CSL 356: Analysis and Design of Algorithms

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Greedy Algorithms: Example

Kruskal's Algorithm(G)
- $S = E; T = \{\}$
- While the edge set T does not connect all the vertices
  - Let e be the minimum weight edge in the set S
  - If e does not create a cycle in T
    - $T = T \cup \{e\}$
  - $S = S - \{e\}$

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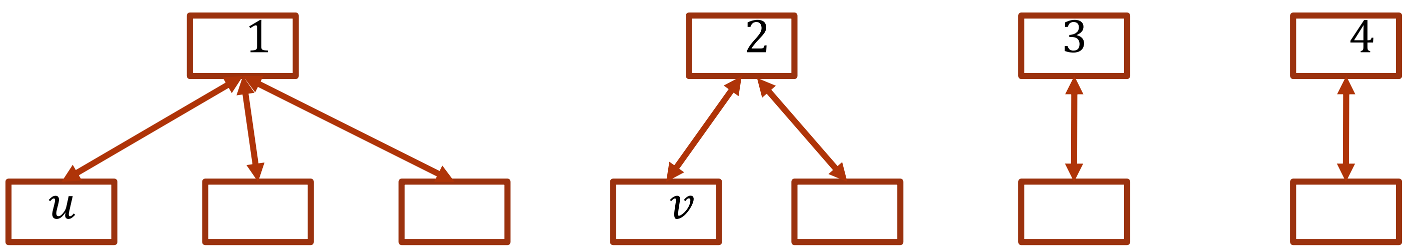
- $S = E; T = \{\}$
- While the edge set T does not connect all the vertices
  - *//Note that* $G' = (V, T)$ *contains disconnected components*
  - Let $e = (u, v)$ be the minimum weight edge in the set S
  - If u and v are in different components
    - $T = T \cup \{e\}$
  - $S = S - \{e\}$

Greedy Algorithms: Example

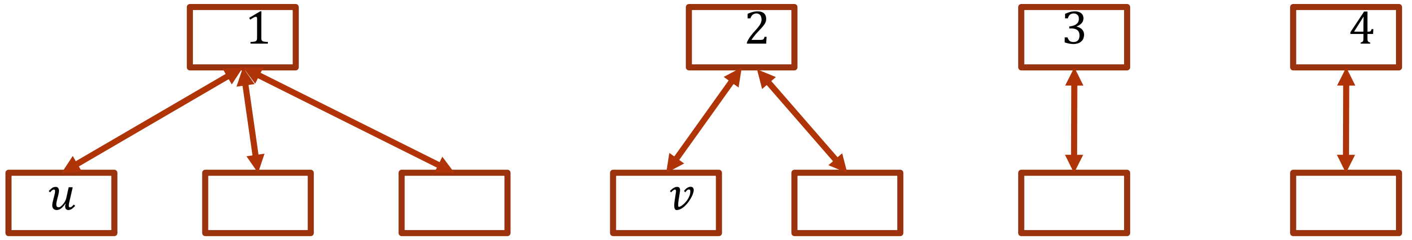
- Union-Find: Used for storing partition of a set of elements. The following two operations are supported.
  - **Find(v)**: Find the partition to which the element v belongs
  - **Union(u, v)**: Merge the partition to which u belongs with the partition to which v belongs.

- Consider the following data structure.



Greedy Algorithms: Example

- Suppose we start from a full partition (each partition contain one element). How much time do the following operations take:
  - **Find($\boldsymbol{v}$)**:
  - **Union($\boldsymbol{u}, \boldsymbol{v}$)**:



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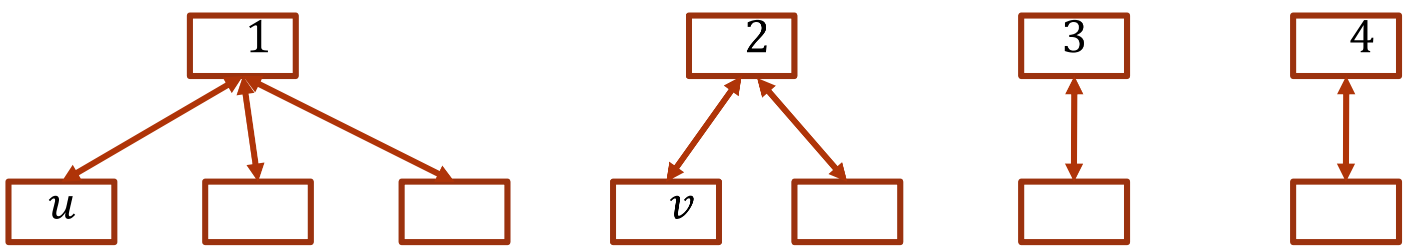

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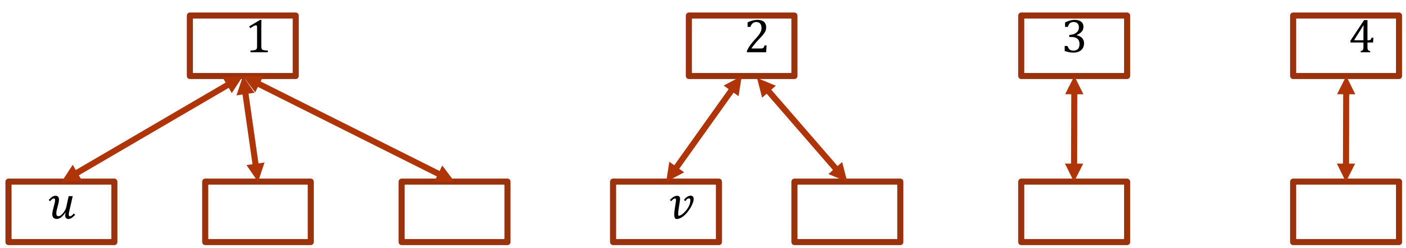
  - **Find($\boldsymbol{v}$)**: $O(1)$
  - **Union($\boldsymbol{u, v}$)**:
    - <u>Claim</u>: Performing k union operations takes $O(k \log k)$ time in the worst case.



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- What is the running time?
  - $O(|E| \cdot \log |V|)$

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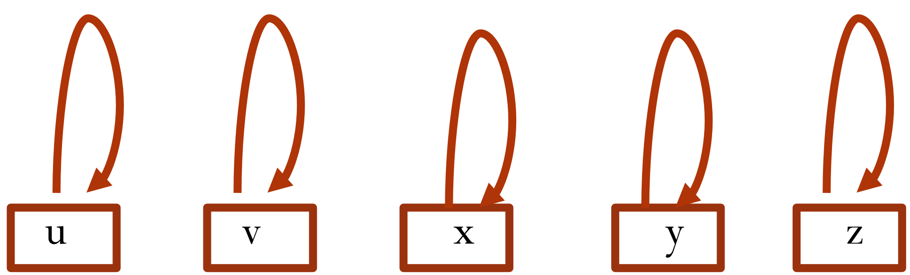

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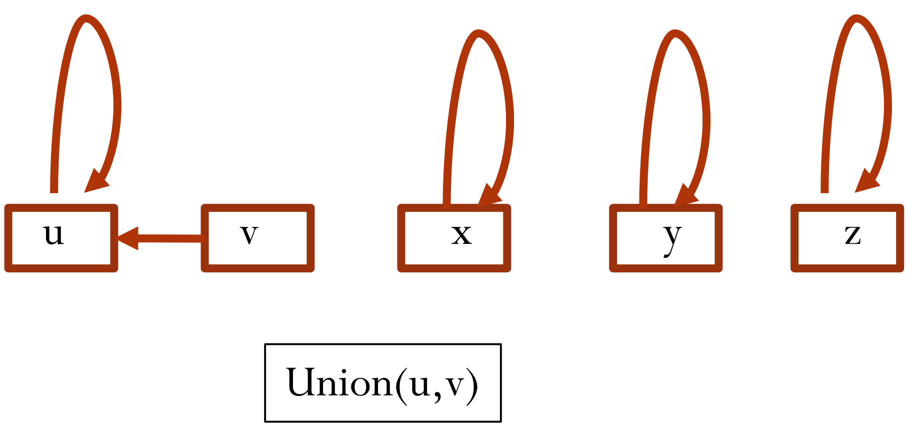

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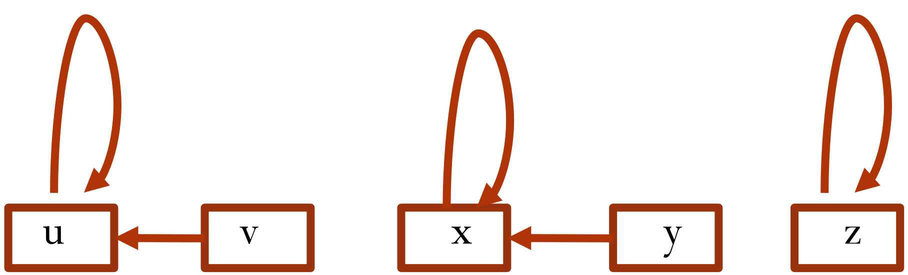

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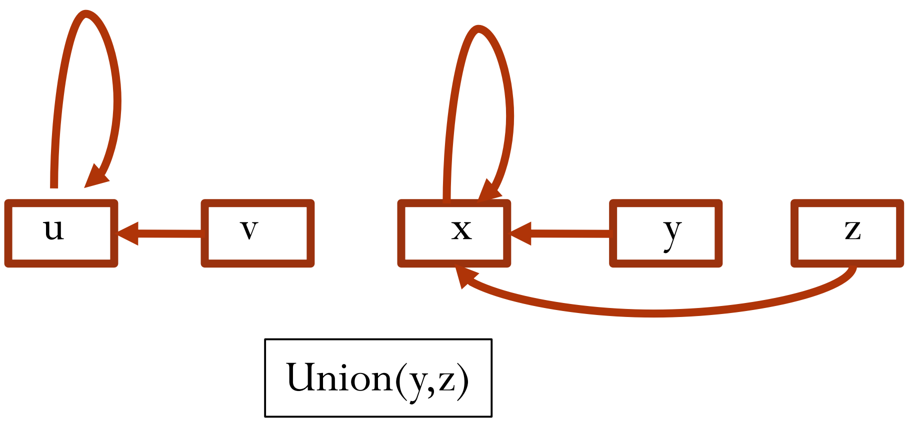

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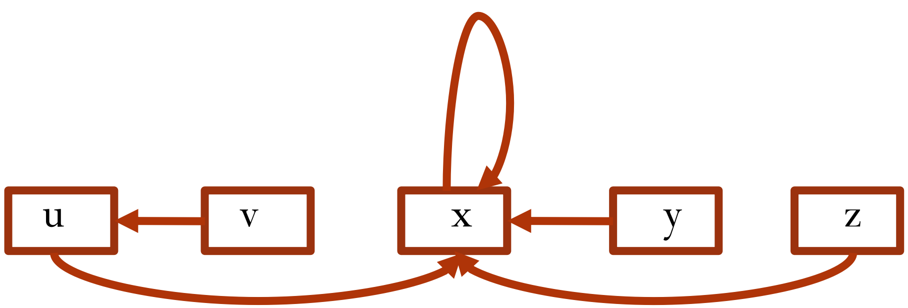
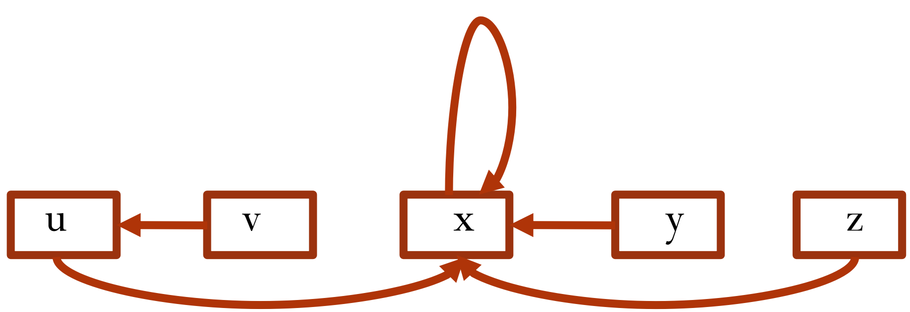


Union(v,y)

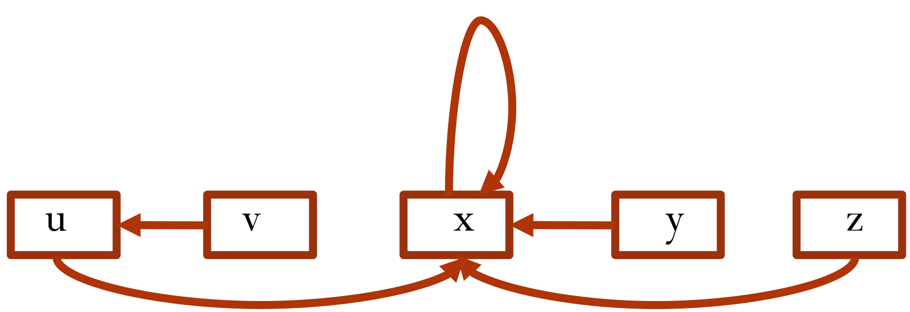
Digression: Union-Find

- Suppose we start from a full partition (each partition contain one element). How much time does the following take:
  - <u>Union</u>:
  - <u>Find</u>:



Digression: Union-Find

- Suppose we start from a full partition (each partition contain one element). How much time does the following take:
  - Union: whatever find takes + $O(1)$
  - Find:



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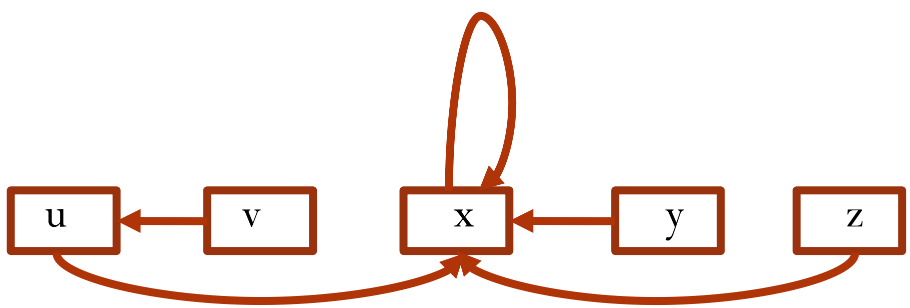
- Suppose we start from a full partition (each partition contain one element). How much time does the following take:
  - Union: whatever find takes + $O(1)$
  - Find: $O(\log n)$



End

Problems to think about:

1. Give an algorithm to find a minimum spanning tree of a weighted graph with minimum multiplicative cost. Multiplicative cost of a spanning tree is the product of edge weights.