- Always try to give algorithm with best possible running time. The points that you obtain will depend on the running time of your algorithm. For example, a student who gives an $O(n)$ algorithm will receive more points than a student who gives an $O(n^2)$ algorithm.
- You are required to give proofs of correctness whenever needed. For example, if you give a greedy algorithm for some problem, then you should also give a proof why this algorithm outputs optimal solution.
- **Use of unfair means will be severely penalized.**

There are 3 questions for a total of 35 points.

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(10) 1. A vertex cover of a given graph is a subset of vertices such that for any edge of the graph at least one of its endpoints is in this subset. Design an algorithm that outputs the vertex cover of minimum cardinality of a given bipartite graph $G = (L, R, E)$.

(5) 2. (a) An edge in a network flow graph is called *downwards critical* if decreasing the capacity of this edge decreases the maximum flow in the network. Give an efficient algorithm to find a downwards critical edge in a network.

(5) (b) An edge in a network flow graph is called *upwards critical* if increasing the capacity of this edge increases the maximum flow in the network. Give an efficient algorithm to find an upwards critical edge in a network in case there exists one.

(15) 3. You are given n pairs of integers $(d_1, d'_1), ..., (d_n, d'_n)$ such that $\forall i, d_i, d'_i \geq 0$. You have to check if there exists a directed graph $G = (\{1, ..., n\}, E)$ such that the in-degree of vertex i is d_i and the out-degree of vertex i is d'_i. Give an algorithm that performs this check. Your algorithm should also output a graph with the given degree sequence, in case there exists one. Discuss running time.