

- Always try to give algorithm with best possible running time. The points that you obtain will depend on the running time of your algorithm. For example, a student who gives an $O(n)$ algorithm will receive more points than a student who gives an $O(n^2)$ algorithm.
- You are required to give proofs of correctness whenever needed. For example, if you give a greedy algorithm for some problem, then you should also give a proof why this algorithm outputs optimal solution.
- **Use of unfair means will be severely penalized.**

There are 3 questions for a total of 40 points.

- (15) 1. If we split a given undirected graph G at all critical vertices, then we get a decomposition of the graph G_1, \dots, G_l such that $\forall i \neq j, G_i$ and G_j do not share any edges even though they may share a vertex (note that this is a critical vertex in G). The figure below shows an example. Design an algorithm that takes as input a graph (assume adjacency list representation) and outputs such a decomposition of the graph.

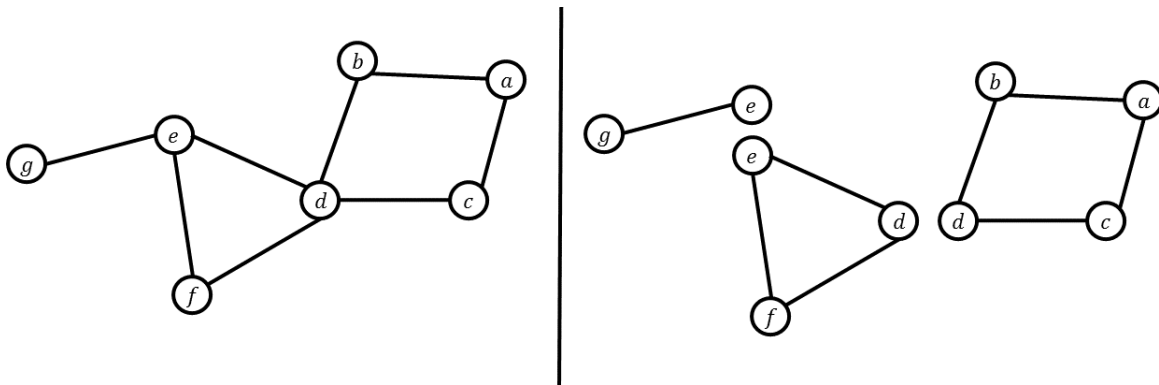


Figure 1: The figure in the left shows the original graph and the figure on the right shows the decomposition.

- (13) 2. Given a directed graph $G = (V, E)$ and a vertex u , design an algorithm that outputs all vertices $S \subseteq V$ such that for all $v \in S$, there is an even length path from u to v in G .
- (12) 3. There are n men with heights m_1, \dots, m_n and n women with heights w_1, \dots, w_n . You have to match men to women for a dance such that the average difference in height of each pair, is minimized. Design an algorithm to solve this problem.