- You have to discuss the running time of your algorithms. Always try to give algorithm with best possible running time.
- You are required to give proofs of correctness whenever needed.
- You may use any of the following known NP-complete problems to show that a given problem is NP-complete: 3-SAT, INDEPENDENT-SET, VERTEX-COVER, SUBSET-SUM, 3-COLORING, 3D-MATCHING, SET-COVER, CLIQUE, HAMILTONIAN-CYCLE, HAMILTONIAN-PATH.
- **Use of unfair means will be severely penalized.**

There are 3 questions for a total of 50 points.

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(15) 1. Consider the following problem:

> EVEN-VC: Given a graph G such that all vertices have even degree and given an integer k, determine if there exists a vertex cover of G of size at most k.

Show that EVEN-VC is NP-complete.

(15) 2. Consider the following problem:

> LONG-PATH: Given a weighted, directed graph $G = (V, E)$, two vertices $s, t \in V$ and a number W, determine if there is a *simple path* between s and t such that the sum of weights of edges in this path is $\geq W$.

Recall that a simple path is a path that does not have any vertices repeated. Show that LONG-PATH is NP-Complete.

(20) 3. Consider the following problem (a version of this problem was discussed in one of the tutorials):

> CUT: Given a rectangular piece of cloth sheet of dimension $h \times w$, n rectangular items with dimension $h_1 \times w_1, ..., h_n \times w_n$ and profits $p_1, ..., p_n$, and an integer P, determine whether it is possible to cut these rectangular items from the sheet such that the total profit is exactly P. You are allowed to cut at most one copy of any item from the sheet. You may assume that arbitrary cuts are possible.

Show that CUT is NP-complete.