CSL759: Cryptography and Computer Security

Ragesh Jaiswal

CSE, IIT Delhi

CPA Security

- Until now, we have seen encryption schemes that are secure in some limited sense:
  - One-time encryption
  - Ciphertext-only adversary.

- We would now like to transition to stronger notions of security for symmetric encryption schemes that allows multiple encryptions and where the adversary can obtain encryptions of its choice (CPA security).

- Pseudorandom function (PRF) and Pseudorandom Permutation (PRP) are Cryptographic primitives that help us to design such schemes that are "CPA-secure".

Pseudorandom Function (PRF)

Pseudorandom Function

- We consider functions of the form $F: \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$.
- These are called *keyed* functions since we have a *collection* of 2^k functions, one for each value of the key $K \in \{0,1\}^k$. This function is denoted by $F_K: \{0,1\}^n \rightarrow \{0,1\}^n$ and is defined as $F_K(x) = F(K, x)$.
- This collection of functions is also known as a *function family*.
- We will use such function families as a primitive in designing symmetric encryption schemes that are CPA-secure.
- Th useful security notion for this primitive is how similar this family is to the family of random functions from $\{0,1\}^n$ to $\{0,1\}^n$.

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- For this, we define the following two *Experiments* and then compare the bahavior of adversaries in these two experiments.

- $Real_{A,F}$
  - Randomly pick $K \leftarrow \{0,1\}^k$.
  - When A queries with an input $x \in \{0,1\}^n$, return $F_K(x)$.
  - Finally A outputs a bit b.
  - Output b.

- $Random_A$
  - Pick a random function f from $\{0,1\}^n$ to $\{0,1\}^n$.
  - When A queries with an input $x \in \{0,1\}^n$, return $f(x)$.
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- Why did we not have to define these "experiments" while discussing the security of PRGs?

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  - When A queries with an input $x \in \{0,1\}^n$, return a random value from $\{0,1\}^n$.
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The adversary is not allowed to repeat a query.

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- Th useful security notion for this primitive is how similar the family is to the family of random functions from $\{0,1\}^n$ to $\{0,1\}^n$.
- The PRF advantage of an adversary A is defined as follows: $Adv_{PRF}(A, F) = \left|\Pr\left[Real_{A,F} = 1\right] - \Pr[Random_A = 1]\right|$

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- A function $F: \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$ is called (t,q,ϵ)-secure PRF if for every adversary A that runs in time $\le t$ and asks $\le q$ queries, $Adv_{PRF}(A,F) \le \epsilon$.

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- We can define asymptotic security for *length-preserving functions*, $F: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$, where the length of the key, input, and output are the same.
  - Such a function is called a secure pseudorandom function (or just PRF) if for every adversary A that runs in polynomial time, and makes polynomial number of queries, there is a negligible function $negl$ such that $Adv_{PRF}(A, F) \leq negl(k)$.

CPA security for Encryption Schemes

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- Borrowing ideas from one-time, ciphertext-only attack scenario, we can try to use message-indistinguishability as our notion of security.
- What is the main issue with this idea?

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- What is the main issue with this idea?
  - In CPA, the adversary is allowed multiple encryptions of messages of its choice.
- How do we define security then?

CPA Security for Encryption Schemes

- Borrowing ideas from one-time, ciphertext-only attack scenario, we can try to use message-indistinguishability as our notion of security.
- A symmetric encryption scheme $SE = (E, D)$ is said to be IND-CPA insecure if an efficient adversary is able to figure out which world it is in.

- $Left_{SE,A}$
  - Randomly pick key $K \leftarrow \{0,1\}^k$.
  - When A queries message pair (M_0^i, M_1^i) return $E(M_0^i)$ to A.
  - Finally A outputs b.
  - Output b.

- $Right_{SE,A}$
  - Randomly pick key $K \leftarrow \{0,1\}^k$.
  - When A queries message pair (M_0^i, M_1^i) return $E_K(M_1^i)$ to A.
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- The IND-CPA advantage of an adversary A is defined as follows:

$$Adv_{ind-cpa}(A, SE) = \left|\Pr\left[Left_{SE,A} = 1\right] - \Pr\left[Right_{SE,A} = 1\right]\right|$$

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  - Randomly pick key $K \leftarrow \{0,1\}^k$.
  - When A queries message pair (M_0^i, M_1^i) return $E_K(M_1^i)$ to A.
  - Finally A outputs b.
  - Output b.

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- A symmetric encryption scheme $SE = (E, D)$ is said to be ind-cpa secure if for every adversary A that runs in polynomial time and makes polynomial number of queries, there exist a negligible function $negl$ such that $Adv_{ind-cpa}(A, SE) \leq negl(k)$.

CPA Security for Encryption Schemes

- IND-CPA allows adversaries to make multiple queries.
- How much advantage do adversaries who is allowed to ask $q > 1$ queries, have over adversaries who can only make 1 "left/right" query?

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- IND-CPA allows adversaries to make multiple queries.
- How much advantage do adversaries who is allowed to ask $q > 1$ queries, have over adversaries who can only make 1 query?

- $Left'_{SE,A}$
  - Randomly pick key $K \leftarrow \{0,1\}^k$.
  - When A queries challenge message pair (M_0, M_1) return $E_K(M_0)$ to A.
  - When A queries a message M^j, then return $E_K(M^j)$ to A
  - Finally A outputs b.
  - Output b.

- $Right'_{SE,A}$
  - Randomly pick key $K \leftarrow \{0,1\}^k$.
  - When A queries challenge message pair (M_0, M_1) return $E_K(M_1)$ to A.
  - When A queries a message M^j, then return $E_K(M^j)$ to A
  - Finally A outputs b.
  - Output b.

- The FTG-CPA advantage of an adversary A is defined as follows:
$$Adv_{ftg-cpa}(A, SE) = \left|\Pr\left[Left'_{SE,A} = 1\right] - \Pr\left[Right'_{SE,A} = 1\right]\right|$$
- A symmetric encryption scheme $SE = (E, D)$ is called (t, q, ϵ)-ftg-cpa secure if for every adversary A that runs in time $\leq t$ and asks $\leq q$ quesries, $Adv_{ftg-cpa}(A, SE) \leq \epsilon$.

CPA Security for Encryption Schemes

- IND-CPA allows adversaries to make multiple queries.
- How much advantage do adversaries who is allowed to ask $q > 1$ queries, have over adversaries who can only make 1 query?
- Theorem: If a symmetric encryption scheme $SE = (E, D)$ is (t, q, ϵ)-ftg-cpa secure, then SE is also $(t, q, \epsilon \cdot q)$-ind-cpa secure.
  - We prove the following: Let A be any ind-cpa adversary that runs in time t and makes q queries, then there exists an ftg-cpa adversary that runs in time t and makes q queries such that
    $$Adv_{ind-cpa}(A, SE) \leq q \cdot Adv_{ftg-cpa}(B, SE).$$

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$$Adv_{ind-cpa}(A, SE) \leq q \cdot Adv_{ftg-cpa}(B, SE).$$
- To prove this, we define hybrid experiments.

$Left_{SE,A}$

$G^0_{SE,A}$
- Randomly pick $K \leftarrow \{0,1\}^k$.
- For A's ith query (M_0^i, M_1^i),
  if $(i \leq q - 0)$, then
  return $E_K(M_0^i)$ to A
  else return $E_K(M_1^i)$ to A
- Finally A outputs b.
- Output b.

$G^1_{SE,A}$
- Randomly pick $K \leftarrow \{0,1\}^k$.
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  if $(i \leq q - 1)$, then
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- Finally A outputs b.
- Output b.

$Right_{SE,A}$

$G^q_{SE,A}$
- Randomly pick $K \leftarrow \{0,1\}^k$.
- For A's ith query (M_0^i, M_1^i),
  if $(i \leq q - q)$, then
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- Output b.

CPA Security for Encryption Schemes

- Theorem: Let A be any ind-cpa adversary that runs in time t and makes q queries, then there exists an ftg-cpa adversary that runs in time t and makes q queries such that

$$Adv_{ind-cpa}(A, SE) \leq q \cdot Adv_{ftg-cpa}(B, SE).$$

- $$Adv_{ind-cpa}(A, SE) = \left|\Pr\left[Left_{SE,A} = 1\right] - \Pr\left[Right_{SE,A} = 1\right]\right|$$
$$= \left|\Pr\left[G^0_{SE,A} = 1\right] - \Pr\left[G^q_{SE,A} = 1\right]\right|$$

- Let $P_0 = \Pr[G^0_{SE,A} = 1], P_1 = \Pr[G^1_{SE,A} = 1], \ldots, P_q = \Pr[G^q_{SE,A} = 1]$

$Left_{SE,A}$

$G^0_{SE,A}$

- Randomly pick $K \leftarrow \{0,1\}^k$.
- For A's ith query (M^i_0, M^i_1),
  if $(i \leq q - 0)$, then
  return $E_K(M^i_0)$ to A
  else return $E_K(M^i_1)$ to A
- Finally A outputs b.
- Output b.

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- Output b.

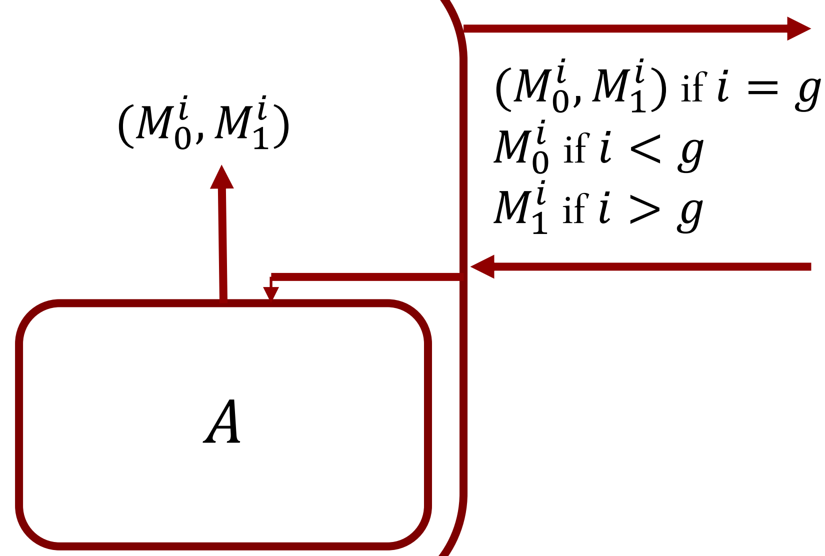
CPA Security for Encryption Schemes

- Theorem: Let A be any ind-cpa adversary that runs in time t and makes q queries, then there exists an ftg-cpa adversary that runs in time t and makes q queries such that
$$Adv_{ind-cpa}(A, SE) \leq q \cdot Adv_{ftg-cpa}(B, SE).$$

- $$\begin{aligned} Adv_{ind-cpa}(A, SE) &= \left|\Pr\left[Left_{SE,A} = 1\right] - \Pr\left[Right_{SE,A} = 1\right]\right| \\ &= \left|\Pr\left[G^0_{SE,A} = 1\right] - \Pr\left[G^q_{SE,A} = 1\right]\right| \end{aligned}$$
- Let $P_0 = \Pr[G^0_{SE,A} = 1], P_1 = \Pr[G^1_{SE,A} = 1], \ldots, P_q = \Pr[G^q_{SE,A} = 1]$

B

- Pick $g \leftarrow [q]$ randomly
- When A makes its i^{th} query (M_0^i, M_1^i):
  - If $(i < g)$ make a query with M_0^i and return the value to A
  - If $(i > g)$ make a query with M_1^i and return the value to A
  - If $(i = g)$ make a query $\left(M_0^i, M_1^i\right)$ and return the value to A
- Output A's result



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- $Adv_{ind-cpa}(A, SE) = \left|\Pr\left[Left_{SE,A} = 1\right] - \Pr\left[Right_{SE,A} = 1\right]\right|$
$$= \left|\Pr\left[G^0_{SE,A} = 1\right] - \Pr\left[G^q_{SE,A} = 1\right]\right|$$
- Let $P_0 = \Pr[G^0_{SE,A} = 1], P_1 = \Pr[G^1_{SE,A} = 1], \ldots, P_q = \Pr[G^q_{SE,A} = 1]$
- $\Pr[Left'_{SE,B} = 1] = ?$
- $\Pr\left[Right'_{SE,B} = 1\right] = ?$

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- Theorem: Let A be any ind-cpa adversary that runs in time t and makes q queries, then there exists an ftg-cpa adversary that runs in time t and makes q queries such that
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- Let $P_0 = \Pr[G^0_{SE,A} = 1], P_1 = \Pr[G^1_{SE,A} = 1], \ldots, P_q = \Pr[G^q_{SE,A} = 1]$
- $\Pr\left[Left'_{SE,B} = 1\right] = \frac{1}{q} \cdot (P_0 + P_1 + \cdots + P_{q-1})$
- $\Pr\left[Right'_{SE,B} = 1\right] = \frac{1}{q} \cdot (P_1 + P_2 + \ \ldots + P_q)$
- $$Adv_{ftg-cpa}(B, SE) = \left|\Pr\left[Left'_{SE,B} = 1\right] - \Pr\left[Right'_{SE,B} = 1\right]\right|$$
$$= \frac{1}{q} \cdot \left|\left((P_0 - P_q)\right)\right|$$
$$= \frac{1}{q} \cdot Adv_{ind-cpa}(A, SE)$$

CPA-Security for Encryption Schemes

- Alternate definition of FTG-CPA security.
- $GuessLR_{SE,A}$
  - Randomly pick a key $K \leftarrow \{0,1\}^n$.
  - Pick a random bit $b \leftarrow \{0,1\}$
  - When A makes a encryption query M^i, return the value $E_K(M^i)$.
  - When A makes the challenge query (M_0, M_1), return the value $E_K(M_b)$.
  - Finally, A outputs a bit b'
  - If $(b = b')$ output 1 else output 0
- Theorem: $\Pr\left[GuessLR_{SE,A} = 1\right] = \frac{1}{2} \pm \frac{1}{2} \cdot Adv_{ftg-cpa}(A, SE)$

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- So, summing up all the discussion until now, for CPA-security of an encryption scheme, we just need to analyse the performance of an adversary in the experiment $GuessLR_{SE,A}$.

CPA-Security for Encryption Schemes

- Suppose we have a *secure* pseudorandom permutation family $F: \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^n$.
  - We saw a few examples(AES,3DES etc.) in the last lecture.
- Consider the following encryption scheme $SE = (E, D)$ that encrypts messages of length n.
  - $E_K(M) = F_K(M)$ and $D_K(C) = F_K^{-1}(C)$

- Is SE ind-cpa secure?
- Is SE ftg-cpa secure?
- Is SE "GuessLR" secure?

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- Is SE "GuessLR" secure?
  - No
  - Adversary A
    - Query the message 0^n and get back $C = E_K(0^n)$.
    - Make the challenge query $(0^n, 1^n)$ and get back C'.
    - If $(C == C')$, then output 0 else output 1
  - $\Pr[GuessLR_{SE,A} = 1] =?$

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- Consider the following encryption scheme $SE = (E, D)$ that encrypts messages of length n.
  - $E_K(M) = F_K(M)$ and $D_K(C) = F_K^{-1}(C)$
- In fact, any deterministic encryption scheme cannot be IND-CPA secure!
  - For SE to be IND-CPA secure, everytime you encrypt a message M, you should get a different ciphertext!

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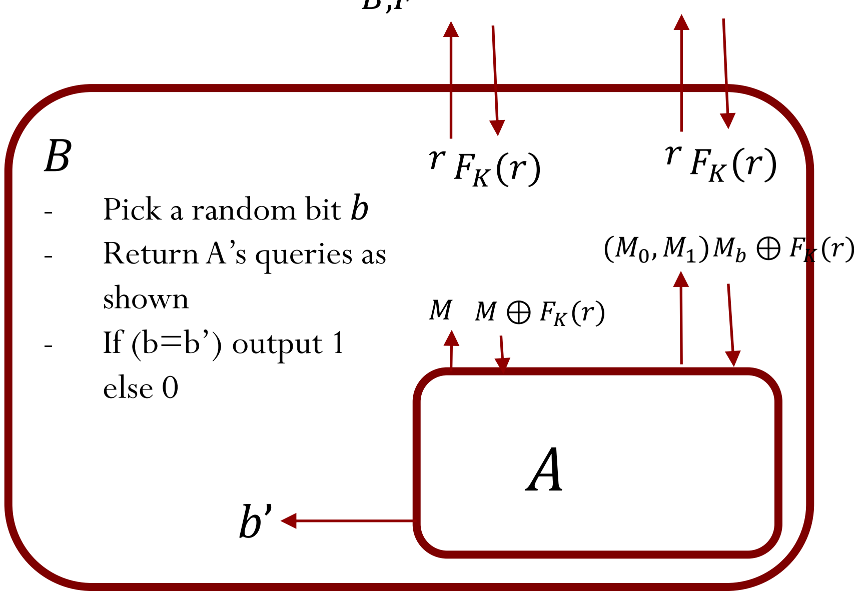
  - We saw a few examples(AES, 3DES etc.) in the last lecture.
- Consider the following encryption scheme $SE = (E, D)$ that encrypts messages of length n.
  - $E_K(M)$
    - Pick a random $r \leftarrow \{0,1\}^n$
    - Output $\mathrm{C} = < r, F_K(r) \oplus M >$
  - $D_K(C)$
    - Parse C as $< r, s >$
    - Output $M = F_K(r) \oplus s$
- <u>Theorem</u>: If F is $\left(2t, q, \frac{\epsilon}{2} - \frac{q}{2^n}\right)$-secure PRF, then SE is (t, q, ϵ)-ftg-cpa secure symmetric encryption scheme.

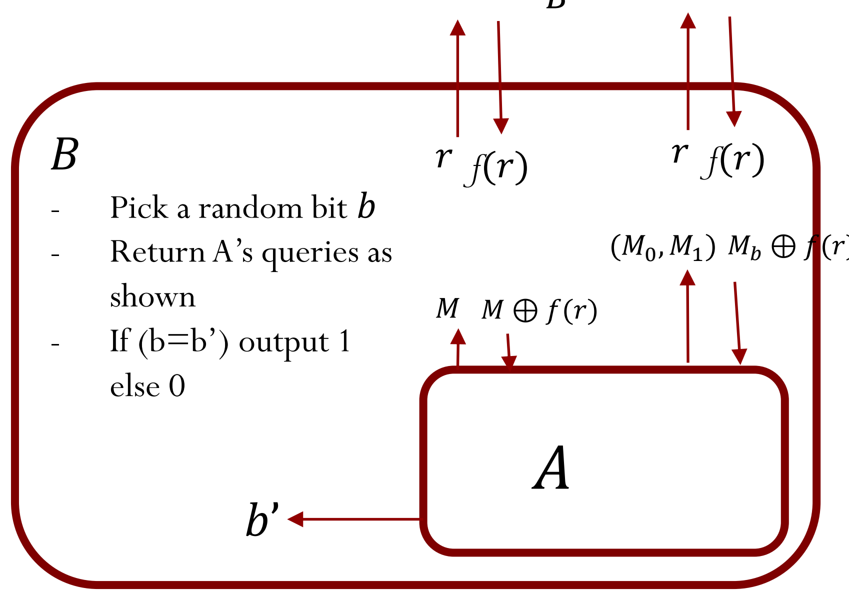
CPA-Security for Encryption Schemes

- Theorem: Consider an adversary A that runs in time t, makes q queries such that $\Pr\left[GuessLR_{SE,A} = 1\right] > \frac{1}{2} + \epsilon$, then there is an adversary B that runs in time at most $2t$, makes $(q+1)$ queries such that $Adv_{PRF}(B, F) > \epsilon - \frac{q}{2^n}$.

$Real_{B,F}$



$Random_B$

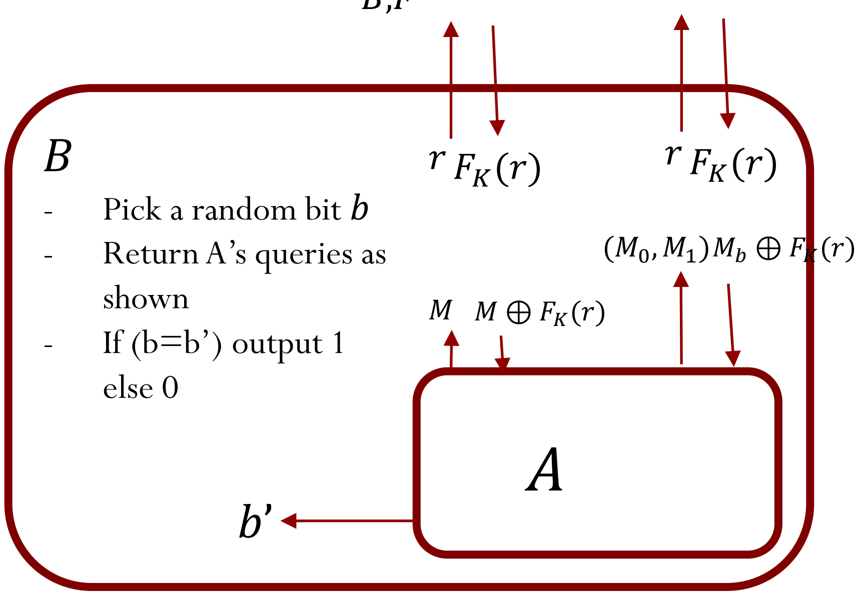


- $\Pr\left[Real_{B,F} = 1\right] = ?$

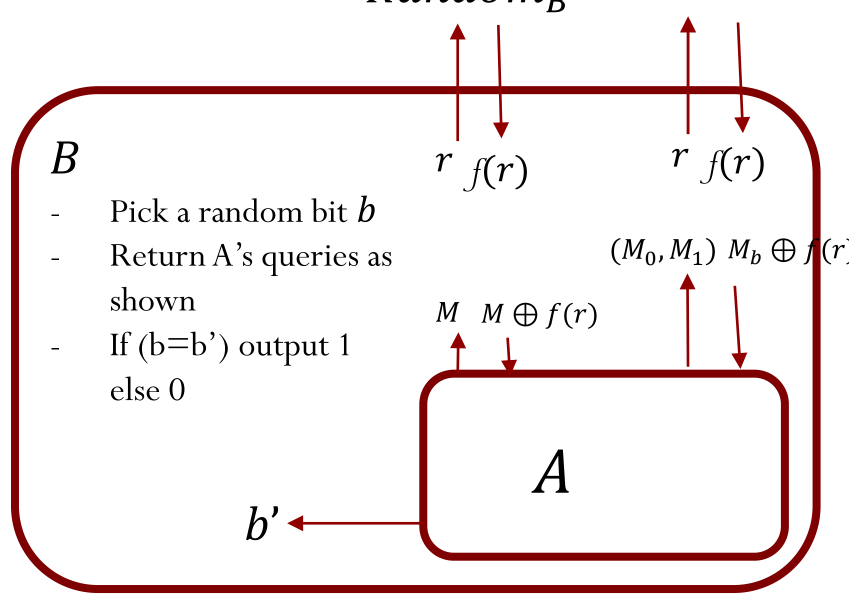
CPA-Security for Encryption Schemes

- Theorem: Consider an adversary A that runs in time t, makes q queries such that $\Pr[GuessLR_{SE,A} = 1] > \frac{1}{2} + \epsilon$, then there is an adversary B that runs in time at most $2t$, makes $(q+1)$ queries such that $Adv_{PRF}(B, F) > \epsilon - \frac{q}{2^n}$.

$Real_{B,F}$



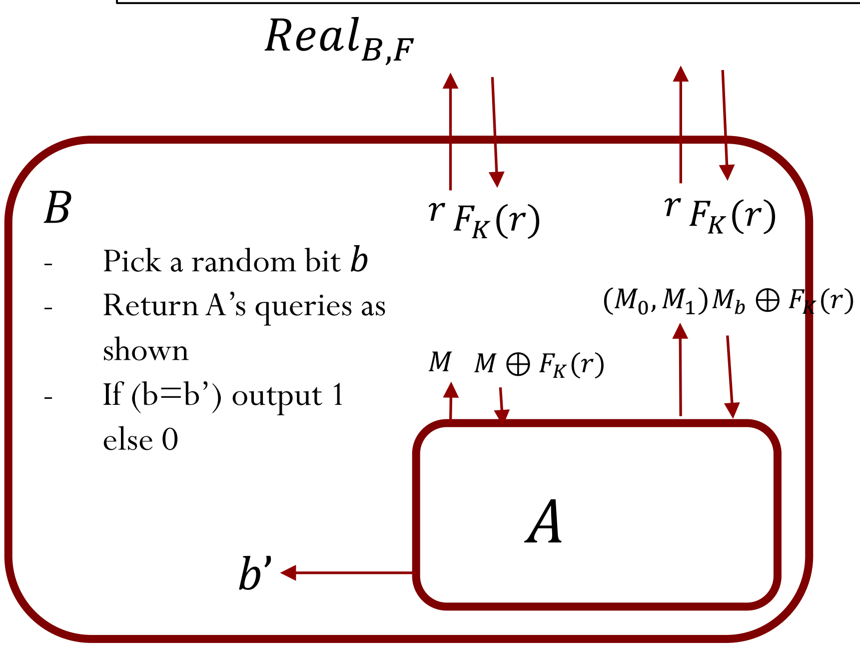
$Random_B$

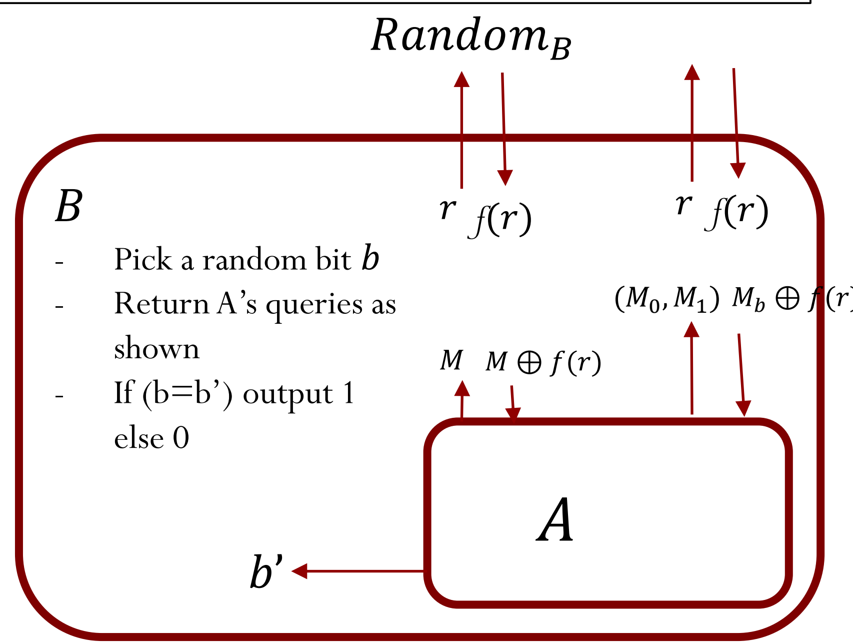


- $\Pr[Real_{B,F} = 1] = \Pr[\text{GuessLR}_{SE,A}]$
- $\Pr[Random_B = 1] \leq ?$

CPA-Security for Encryption Schemes

- Theorem: Consider an adversary A that runs in time t, makes q queries such that $\Pr[GuessLR_{SE,A} = 1] > \frac{1}{2} + \epsilon$, then there is an adversary B that runs in time at most $2t$, makes $(q+1)$ queries such that $Adv_{PRF}(B,F) > \epsilon - \frac{q}{2^n}$.





- $\Pr[Real_{B,F} = 1] = \Pr[\mathrm{GuessLR}_{SE,A}]$
- $\Pr[Random_B = 1] \leq \frac{1}{2} + \frac{q}{2^n}$

End