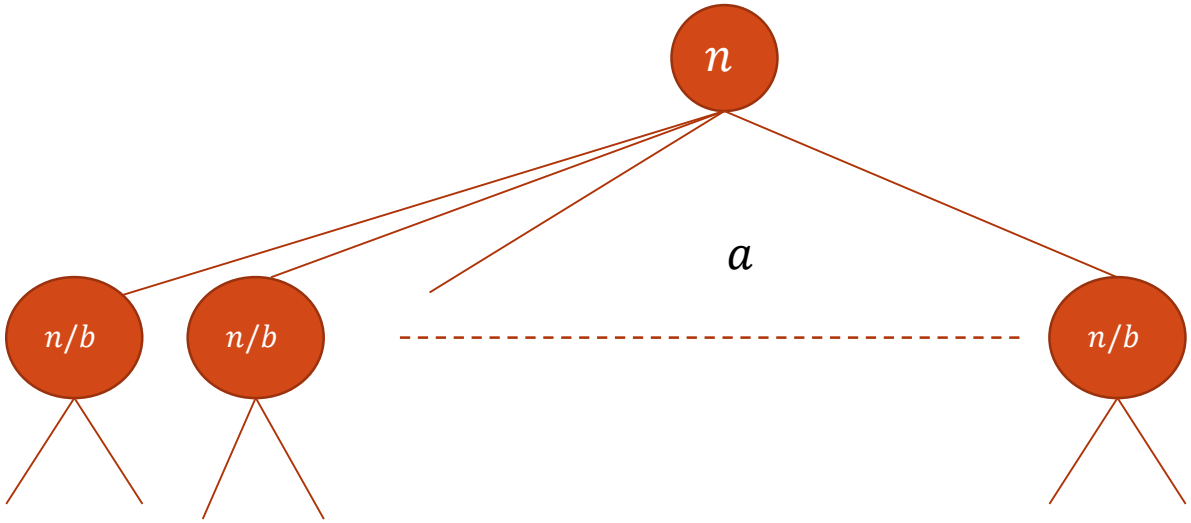
Then,

$$T(n) = \quad \text{if } a < b^k$$

$$= \quad \text{if } a = b^k$$

$$= \quad \text{if } a > b^k$$

Divide and Conquer: Recurrence Relation



$c \cdot n^k$

$c \cdot a \cdot (n/b)^k$

$c \cdot n^k \cdot (a/b^k)^2$

$c \cdot n^k \cdot (a/b^k)^i$

$$c \cdot n^k \cdot (1 + r + r^2 + \cdots + r^{\log(n)/\log(b)}), r = a/b^k$$

Divide and Conquer: Recurrence Relation

- Master Theorem: Let

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + c \cdot n^k \quad \text{and} \quad T(1) = c$$

Then,

$$\begin{aligned} T(n) &= O(n^k) && \text{if } a < b^k \\ &= O(n^k \cdot log_b(n)) && \text{if } a = b^k \\ &= O(n^{\log(a)/\log(b)}) && \text{if } a > b^k \end{aligned}$$

Problem Session

Problem Session

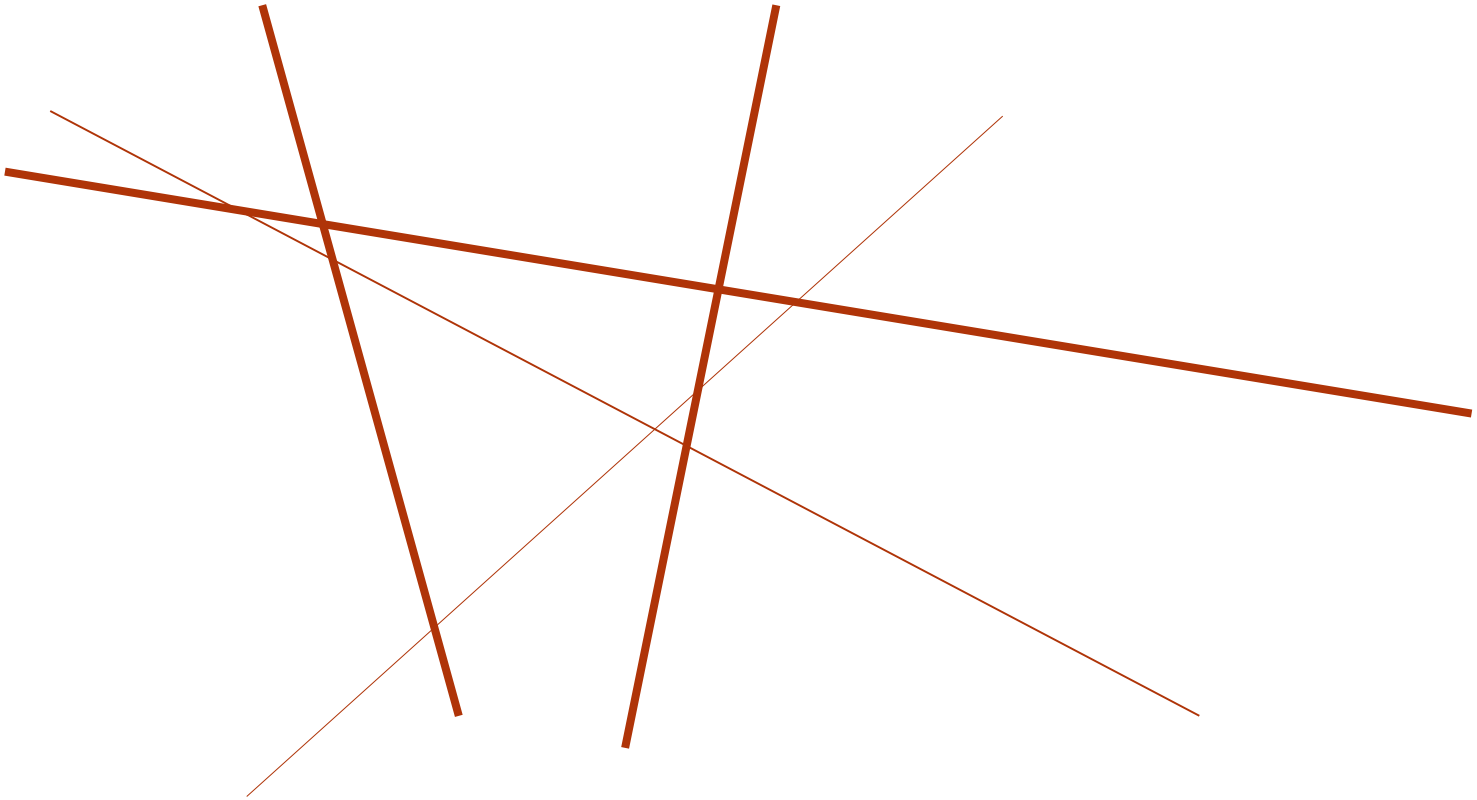
- Bookshelf Problem: There is a bookshelf with multiple shelves and n books that are to be places in a *fixed* order in the bookshelf. Design an algorithm to place the books such that the sum of *leftover space* in each of the shelves is minimized.

Problem Session

- Valid Pair Set: Given n integers $x_1, \ldots, x_n$ and an integer P, a set $S = \{(i, j): i < j \; and \; x_i + x_j \geq P\}$ is said to be a valid pair set if each index is present in at most one pair in S. Design an algorithm that outputs a valid pair set with maximum cardinality.

Problem Session

- Hidden Surface Removal: You are given n non-verticle lines on a plane. A line is said to be "visible" if there is some x-coordinate at which this line is the *uppermost* line. Give an algorithm that outputs all the "visible" lines.



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